THE KONDO MODEL

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History

Resistivity of a metal

- determined by different scattering mechanisms:
  - between conduction elelctrons and lattice distortions
    - $\rho^{el}_{Phonon} \propto T^5$
  - electron-electron
    - $\rho^{el}_{el-el} \propto T^2$
  - between electrons and static impurities

History

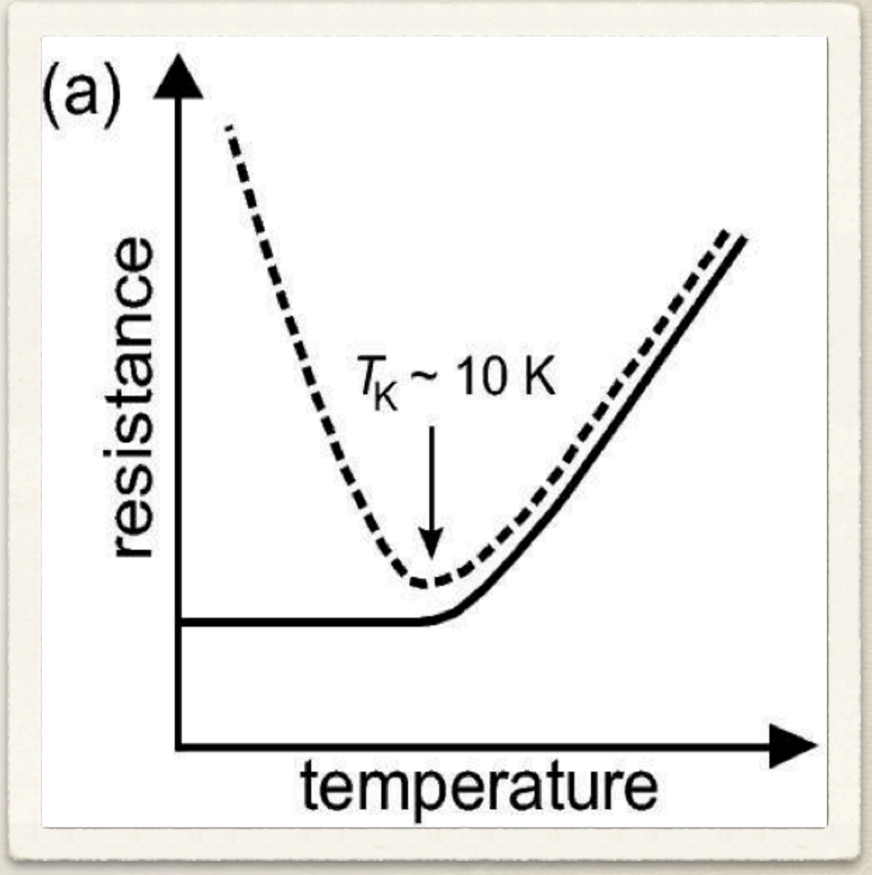
Resistivity of a metal

- monotonic temperature dependence
  - $\rho^{el}(T) = ac_{imp}\rho_0^{el} + bT^2 + cT^5$
- saturation in the limit $T \rightarrow 0$
  - $lim_{T\rightarrow 0}\rho^{el}(T) = ac_{imp}\rho_0^{el}$

History

Kondo effect

- 1934 de Haas, de Boer and van den Berg
- electrical resistivity of Au
- unexpected local minimum



History

Kondo effect

- 1964 solved by J. Kondo
- minimum associated with magnetic impurities
- novel scattering mechanism
  - spin-flip scattering
  - temperature dependant

History

Kondo effect

- new energy scale
  - Kondo temperature T_K
- Kondo effect dominates for temperature near T_K
- $\rho^{el}(T) = ac_{imp}\rho_0^{el} + bT^2 + cT^5 + c_{imp}\rho_1^{el} \ln\left[\frac{T_K}{T}\right]$
  - characteristic resistivity ρ_1^{el}

The Kondo model

perturbation method

- scattering of conduction electrons from a localized magnetic impurity
  - $\hat{H}_K = \hat{H}_l + \sum_{kk'} J \left[S^z \left(\alpha^\dagger_{sk\uparrow} \alpha_{sk'\uparrow} - \alpha^\dagger_{sk\downarrow} \alpha_{sk'\downarrow} \right) + S^+ \alpha^\dagger_{sk\downarrow} \alpha_{sk'\uparrow} + S^- \alpha^\dagger_{sk\uparrow} \alpha_{sk'\downarrow} \right]$
  - $\hat{H}_K = \hat{H}_l + \hat{H}'$
  - $\hat{H}' = 2J\hat{S} \cdot \hat{s}_0$

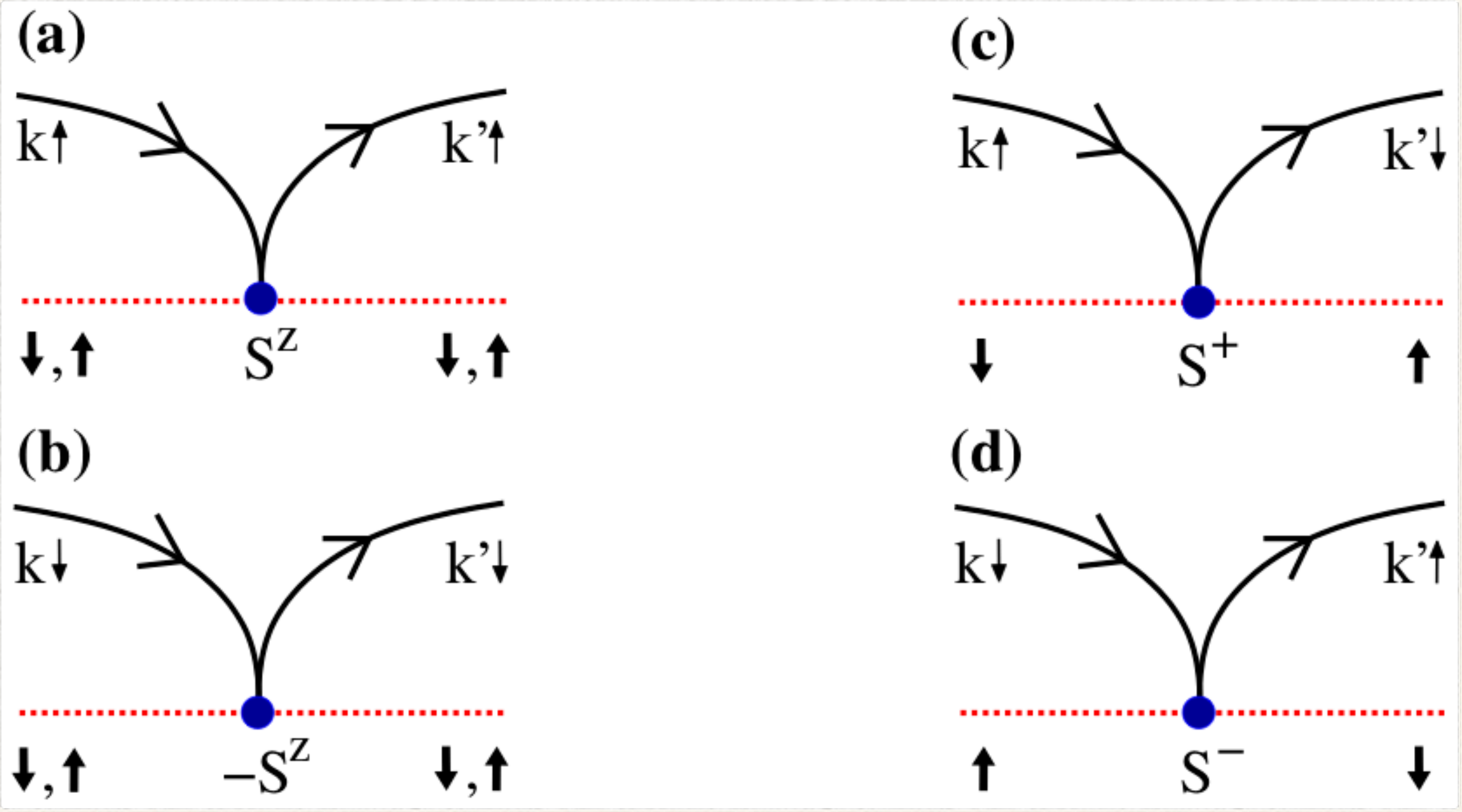
The Kondo model

perturbation method

- Schrödinger equation
  - $\left(\epsilon - \hat{H}_l\right) \mid \Psi\rangle = \hat{H}' \mid \Psi\rangle$
- formal solution
  - $\mid \Psi\rangle = \mid \Psi_0\rangle + \frac{1}{\epsilon + i0^+ - \hat{H}_l}\hat{H}' \mid \Psi\rangle$
- $\mathcal{T} = \hat{H}' + \hat{H}'\frac{1}{\epsilon + i0^+ - \hat{H}_l}\hat{H}' + \hat{H}'\frac{1}{\epsilon + i0^+ - \hat{H}_l}\hat{H}'\frac{1}{\epsilon + i0^+ - \hat{H}_l}\hat{H}' + \dots$

The Kondo model

perturbation method



The Kondo model

perturbation method

- first order contributions
  - $\langle k' \uparrow | \mathcal{T}^{(1)} | k \uparrow \rangle = JS^z$

    $\langle k' \downarrow | \mathcal{T}^{(1)} | k \downarrow \rangle = -JS^z$

    $\langle k' \uparrow | \mathcal{T}^{(1)} | k \downarrow \rangle = JS^-$

    $\langle k' \downarrow | \mathcal{T}^{(1)} | k \uparrow \rangle = JS^+$

The Kondo model

perturbation method

- second order in J
- scattering propability
  - $W_{kk'} = \frac{2\pi N_{imp}}{\hbar} \|T^{(1)}_{kk'}\|^2 = |J|^2 \frac{2\pi N_{imp}}{\hbar} S(S+1)$
  - $S = \langle S^z \rangle$
  - N_{imp} number of magnetic impurities

The Kondo model

perturbation method

- $\rho_{imp}^{el} = \dfrac{m}{ne^2\tau\left(k_F\right)}$
- $\left[\tau\left(k_F\right)\right]^{-1} = \sum\limits_{\overrightarrow{k}\,'} W_{\overrightarrow{k}\,\overrightarrow{k}\,'}\left(1-\cos\theta'\right)\delta\left(\epsilon_{\overrightarrow{k}} - \epsilon_{\overrightarrow{k}\,'}\right)$
- $\left[\tau\left(k_F\right)\right]^{-1} = \dfrac{3\pi J^2 S(S+1)c_{imp}n}{2\epsilon_F\hbar}$

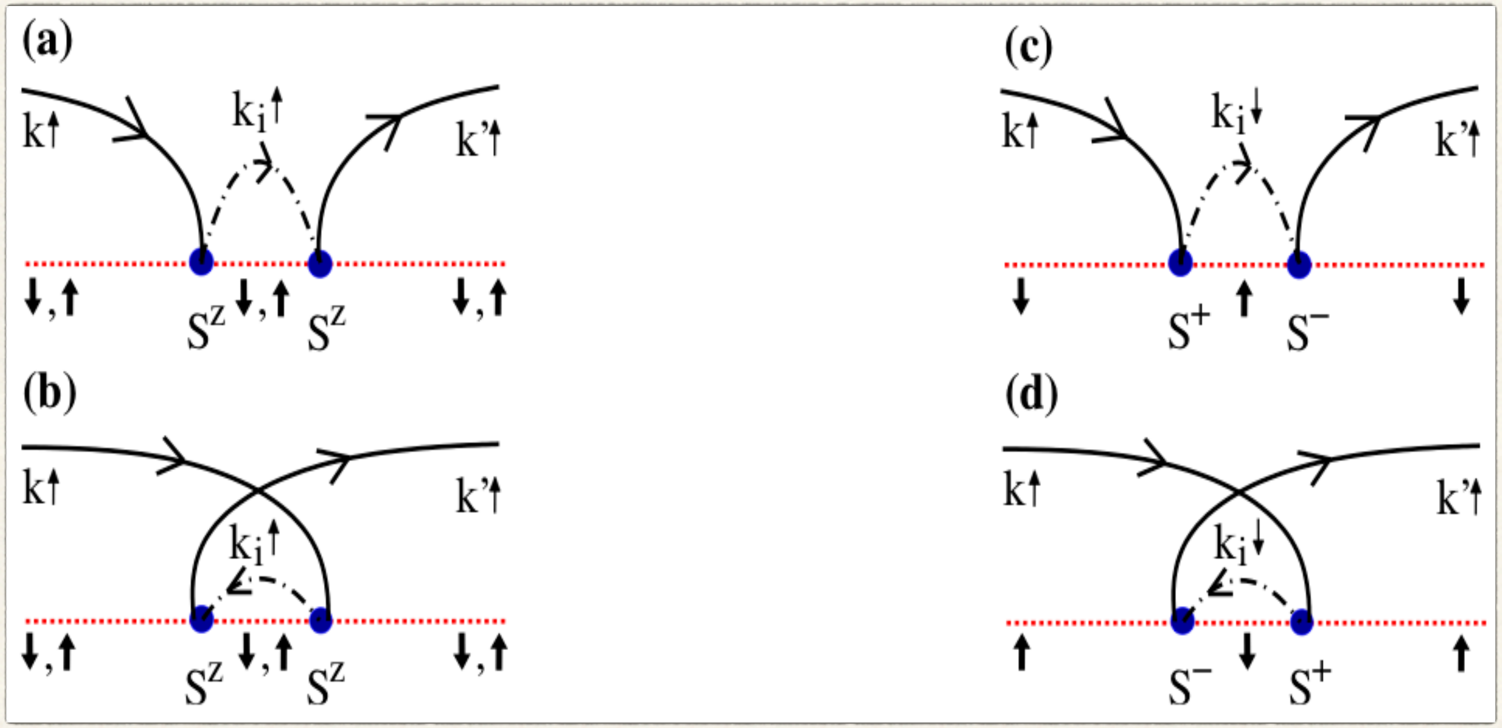
The Kondo model

perturbation method

- second order contribution to the resistivity
  - $\rho_{imp}^{el,(2)} = \dfrac{3\pi m J^2 S(S+1) c_{imp}}{2e^2 \epsilon_F \hbar}$
  - temperature independent

The Kondo model

perturbation method



The Kondo model

perturbation method

- second order processes (c)
  - $\langle k' \uparrow | \mathcal{T}^{(2)} \mid k \uparrow \rangle = \sum_{k_i} J^2 \langle k' \uparrow | \, \alpha^\dagger_{sk'\uparrow} \alpha_{sk_i\downarrow} S^- \frac{1}{\epsilon + i0^+ - \hat{H}_l} S^+ \alpha^\dagger_{sk_i\downarrow} \alpha_{sk\uparrow} \mid k \uparrow \rangle$

  - $\langle k' \uparrow | \mathcal{T}^{(2)} \mid k \uparrow \rangle = \sum_{k_i} J^2 \frac{S^- S^+ \left[1 - f(\epsilon_{k_i}\right]}{\epsilon - \epsilon_{k_i} + i0^+}$

The Kondo model

perturbation method

- transport relaxation time
  - $$[\tau\left(k_F\right)]^{-1} = \frac{3\pi J^2 S(S+1) c_{imp} n}{2\epsilon_F \hbar}\left[1 + 4J\rho(0)\ln\frac{D}{\max\left(|\epsilon|, kT\right)}\right]$$
  - $$\rho_{imp}^{el,(3)} = \frac{3\pi m J^2 S(S+1) c_{imp}}{2e^2\epsilon_F \hbar}\left[1 - 4J\rho(0)\ln\left(\frac{kT}{D}\right)\right]$$

The Kondo model

perturbation method

- resistivity:
  - $\rho^{el}(T) = ac_{imp}\rho_0^{el} + bT^2 + cT^5 + c_{imp}\rho_1^{el} \ln\left[\frac{T_K}{T}\right]$
  - unphysical, divergent resistivity
  - perturbative method fails for sufficiently small temperatures

The Kondo model

PMS

- Poor man‘s scaling (PMS)
  - 1970 P.W. Anderson
  - effective Hamiltonian that captures the low-energy properties of a given system
- scaling equation of the Kondo
  - $\frac{dJ}{d\ln D} = -2\rho J^2$

Summary

- The non-trivial physics associated with the presence of magnetic impurities in a solid is referred to as the Kondo effect

References

- Introduction to the Kondo effect
- Kondo Effekt in Supraleitender Umgebung
  Diplomarbeit von Julia Sabelin