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Fachbereich Physik

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Einführung in die Theoretische Festkörperphysik

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Exercise 4

(Due date: 18.11.2019)

Problem 1 (**Phonons in a one-dimensional chain with a single-atom basis**) (4 points)

A one dimensional chain consists of atoms of only a single species. The atoms are coupled to their nearest neighbours with a spring constant α. The distance between the neighbouring atoms is a.

a) Give the potential energy of the system.
b) Calculate the dynamical matrix $D(\vec{r})$ of the system.
c) Calculate the dispersion relation $\omega(q)$ of the phonons of the system. How does $\omega(\vec{q})$ scale with $\vec{q}$ in the limit $q \to 0$?
d) Sketch the dispersion relation $\omega(q)$. Write the values $\omega(q=0)$ and $\omega(q=\pi/a)$.

Problem 2 (**One dimensional chain with a two-atom basis**) (4 points)

In the one dimensional chain from problem 5.1 we replace every second atom with an atom with mass $M_2 \neq M_1$. All atoms are coupled to their nearest neighbours with a spring constant α. The distance between the neighbouring atoms is a.

a) Give the potential energy of the system.
b) Calculate the dynamical matrix $D(\vec{r})$ of the system. What is the dimension of this matrix?
c) Calculate the dispersion relation $\omega(q)$ of the phonons of the system. How many phonon bands are there? How does $\omega(\vec{q})$ scale with $\vec{q}$ in the limit $q \to 0$ for each band?
d) Sketch the dispersion relation for the case $M_1 = M_2$. Give the values $\omega(q=0)$ and $\omega(q=\pi/2a)$ for each phonon band. Compare the results with exercise 5.1.
e) Sketch the dispersion relation for the case $M_1 = \frac{2}{3}M_2$. Give the values $\omega(q=0)$ and $\omega(q=\pi/2a)$ for each phonon band. What is the qualitative difference to the case $M_1 = M_2$?

Problem 3 (**Bose and Fermi Operators**) (2 points)

Consider a Hermitian operator H, which satisfies the following commutation relations with the boson operators:

(1) $$[H, b_l^\dagger] = E_l b_l^\dagger$$

a) Show that $H$is diagonal in the boson operators, i.e.

(2) $$H = \sum_l E_l b_l^\dagger b_l + H',$$

where H' satisfies the following relations for all l:

(3) $$[H', b_l^\dagger] = [H', b_l] = 0.$$

b) What is changing for the case of Fermions?