Interaction effects in graphene and 3D Weyl semimetals

Anand Sharma

RWTH Aachen

CMT Seminar

March 12, 2019
Plan

1. **Graphene**
   - Motivation
   - FRG approach
   - Results

2. **3D Weyl semimetals**
   - Introduction
   - Results

3. **Summary**
Plan

1. Graphene
   - Motivation
   - FRG approach
   - Results

2. 3D Weyl semimetals
   - Introduction
   - Results

3. Summary
Graphene electronic properties\textsuperscript{1}

- No. of fermionic flavors: $N_f = 4$
- Bare Hamiltonian: $H_k = v_F \sigma \cdot k$
- Energy dispersion: $E(k) = \pm v_F |k|$  
- Momentum cutoff: $\Lambda_0 \approx \frac{1}{a}$
- Long-range interaction: $V(r) = \frac{e^2}{\epsilon_0 r}$
- Effective interaction: $\alpha = \frac{e^2}{\epsilon_0 v_F} \approx 2.2$

\textsuperscript{1}A. H. Castro Neto et al., Rev. Mod. Phys. 81, 109 (2009).
Graphene many-body interaction - experiment

What are the experimental signatures of electron-electron interaction?
Graphene many-body interaction - experiment

What are the experimental signatures of electron-electron interaction?

X. Du et al., Nature 462, 192 (2009)

Graphene many-body interaction - experiment

What are the experimental signatures of electron-electron interaction?

X. Du et al., Nature 462, 192 (2009)


D. C. Elias et al., Nat. Phys. 7, 701 (2011)
Graphene many-body interaction - experiment

How does the velocity gets renormalized?

\[
\frac{v_F(n)}{v_F(n_0)} = \left[1 + \frac{\alpha}{8\varepsilon_G} \ln \left(\frac{n_0}{n}\right)\right] = \left[1 + \frac{\alpha}{4\varepsilon_G} \ln \left(\frac{\Lambda_0}{\Lambda}\right)\right]
\]

where

\[
\varepsilon_G = 1 + \frac{\pi N_f \alpha}{8} \quad \text{and} \quad \Lambda = k_F = \sqrt{\pi n}
\]

D. C. Elias et al., Nat. Phys. 7, 701 (2011)
Graphene many-body interaction - theory
What is the theory of electron-electron interaction?
Graphene many-body interaction - theory

What is the theory of electron-electron interaction?

- 1st order pert. theory (PT) [leading order (LO) in $\alpha$]: J. González et al., Nucl. Phys. B 424, 595 (1994)

$$\frac{\nu(k)}{\nu(\Lambda_0)} = 1 + \frac{\alpha}{4} \ln \left( \frac{\Lambda_0}{k} \right)$$
Graphene many-body interaction - theory

What is the theory of electron-electron interaction?

- 1st order pert. theory (PT) [leading order (LO) in $\alpha$]: J. González et al., Nucl. Phys. B 424, 595 (1994)

\[
\frac{\nu(k)}{\nu(\Lambda_0)} = 1 + \frac{\alpha}{4} \ln \left( \frac{\Lambda_0}{k} \right)
\]

- 2nd order PT [next LO (NLO) in $\alpha$]: E. Barnes et al., Phys. Rev. B 89, 235431 (2014)

\[
\frac{\nu(k)}{\nu(\Lambda_0)} = 1 + \left[ \frac{\alpha}{4} + \left( \frac{\ln(2)}{2} - \frac{2}{3} \right) \alpha^2 \right] \ln \left( \frac{\Lambda_0}{k} \right)
\]
Graphene many-body interaction - theory

What is the theory of electron-electron interaction?

- 1st order pert. theory (PT) [leading order (LO) in $\alpha$]: J. González et al., Nucl. Phys. B 424, 595 (1994)

\[
\frac{\nu(k)}{\nu(\Lambda_0)} = 1 + \frac{\alpha}{4} \ln \left( \frac{\Lambda_0}{k} \right)
\]

- 2nd order PT [next LO (NLO) in $\alpha$]: E. Barnes et al., Phys. Rev. B 89, 235431 (2014)

\[
\frac{\nu(k)}{\nu(\Lambda_0)} = 1 + \left[ \frac{\alpha}{4} + \left( \frac{\ln(2)}{2} - \frac{2}{3} \right) \alpha^2 \right] \ln \left( \frac{\Lambda_0}{k} \right)
\]

Graphene many-body interaction

At low density in free standing graphene, what causes screening due to RPA?
At low density in free standing graphene, what causes screening due to RPA?
Plan

1 Graphene
   - Motivation
   - FRG approach
   - Results

2 3D Weyl semimetals
   - Introduction
   - Results

3 Summary
Methodology

- Exact flow equation of $n-$leg irreducible vertex.

\[
\partial_{\Lambda} \Gamma_{\lambda, \alpha_1 \cdots \alpha_n}^{(n)} = -\frac{1}{2} \sum_{\nu=1}^{\infty} \sum_{n_1=1}^{\infty} \cdots \sum_{n_\nu=1}^{\infty} \delta_{n_1+n_2+\cdots+n_\nu} \\
\times S_{\alpha_1 \cdots \alpha_{n_1} \alpha_{n_1+1} \cdots \alpha_{n_\nu+1} \cdots \alpha_{n_\nu}} \mathrm{Tr} \left( Z \Gamma_{\lambda, \alpha_{n_\nu+1}}^{(n_\nu+2)} \Gamma_{\lambda, \alpha_{n_\nu-n_\nu+1}}^{(n_\nu-1+2)} \cdots \Gamma_{\lambda, \alpha_{n-1}}^{(n_\nu+2)} \right).
\]

- Cutoff scheme (smooth, sharp, Litim, etc...) for fermionic or bosonic Green's function.

- Truncation scheme based on symmetry arguments and relevance of the vertices.

- Solve the self-consistent flow equations of the vertices.

---

Effective action

\[ S_{\Lambda_0}[\bar{\psi}, \psi] = -\sum_p \int \psi_p^\dagger (K) [G_p^0 (K)]^{-1} \psi_p (K) + \frac{1}{2} \int_Q f_q \rho (-Q) \rho (Q) \]
Effective action

\[ S_{\Lambda_0}[\bar{\psi}, \psi] = - \sum_p \int_K \psi_p^\dagger(K) [G_p^0(K)]^{-1} \psi_p(K) + \frac{1}{2} \int_Q f_q \rho(-Q) \rho(Q) \]

\[ G_p^0(K) = \left[ i \omega - \nu_p \sigma \cdot k \right]^{-1} \]

\[ \nu_p = \rho v_F \]

\[ p = \pm \text{ (Dirac points)} \]

\[ f_q = \frac{2\pi e^2}{\epsilon_0 q} \]

\[ \rho(Q) = \sum_p \int_K \psi_n^\dagger(K) \psi_n(K+Q) \]
Hubbard-Stratonovich (HS) effective action

\[
S_{\Lambda_0}[\bar{\psi}, \psi, \phi] = - \sum_p \int_K \psi_p^\dagger(K) [G_p^0(K)]^{-1} \psi_p(K)
+ \frac{1}{2} \int_Q \left[ f_q^{-1} \phi(-Q) \phi(Q) + 2i \rho(-Q) \phi(Q) \right]
\]
Hubbard-Stratonovich (HS) effective action

\[
S_{\Lambda_0}[\bar{\psi}, \psi, \phi] = - \sum_p \int_K \psi_p^\dagger(K)[G^0_p(K)]^{-1}\psi_p(K)
+ \frac{1}{2} \int_Q \left[ f_{-Q}^{-1}\phi(-Q)\phi(Q) + 2i\rho(-Q)\phi(Q) \right]
\]

\[G^0_p(K) = [i\omega - \nu_p\sigma \cdot k]^{-1}\]

\[\nu_p = p\nu_F\]

\[p = \pm \text{ (Dirac points)}\]

\[f_q = \frac{2\pi e^2}{\epsilon_0 q}\]

\[\rho(Q) = \sum_p \int_K \psi_p^\dagger(K)\psi_p(K + Q)\]
Sharp regulator/ fermionic cutoff

\[ \Gamma_{\Lambda}[\bar{\psi}, \psi, \phi] = -\sum_p \int_K \psi_p^\dagger(K) G_{p,\Lambda}^{-1}(K) \psi_p(K) + \frac{1}{2} \int_Q \phi(-Q) F_{\Lambda}^{-1}(Q) \phi(Q) \]

\[ + \sum_{p,s} \int_K \int_Q \Gamma_{p,\Lambda}^s(K + Q, K, Q) \bar{\psi}_p^s(K + Q) \psi_p^s(K) \phi(Q) \]
Sharp regulator/ fermionic cutoff

\[ \Gamma_{\Lambda}[\bar{\psi}, \psi, \phi] = -\sum_p \int_K \psi^\dagger_p(K) G^{-1}_{p,\Lambda}(K) \psi_p(K) + \frac{1}{2} \int_Q \phi(-Q) F^{-1}_{\Lambda}(Q) \phi(Q) \]

\[ + \sum_{p, s} \int_K \int_Q \Gamma^s_{p,\Lambda}(K + Q, K, Q) \bar{\psi}^s_p(K + Q) \psi^s_p(K) \phi(Q) \]

where \( s = A \) or \( B \) and

\[ G^{-1}_{p,\Lambda}(K) = [G^0_{p,\Lambda}(K)]^{-1} - \Sigma_{p,\Lambda}(K) \]

\[ G^0_{p,\Lambda}(K) = \left[ i\omega - v_p \sigma \cdot k - R^\psi_{\Lambda}(K) \right]^{-1} \]

\[ = \Theta(k - \Lambda) \left[ i\omega - v_p \sigma \cdot k \right]^{-1} \]

\[ R^\psi_{\Lambda}(K) = [G^0_p(K)]^{-1} (1 - \Theta^{-1}(k - \Lambda)) \]

\[ F^{-1}_{\Lambda}(Q) = f^{-1}_q + \Pi_{\Lambda}(Q) \]
Relevance of vertices in RG sense

Let $\Gamma^{(f,b)}$ be a bare vertex with $f$ fermionic and $b$ bosonic external legs.
Relevance of vertices in RG sense

Let $\Gamma^{(f,b)}$ be a bare vertex with $f$ fermionic and $b$ bosonic external legs.

<table>
<thead>
<tr>
<th>Vertices</th>
<th>2D graphene</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma^{(2,1)}$</td>
<td>marginal</td>
</tr>
<tr>
<td>$\Gamma^{(0,3)}$</td>
<td>marginal</td>
</tr>
<tr>
<td>$\Gamma^{(4,0)}$</td>
<td>irrelevant</td>
</tr>
<tr>
<td>$\Gamma^{(0,4)}$</td>
<td>irrelevant</td>
</tr>
<tr>
<td>$\Gamma^{(2,2)}$</td>
<td>irrelevant</td>
</tr>
</tbody>
</table>
Relevance of vertices in RG sense

Let $\Gamma^{(f,b)}$ be a bare vertex with $f$ fermionic and $b$ bosonic external legs.

<table>
<thead>
<tr>
<th>Vertices</th>
<th>2D graphene</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma^{(2,1)}$</td>
<td>marginal</td>
</tr>
<tr>
<td>$\Gamma^{(0,3)}$</td>
<td>marginal</td>
</tr>
<tr>
<td>$\Gamma^{(4,0)}$</td>
<td>irrelevant</td>
</tr>
<tr>
<td>$\Gamma^{(0,4)}$</td>
<td>irrelevant</td>
</tr>
<tr>
<td>$\Gamma^{(2,2)}$</td>
<td>irrelevant</td>
</tr>
</tbody>
</table>

- Neglect vertices which are irrelevant in RG sense and/or vanish at the initial scale
- Retain only those which couple to the fermionic and bosonic self-energies.

Our truncation scheme is valid for any value of $\alpha = \frac{e^2}{\epsilon_0 v_F}$. 
Flow equation of fermionic self-energy

\[ \partial \Lambda \Sigma_{ss'}^{s'}(K) = \int_Q F_\Lambda(Q) \dot{G}_{ss'}^{s'}(K - Q) \Gamma^s_{p,\Lambda}(K, K - Q, Q) \Gamma^{s'}_{p,\Lambda}(K - Q, K, -Q) \]
Flow equation of bosonic self-energy

\[ \partial_{\Lambda} \Pi_{\Lambda}(Q) = \sum_{ss'} \sum_{p} \int_{K} \left[ G_{ss',p,\Lambda}(K) G_{s',s}(K - Q) + G_{ss',p,\Lambda}(K) \dot{G}_{s',s}(K - Q) \right] \]

\[ \Gamma_{p,\Lambda}^{s}(K - Q, K, -Q) \Gamma_{p,\Lambda}^{s'}(K, K - Q, Q) \]
Flow equation of three-legged vertex

\[
\partial_\Lambda \Gamma^s_{p,\Lambda}(K, K - Q, Q) = \cdots
\]
Flow equation of three-legged vertex

\[ \partial_{\Lambda} \Gamma_{p,\Lambda}^{s}(K, K - Q, Q) = \cdots \]

\[ \dot{G}_{p,\Lambda}(K) = -G_{p,\Lambda}(K) \left[ \partial_{\Lambda} \left[ G_{\Lambda}^{0}(K) \right]^{-1} \right] G_{p,\Lambda}(K) \]

\[ = -\delta(k - \Lambda) \left[ i\omega - v_{p}\sigma \cdot k - \Sigma_{p,\Lambda}(K) \right]^{-1} \]
Renormalized quantities

Velocity:

\[ \Sigma_{p,\Lambda}(K) = pV_\Lambda(k) \sigma \cdot k + (1 - Z_\Lambda^{-1})i\omega + O(\omega^2) \]
\[ G_{p,\Lambda}(K) = -\Theta(k - \Lambda)Z_\Lambda \frac{i\omega + pv_\Lambda(k)\sigma \cdot k}{\omega^2 + \xi_\Lambda^2(k)} \]
\[ \xi_\Lambda(k) = v_\Lambda(k)k \]
\[ v_\Lambda(k) = Z_\Lambda[v_F + V_\Lambda(k)] \]
Renormalized quantities

Velocity:

\[ \Sigma_{p,\Lambda}(K) = pV_{\Lambda}(k)\sigma \cdot k + \left(1 - Z_{\Lambda}^{-1}\right)i\omega + \mathcal{O}(\omega^2) \]

\[ G_{p,\Lambda}(K) = -\Theta(k - \Lambda)Z_{\Lambda} \frac{i\omega + p\nu_{\Lambda}(k)\sigma \cdot k}{\omega^2 + \xi_{\Lambda}^2(k)} \]

\[ \xi_{\Lambda}(k) = \nu_{\Lambda}(k)k \]

\[ \nu_{\Lambda}(k) = Z_{\Lambda}[\nu_F + V_{\Lambda}(k)] \]

Dielectric function:

\[ \epsilon_{\Lambda}(Q) = 1 + f_q \Pi_{\Lambda}(Q) \]
Renormalized quantities

Velocity :

\[ \Sigma_{p,\Lambda}(K) = pV_{\Lambda}(k)\sigma \cdot k + (1 - Z_{\Lambda}^{-1})i\omega + \mathcal{O}(\omega^2) \]

\[ G_{p,\Lambda}(K) = -\Theta(k - \Lambda)Z_{\Lambda} \frac{i\omega + pv_{\Lambda}(k)\sigma \cdot k}{\omega^2 + \xi_{\Lambda}^2(k)} \]

\[ \xi_{\Lambda}(k) = v_{\Lambda}(k)k \]

\[ v_{\Lambda}(k) = Z_{\Lambda}[v_F + V_{\Lambda}(k)] \]

Dielectric function :

\[ \epsilon_{\Lambda}(Q) = 1 + f_q \Pi_{\Lambda}(Q) \]

Vertex :

\[ \Gamma_{\Lambda}(0, 0, 0) = i\gamma_{\Lambda} \]
Renormalized quantities

Using Ward identity: $Z_\Lambda = 1/\gamma_\Lambda$ we get

**Dynamical flow equations**

\[
\lambda \partial_\lambda \nu_\lambda(k) = \eta_\lambda \nu_\lambda(k) + \lambda Z_\lambda \left( \partial_\lambda V_\lambda \right)
\]

\[
\lambda \partial_\lambda \epsilon_\lambda(Q) = \lambda f_q \partial_\lambda \Pi_\lambda(Q)
\]

\[
\lambda \partial_\lambda Z_\lambda = \eta_\lambda Z_\lambda
\]
Renormalized quantities

Using Ward identity: $Z_{\Lambda} = 1/\gamma_{\Lambda}$ we get

Dynamical flow equations

$$\Lambda \partial_{\Lambda} \nu_{\Lambda}(k) = \eta_{\Lambda} \nu_{\Lambda}(k) + \Lambda Z_{\Lambda} (\partial_{\Lambda} V_{\Lambda})$$

$$\Lambda \partial_{\Lambda} \epsilon_{\Lambda}(Q) = \Lambda f_{q} \partial_{\Lambda} \Pi_{\Lambda}(Q)$$

$$\Lambda \partial_{\Lambda} Z_{\Lambda} = \eta_{\Lambda} Z_{\Lambda}$$

where anomalous dimension related to frequency is

$$\eta_{\Lambda} = \Lambda Z_{\Lambda} \lim_{\omega \to 0} \frac{\partial}{\partial (i\omega)} \partial_{\Lambda} \Sigma_{\Lambda}(0, i\omega)$$
Renormalized quantities

In static approximation $Z_\Lambda = \gamma_\Lambda = 1$ and we get

**Static flow equations**

\[
\Lambda \partial_\Lambda v_\Lambda(k) = -\frac{e^2}{2} \frac{\Lambda}{k} \int_0^\pi \frac{d\varphi}{\pi} \frac{\cos \varphi}{\sqrt{1 - 2(k/\Lambda)\cos \varphi + (k/\Lambda)^2}} \frac{1}{\epsilon_\Lambda(\sqrt{\Lambda^2 - 2k\Lambda \cos \varphi + k^2})}
\]

\[
\Lambda \partial_\Lambda \epsilon_\Lambda(q) = -2N_s e^2 \frac{q}{\Lambda} \int_0^{\pi/2} \frac{d\varphi}{\pi} \frac{\Theta(1 + \frac{q}{2\Lambda} \cos \varphi - \frac{q}{2\Lambda})}{\sqrt{[1 + (q/2\Lambda) \cos \varphi]^2 - [q/2\Lambda]^2}} \frac{\sin^2 \varphi}{[v_\Lambda(\Lambda) + (1 + (q/\Lambda) \cos \varphi) \nu_\Lambda(\Lambda + q \cos \varphi)]}
\]

where $N_s = 2$ is spin degeneracy.
Plan

1. Graphene
   - Motivation
   - FRG approach
   - Results

2. 3D Weyl semimetals
   - Introduction
   - Results

3. Summary
Results

\[ \epsilon_{\text{RPA}} = 1 + \frac{\pi N_f \alpha}{16} \approx 4.46 \quad (N_f = 8) \]

\[ v(k) = A(\alpha) + B(\alpha) \ln(\Lambda_0 / k) \]

\[ \alpha = \frac{e^2}{v_F} \approx 2.2 \]
Velocity renormalization

Can the screening effects due to the substrate explain the RPA result?
Can the screening effects due to the substrate explain the RPA result?
Velocity renormalization

\[ \varepsilon = \frac{1 + \varepsilon_s}{2} \]

Graphene/quartz, \( \varepsilon_s = 1.82 \)
Graphene/suspended, \( \varepsilon_s = 1.0 \)
Graphene/SiC(000-1), \( \varepsilon_s = 4.22 \)

What about 3D case with large number of non-degenerate nodes?
Velocity renormalization

\[ \varepsilon = \frac{1 + \varepsilon_s}{2} \]

Graphene/quartz, \( \varepsilon_s = 1.82 \)
Graphene/suspended, \( \varepsilon_s = 1.0 \)
Graphene/SiC(000-1), \( \varepsilon_s = 4.22 \)

\[ \alpha/\varepsilon = 2.2 \]
\[ \alpha/\varepsilon = 1.5 \]
\[ \alpha/\varepsilon = 0.5 \]

What about 3D case with large number of non-degenerate nodes?
Plan

1. Graphene
   - Motivation
   - FRG approach
   - Results

2. 3D Weyl semimetals
   - Introduction
   - Results

3. Summary
**Introduction**

Broken time-reversal or inversion symmetry in Dirac SM

Bare Hamiltonian: \( H_k = \nu_F \sigma \cdot k \)

Energy dispersion: \( E(k) = \pm \nu_F |k| \)

Long-range interaction: \( V(r) = \frac{e^2}{\epsilon_0 r} \)

Effective interaction: \( \alpha = \frac{e^2}{\epsilon_0 \nu_F} \)

---

Abrikosov-Beneslavskii theory
Interacting gapless semiconductors of first kind

SOVIET PHYSICS JETP VOLUME 32, NUMBER 4 APRIL, 1971

POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND DIELECTRICS

A. A. ABRIKOSOV and S. D. BENESLAVSKII

L. D. Landau Institute of Theoretical Physics
Submitted April 13, 1970

The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.

The role of the Coulomb interaction is considered for both types of spectra. In the case of a linear dispersion law a slowly varying (logarithmic) factor appears in the spectrum. In the case of a quadratic spectrum the effective interaction becomes strong for small momenta, and the concept of the one-particle spectrum turns out to be inapplicable. The behavior of the Green's functions is determined by similarity laws analogous to those obtained in field theory with strong coupling and in the neighborhood of a phase transition point of the second kind (scaling). Hence follow power laws for the electronic heat capacity and for the momentum distribution of the electrons.
Abrikosov-Beneslavskii theory

*Static* effective Coulomb interaction and scaling at *single node* provides renormalized quantities.

**Fermionic self-energy**

\[ v(k) = v_F f\left(\ln\left(\frac{k_{\text{max}}}{k}\right)\right) \]

**Bosonic self-energy**

\[ \varepsilon(q) = 1 + \Pi(q) V(q) = \left[ 1 + (a + b) l \right]^b a + b \]

What about dynamic interaction and large number of nodes?
Abrikosov-Beneslavskii theory

*Static* effective Coulomb interaction and scaling at *single node* provides renormalized quantities.

\[
\text{Fermionic self-energy}
\]

\[
\psi = \psi_\text{F} \psi_\text{f}
\]

\[
\text{Velocity :}
\]

\[
v(k) = v_F f\left(\ln\left(\frac{k_{\text{max}}}{k}\right)\right)
\]

\[
f(l) = \left[1 + (a + b)l\right]^{\frac{a}{a+b}}
\]
Abrikosov-Beneslavskiǐ theory

*Static* effective Coulomb interaction and scaling at *single node* provides renormalized quantities.

**Fermionic self-energy**

\[ v(k) = v_F f\left(\ln\left(\frac{k_{\text{max}}}{k}\right)\right) \]

\[ f(l) = \left[1 + (a + b)l\right]^\frac{a}{a+b} \]

**Bosonic self-energy**

\[ \varepsilon(q) = 1 + \Pi(q) V(q) \]

\[ = \left[1 + (a + b)l\right]^\frac{b}{a+b} \]

What about dynamic interaction and large number of nodes?
Abrikosov-Beneslavskii theory

*Static* effective Coulomb interaction and scaling at *single node* provides renormalized quantities.

**Fermionic self-energy**

\[ v(k) = v_F \, f \left( \ln \left( \frac{k_{\text{max}}}{k} \right) \right) \]

\[ f(l) = \left[ 1 + (a + b)l \right]^{\frac{a}{a+b}} \]

**Bosonic self-energy**

**Dielectric function**

\[ \varepsilon(q) = 1 + \Pi(q) \, V(q) \]

\[ = \left[ 1 + (a + b)l \right]^{\frac{b}{a+b}} \]

What about dynamic interaction and large number of nodes?
Plan

1. **Graphene**
   - Motivation
   - FRG approach
   - Results

2. **3D Weyl semimetals**
   - Introduction
   - Results

3. **Summary**
Methodology - relevance of vertices in RG sense

Let $\Gamma^{(f,b)}$ be a bare vertex with $f$ fermionic and $b$ bosonic external legs.

<table>
<thead>
<tr>
<th>Vertices</th>
<th>3D Weyl semimetals</th>
<th>2D graphene</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma^{(2,1)}$</td>
<td>marginal</td>
<td>marginal</td>
</tr>
<tr>
<td>$\Gamma^{(0,3)}$</td>
<td>relevant</td>
<td>marginal</td>
</tr>
<tr>
<td>$\Gamma^{(4,0)}$</td>
<td>irrelevant</td>
<td>irrelevant</td>
</tr>
<tr>
<td>$\Gamma^{(0,4)}$</td>
<td>marginal</td>
<td>irrelevant</td>
</tr>
<tr>
<td>$\Gamma^{(2,2)}$</td>
<td>irrelevant</td>
<td>irrelevant</td>
</tr>
</tbody>
</table>

- Neglect vertices which are irrelevant in RG sense and/or vanish at the initial scale
- Retain only those which couple to the fermionic and bosonic self-energies.

*Our truncation scheme is valid for any value of $\alpha = \frac{e^2}{\epsilon_0 v_F}$ and $N_W$.***
Flow equation of three-legged vertex

\[ \partial_\Lambda \Gamma^b_{n,\Lambda}(K, K - Q, Q) = \cdots \]

with \( b = C \) or \( V \) and \( n = 1, \cdots, N_W \).
Flow equation of three-legged vertex

\[ \partial_\Lambda \Gamma_{n,\Lambda}^b(K, K - Q, Q) = \cdots \]
Flow equation of three-legged vertex

$$\partial_{\Lambda} \Gamma_{n,\Lambda}^b(K, K - Q, Q) = \cdots$$
Renormalized quantities

In static approximation $Z_\Lambda = \gamma_\Lambda = 1$ and we get

**Static flow equations**

\[
\Lambda \partial_\Lambda \left[ \frac{v_\Lambda(k)}{v_F} \right] = -\frac{\alpha}{2} \Lambda \left[ \frac{\sin \varphi \cos \varphi}{\pi} \right] \frac{1}{\left[ 1 - 2 \left( \frac{k}{\Lambda} \right) \cos \varphi + \left( \frac{k}{\Lambda} \right)^2 \right]} \varepsilon_\Lambda \left( \sqrt{\Lambda^2 - 2k\Lambda \cos \varphi + k^2} \right)
\]

\[
\Lambda \partial_\Lambda \varepsilon_\Lambda(q) = -\alpha N_s N_W \left[ \frac{\sin^3 \varphi \Theta(1 + \frac{q}{\Lambda} \cos \varphi - \frac{q}{\Lambda})}{\pi} \right] \left\{ \left[ \frac{v_\Lambda(\Lambda)}{v_F} \right] + \left( 1 + \frac{q}{\Lambda} \cos \varphi \right) \left[ \frac{v_\Lambda(\Lambda+q \cos \varphi)}{v_F} \right] \right\}
\]

where $N_s = 2$ is spin degeneracy, $N_W$ being the number of Weyl nodes, and $\alpha$ is a parameter.
Renormalized velocity

\[ \frac{v_{\Lambda \to 0}(k)}{v_F} \]

\[ k / \Lambda_0 \]

\[ N_W = 2 \]

\[ \alpha = 2.0 \]
\[ \alpha = 1.0 \]
\[ \alpha = 0.5 \]
\[ \alpha = 0.2 \]

\[ NW = 2 \]

\[ \alpha \]

Renormalized velocity

\[ \frac{v_{\Lambda \to 0}(k)}{v_F} \]

\( k / \Lambda_0 \) vs. \( \alpha = 2.0, 1.0, 0.5, 0.2 \)

\( N_W = 2 \)

\( N_W = 12 \)

\( N_W = 24 \)
Renormalized dielectric function

\[ \varepsilon(q) = \frac{\Lambda_0^n}{q^2} \]

\[ N_W = 2 \]

\[ \alpha = 2.0 \quad \alpha = 1.0 \quad \alpha = 0.5 \quad \alpha = 0.2 \]
Renormalized dielectric function

\[ \varepsilon_{\Lambda_0}(q) \rightarrow 0(q) \]

\[ N_W = 2 \]

\[ \alpha = 2.0 \]
\[ \alpha = 1.0 \]
\[ \alpha = 0.5 \]
\[ \alpha = 0.2 \]

\[ N_W = 12 \]

\[ N_W = 24 \]
Power-law fit

In limit of vanishing momentum, $k \rightarrow 0$,

\[ \Lambda \partial_\Lambda [v_\Lambda / v_F] = - \frac{2\alpha}{3\pi \varepsilon_\Lambda} \frac{1}{[v_\Lambda / v_F]} \quad \Rightarrow \quad \partial_l [v_l / v_F] = \frac{a}{\varepsilon_l} \quad \text{where} \quad a = \frac{2\alpha}{3\pi} \]

\[ \Lambda \partial_\Lambda \varepsilon_\Lambda = - \frac{\alpha N_s N_W}{3\pi} \frac{1}{[v_\Lambda / v_F]} \quad \Rightarrow \quad \partial_l \varepsilon_l = \frac{b}{[v_l / v_F]} \quad \text{where} \quad b = \frac{\alpha N_s N_W}{3\pi} \]
Power-law fit

In limit of vanishing momentum, \( k \rightarrow 0 \),

\[
\Lambda \partial_{\Lambda} \left[ \frac{v_{\Lambda}}{v_{F}} \right] = -\frac{2\alpha}{3\pi} \frac{1}{\varepsilon_{\Lambda}} \quad \implies \quad \partial_{l} \left[ \frac{v_{l}}{v_{F}} \right] = \frac{a}{\varepsilon_{l}} \quad \text{where} \quad a = \frac{2\alpha}{3\pi}
\]

\[
\Lambda \partial_{\Lambda} \varepsilon_{\Lambda} = -\frac{\alpha N_{s} N_{W}}{3\pi} \frac{1}{\left[ \frac{v_{\Lambda}}{v_{F}} \right]} \quad \implies \quad \partial_{l} \varepsilon_{l} = \frac{b}{\left[ \frac{v_{l}}{v_{F}} \right]} \quad \text{where} \quad b = \frac{\alpha N_{s} N_{W}}{3\pi}
\]

With \( \partial_{l} \left\{ \left[ \frac{v_{l}}{v_{F}} \right] \varepsilon_{l} \right\} = a + b \)

Generalized Abrikosov-Beneslavskii theory

\[
\left[ \frac{v_{l}}{v_{F}} \right] = \left[ 1 + (a + b) l \right]^{\frac{a}{a+b}} = \left[ 1 + \frac{\alpha(2 + N_{s} N_{W})}{3\pi} l \right]^{2\frac{N_{s} N_{W}}{2+N_{s} N_{W}}}
\]

\[
\varepsilon_{l} = \left[ 1 + (a + b) l \right]^{\frac{b}{a+b}} = \left[ 1 + \frac{\alpha(2 + N_{s} N_{W})}{3\pi} l \right]^{\frac{N_{s} N_{W}}{2+N_{s} N_{W}}}
\]
Power-law fit

Renormalized velocity and dielectric function

\[ v_l = \left[ A(\alpha, N_W) + B(\alpha, N_W) \right]^{\frac{1}{1+N_W}} \]

\[ \varepsilon_l = \left[ C(\alpha, N_W) + D(\alpha, N_W) \right]^{\frac{N_W}{1+N_W}} \]
Renormalized velocity

$V_{\Lambda \rightarrow 0}(k) / V_F$

$k / \Lambda_0$

$N_W = 2$

$\alpha = 2.0$
$\alpha = 1.0$
$\alpha = 0.5$
$\alpha = 0.2$

$N_W = 12$

$NW = 24$

$\alpha = 2.0$
$\alpha = 1.0$
$\alpha = 0.5$
$\alpha = 0.2$
Renormalized dielectric function

\[ \varepsilon(q/\Lambda_0) \rightarrow 0(q) \]

\[ N_W = 2 \]

\[ N_W = 12 \]

\[ N_W = 24 \]
Plan

1 Graphene
   - Motivation
   - FRG approach
   - Results

2 3D Weyl semimetals
   - Introduction
   - Results

3 Summary
Summary

- Nonperturbative theory of Coulomb interacting 2D and 3D topological semimetals.
- The renormalized velocity shows logarithmic divergence approaching the Dirac (Weyl) point in 2D (3D).
- In 3D Weyl semimetal, the velocity increases with increasing effective coupling, $\alpha$, but decreases with increasing number of flavors, $N_W$, while dielectric function always increases.
- The dielectric function in graphene properly takes account for the lack of screening by summing up the relevant diagrams.
- In static limit, for graphene our results agree with the experimental data and for 3D Weyl semimetals, we generalize the Abrikosov-Beneslavskii theory.
Collaborators

- Carsten Bauer (PhD student at ITP, Köln)
- Andreas Rückriegel (Postdoc at ITP, Uni Utrecht)
- Arthur Scammell (PhD student at ITP, Frankfurt)
- Jan Krieg (PhD student at ITP, Frankfurt)
- Peter Kopietz (ITP, Frankfurt)
Collaborators

- Carsten Bauer (PhD student at ITP, Köln)
- Andreas Rückriegel (Postdoc at ITP, Uni Utrecht)
- Arthur Scammell (PhD student at ITP, Frankfurt)
- Jan Krieg (PhD student at ITP, Frankfurt)
- Peter Kopietz (ITP, Frankfurt)

Thank you for your attention!
Extra slides
Graphene
Dynamic renormalized velocity\textsuperscript{6}

\[ \frac{v_\Lambda}{v_F} = 1 + \frac{\alpha}{4} \ln \left( \frac{C_\Lambda(\alpha)}{\Lambda} \right) \]

\textsuperscript{6} AS and P. Kopietz, Phys. Rev. B 93, 235425 (2016).
Wavefunction renormalization

\[ Z_\Lambda = 0.4772 \]

\[ l = \ln(\Lambda_0/\Lambda) \]
Cutoff function, $C_{\Lambda}(\alpha)$

\[\Lambda_{0} = l \frac{\Lambda_{0}}{l} \]

\[C_{\Lambda} = \Lambda_{0} / l^{2.81} \]

\[C_{\Lambda} = \Lambda_{0} / l^{6.28} \]
3D Weyl semimetals - Static
Fitting values: $A, B, C, and D$

\[ v_I = \left[ A(\alpha, N_W) + B(\alpha, N_W) \right]^{\frac{1}{1+N_W}} \]

\[ \varepsilon_I = \left[ C(\alpha, N_W) + D(\alpha, N_W) \right]^{\frac{N_W}{1+N_W}} \]
3D Weyl semimetals - Dynamic
Sharp regulator/ bosonic cutoff

\[
F^0_\Lambda(Q) = \left[ f_q^{-1} - R^\phi_\Lambda(Q) \right]^{-1} \\
= \Theta(q - \Lambda)f_q \\
R^\phi_\Lambda(Q) = f_q^{-1}(1 - \Theta^{-1}(q - \Lambda)) \\
\dot{F}_\Lambda(Q) = -F^2_\Lambda(Q)\partial_\Lambda \left[ F^0_\Lambda(Q) \right]^{-1} \\
= -\delta(q - \Lambda) \left[ f_q^{-1} - \Pi_\Lambda(Q) \right]^{-1}
\]

\[
[G_\Lambda(K)]^{-1} = [G^0(K)]^{-1} - \Sigma_\Lambda(K) \\
\Sigma_\Lambda(K) = (1 - Z^{-1}_\Lambda)i\omega - (1 - Y^{-1}_\Lambda)v_F\sigma \cdot k + O(\omega^2) \\
G_\Lambda(K) = -Z_\Lambda \frac{i\omega + v_\Lambda\sigma \cdot k}{\omega^2 + v^2_\Lambda k^2} \\
v_\Lambda = Z_\Lambda Y^{-1}_\Lambda v_F
\]
Flow equation of fermionic self-energy

\[
\partial_{\Lambda} \Sigma_{\Lambda}^{bb'}(K) = \sum_{b_1 b_2} \int_Q \dot{F}_{\Lambda}(Q) G_{\Lambda}^{b_1 b_2}(K - Q) \times \Gamma_{\Lambda}^{bb_1 \phi}(K, K - Q, Q) \Gamma_{\Lambda}^{b_2 b' \phi}(K - Q, K, -Q)
\]
Dyson-Schwinger equation for bosonic self-energy

\[ \Pi_\Lambda(Q) = i N_s N_W \sum_{bb'} \int_K G_\Lambda^{bb'}(K) G_\Lambda^{b'b}(K - Q) \Gamma_\Lambda^{b'b'}(K, K - Q, Q) \]
Renormalized quantities

Dynamical flow equations

\[ \Lambda \partial_\Lambda v_\Lambda = (\eta_\Lambda - \tilde{\eta}_\Lambda)v_\Lambda \]

\[ \Lambda \partial_\Lambda Z_\Lambda = \eta_\Lambda Z_\Lambda \]
Renormalized quantities

Dynamical flow equations

\[ \Lambda \partial_{\Lambda} v_{\Lambda} = (\eta_{\Lambda} - \tilde{\eta}_{\Lambda}) v_{\Lambda} \]
\[ \Lambda \partial_{\Lambda} Z_{\Lambda} = \eta_{\Lambda} Z_{\Lambda} \]

where anomalous dimension related to frequency and momentum are

\[ \eta_{\Lambda} = \Lambda Z_{\Lambda} \lim_{\omega \to 0} \frac{\partial}{\partial (i\omega)} \partial_{\Lambda} \Sigma_{bb}^{bb}(0, i\omega) \]
\[ = \Lambda \int_{Q} \frac{\delta(q - \Lambda)}{\frac{\epsilon_{0} \Lambda}{4\pi e^{2}} + \Pi_{\Lambda}(Q)} \frac{\bar{\omega}^{2} - (v_{\Lambda} q)^{2}}{[\bar{\omega}^{2} + (v_{\Lambda} q)^{2}]^{2}} \]

\[ (\sigma \cdot \hat{k}) \tilde{\eta}_{\Lambda} = -\Lambda Y_{\Lambda} \lim_{|k| \to 0} \frac{\partial}{\partial (v_{F}|k|)} \partial_{\Lambda} \Sigma_{bb}'(k, 0) \]
\[ = \Lambda \int_{Q} \frac{\delta(q - \Lambda)}{\frac{\epsilon_{0} \Lambda}{4\pi e^{2}} + \Pi_{\Lambda}(Q)} \frac{\bar{\omega}^{2}}{[\bar{\omega}^{2} + (v_{\Lambda} q)^{2}]^{2}} \]
Dynamic renormalized velocity

\[ l = \ln\left(\frac{\Lambda_0}{\Lambda}\right) \]

- For \( \alpha = 0.2 \), \( N_W = 2 \)
- For \( \alpha = 2.0 \), \( N_W = 24 \)

The graph compares dynamic and static renormalized velocity with different parameters.
Renormalized wavefunction renormalization

\[ l = \ln\left(\frac{\Lambda_0}{\Lambda}\right) \]

\[ Z_{\Lambda} = 0.2, \ N_W = 2 \]
\[ \alpha = 2.0, \ N_W = 24 \]