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Einführung in die Theoretische Festkörperphysik

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Exercise 11

(Due date: 27.01.2020)

Problem 1 (**Specific heat of a semiconductor**) (3 points)

Consider a semiconductor model where the density of states for valence and conducting band are constant, i.e.

$$\rho(E) = \begin{cases} \frac{1}{E_0} & , \ 0 < E < E_0 \text{ and } E_0 + \Delta < E < 2E_0 + \Delta \\ 0 & , \ \text{else} \end{cases} .$$

Moreover, we assume one electron per unit cell in the system. How does the specific heat depend on the temperature in the limit of low temperatures?

Problem 2 (**Extrinsic Semiconductor**) (4 points)

Consider a semiconductor with gap Δ, doped with donor atoms with concentration n_D. The doping introduces energy levels within the band gap, with distance ϵ_D from the conduction band edge. Let the chemical potential be located close to the conduction band edge, $\mu \approx \Delta$ (with respect to the upper valence band edge which defines $E = 0$ here).

Hint: We consider temperatures $k_B T \ll \Delta$, which is fulfilled at room temperature for semiconductors with a large gap Δ. Electrons and holes are treated as free particles with same effective mass $m_e = m_L$. Furthermore, we approximate the dispersive bands as constant for simplicity.

a) How many electrons are provided by the donor atoms on average (assuming single ionization)?

b) From the neutrality condition $n_e = n_L$, derive the relation

$$n_D = 2A \left(A e^{\frac{\epsilon_D}{k_B T}} + 1 \right) \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} \tag{1}$$

Hint: Show that the valence band almost doesn't provide any conduction electrons.

Problem 3 (**Slater determinant**) (3 points)

We consider the fermionic creation operator $c_k^\dagger = \sum_{r_i} \mathrm{e}^{ikr_i} c_{r_i}^\dagger$ that creates states $|k\rangle$ with wave function $\psi_k(r) = \langle r|k\rangle = \langle 0|c_r c_k^\dagger|0\rangle$. Rewrite the wave function

$$\psi_{k_1,k_2,k_3}(r_1, r_2, r_3) = \langle 0|c_{r_1} c_{r_2} c_{r_3} c_{k_1}^\dagger c_{k_2}^\dagger c_{k_3}^\dagger|0\rangle$$

of a system of 3 fermions in terms of a Slater determinant.