Relativistic Hydrodynamics

Luciano Rezzolla
Institute for Theoretical Physics, Frankfurt am Main, Germany

Olindo Zanotti
Laboratory of Applied Mathematics, University of Trento, Italy
Errata/Corrige

Notes

• All of the typos reported in black have been fixed in the revised paperback version, but are still present in the hardback version till a new version is published.

• The page and equation numbering varies slightly between the paperback and the hardback versions. All of the numbering reported in this errata refers to the hardback version.

• All of the typos reported in blue have been found after the paperback version was published and hence are present only on the paperback version. The page numbering refers therefore to the paperback version.

Last update: 7 April 2024
1
A Brief Review of General Relativity

• Page 28, Eq. (1.115). Change

\[ v^x = \frac{v^x + V}{1 + v^x V}, \quad v^y = \frac{Wv^y}{1 + v^x V}, \quad v^z = \frac{Wv^z}{1 + v^x V}. \]

\[ \rightarrow \]

\[ v^x = \frac{v^x + V}{1 + v^x V}, \quad v^y = \frac{v^y}{W(1 + v^x V)}, \quad v^z = \frac{v^z}{W(1 + v^x V)}. \]

• Page 32, Eq. (1.140). Change

\[ \mathcal{L}_\phi V T = \phi \mathcal{L}_V T, \]

\[ \rightarrow \]

\[ \mathcal{L}_\phi V T = \phi \mathcal{L}_V T - V \mathcal{L}_T \phi, \]

• Page 32, Eq. (1.141). Change

\[ \mathcal{L}_V \phi = V^\nu \partial_\nu \phi = \frac{d\phi}{d\lambda}, \]

\[ \rightarrow \]

\[ \mathcal{L}_V \phi = V^\nu \partial_\nu \phi = \frac{d\phi}{d\lambda}. \]

• Page 33, four lines before Eq. (1.147). Change

...not all bases are such that \( e_\mu \cdot e_\nu \neq \eta_{\mu\nu} \)

\[ \rightarrow \]

...not all bases are such that \( e_\mu \cdot e_\nu = \eta_{\mu\nu} \)

• Page 38, Eq. (1.174). Change

\[ \mathcal{L}_\eta \xi = \mathcal{L}_\xi \eta = 0, \]

\[ \rightarrow \]

\[ \mathcal{L}_\eta \xi = -\mathcal{L}_\xi \eta = 0, \]
• Page 40, Eq. (1.183). Change

\[ S := \int_{\mathcal{P}_1} 2\mathcal{L}d\lambda = \ldots \]

\[ \rightarrow \]

\[ S := \int_{\mathcal{P}_1} \mathcal{L}d\lambda = \ldots \]

• Page 43, Eq. (1.196). Change

\[ \frac{d^2(x^\mu + \xi^\mu)}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}(x + \xi) \frac{d(x^\alpha + \xi^\alpha)}{d\lambda} \frac{d(x^\beta + \xi^\beta)}{d\lambda} = 0. \]

\[ \rightarrow \]

\[ \frac{d^2(x^\mu + \xi^\mu)}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}(x^\mu + \xi^\alpha) \frac{d(x^\alpha + \xi^\alpha)}{d\lambda} \frac{d(x^\beta + \xi^\beta)}{d\lambda} = 0. \]

• Page 44, Eq. (1.202). Change

\[ R_{\alpha[\beta\gamma\delta]} = 2(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0. \]

\[ \rightarrow \]

\[ 3!R_{\alpha[\beta\gamma\delta]} = 2(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0. \]

• Page 45, Eq. (1.207). Change

\[ \Gamma^r_{\theta\theta} = -\sin \theta \cos \theta, \quad \Gamma^\theta_{r\theta} = \cot \theta. \]

\[ \rightarrow \]

\[ \Gamma^\phi_{\theta\phi} = -\sin \theta \cos \theta, \quad \Gamma^\theta_{\phi\phi} = \cot \theta. \]

• Page 45, Eq. (1.208). Change

\[ R^r_{\theta\theta r} = -\frac{1}{R_s^2} g_{\theta\theta} = -\sin^2 \theta, \quad R^r_{\theta\theta r} = \frac{1}{R_s^2} g_{rr} = 1, \]

\[ \rightarrow \]

\[ R^\phi_{\phi\phi \theta} = -\frac{1}{R_s^2} g_{\phi\phi} = -\sin^2 \theta, \quad R^\phi_{\theta\phi \theta} = \frac{1}{R_s^2} g_{\phi\phi} = 1, \]

• Page 45, after Eq. (1.208). Change

while the Ricci scalar is simply \( R = 1/R_s^2 \).

\[ \rightarrow \]

while the Ricci tensor is \( R_{ij} = g_{ij}/R_s^2 \) and the Ricci scalar is simply given by \( R = 2/R_s^2 \).
• Page 48, Eq. (1.220). Change

\[ R_{\mu\nu} = 8\pi \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \frac{1}{4\pi} \Lambda g_{\mu\nu} \right), \]

\[ \rightarrow \]

\[ R_{\mu\nu} = 8\pi \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \frac{1}{8\pi} \Lambda g_{\mu\nu} \right), \]

• Page 49, one line after Eq. (1.223). Change

...the coordinate time runs slower than the proper time.

\[ \rightarrow \]

...the proper time runs slower than the coordinate time.

• Page 51, Eq. (1.229). Change

\[ \frac{d}{d\tau} \left[ \left( 1 - \frac{2M}{r} \right)^{-1} \frac{dr}{d\tau} \right] = r \left[ \left( \frac{d\theta}{d\tau} \right)^2 + \sin^2 \theta \left( \frac{d\phi}{d\tau} \right)^2 \right], \]

\[ \rightarrow \]

\[ \frac{d}{d\tau} \left[ \left( 1 - \frac{2M}{r} \right)^{-1} \frac{dr}{d\tau} \right] = r \left[ \left( \frac{d\theta}{d\tau} \right)^2 + \sin^2 \theta \left( \frac{d\phi}{d\tau} \right)^2 \right] - \frac{M}{r^2} \left[ \left( \frac{dt}{d\tau} \right)^2 + \left( 1 - \frac{2M}{r} \right)^{-2} \left( \frac{dr}{d\tau} \right)^2 \right], \]
2

A Kinetic-Theory Description of Fluids

• Page 77, Eq. (2.30). Change

\[ S(t) := -k_B V H(t) = -k_B V \int f(t, \vec{x}, \vec{u}) \ln(f(t, \vec{x}, \vec{u})) \, d^3u, \]

\[ \rightarrow \]

\[ S(t) := -k_B V H(t) = -k_B V \int f(t, \vec{u}) \ln(f(t, \vec{u})) \, d^3u, \]

• Page 79, after Eq. (2.39). Change

“...through the evolution of the momentum flux \( \rho v_j \) (i.e., the rate of change of linear momentum per unit time and unit area), ...”

\[ \rightarrow \]

“...through the flux of the momentum density tensor \( \rho v_i v_j + P_{ij} \) (i.e., the rate of change per unit time and unit area orthogonal to the \( i \)-th direction of the \( j \)-th component of the linear momentum), ...”

• Page 80, after Eq. (2.42). Change

“...unlike the kinetic energy, \( \frac{1}{2} \rho v^2 \)....”

\[ \rightarrow \]

“...unlike the kinetic energy density, \( \frac{1}{2} \rho v_i v_i \)....”

• Page 81, Eq. (2.47). Change

\[ \epsilon = \frac{3 k_B T}{2 m}, \quad p = \frac{2}{3} n m \epsilon = n k_B T, \]

\[ \rightarrow \]

\[ \epsilon = \frac{3 k_n T}{2 m}, \quad p = \frac{2}{3} n m \epsilon = n k_B T, \]

• Page 82. Change Eq. (2.51)

\[ \langle \vec{u}^2 \rangle = \frac{3k_B T}{m} - \langle \vec{u} \rangle^2 = \frac{3k_B T}{m} - \vec{v}^2, \]

\[ \rightarrow \]

\[ \langle \vec{u}^2 \rangle = \frac{3k_n T}{m} + \langle \vec{u} \rangle^2 = \frac{3k_B T}{m} + \vec{v}^2, \]
• Page 83, Eq. (2.54). Change

\[ f_0(u) = 4\pi nu^2 \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left( -\frac{mu^2}{2k_B T} \right). \]

\[ \rightarrow \]

\[ 4\pi u^2 f_0(u) = 4\pi nu^2 \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left( -\frac{mu^2}{2k_B T} \right). \]

• Page 84, Fig. 2.5. Change the labels on the axes: \( v \rightarrow u \).

• Page 84, Eq. (2.59). Change

\[ T = \frac{mn}{3k_B} \int (\vec{u} - \vec{v})^2 f_0 \, d^3u = \frac{m}{3k_B} \langle (\vec{u} - \vec{v})^2 \rangle. \]

\[ \rightarrow \]

\[ T = \frac{m}{3nk_B} \int (\vec{u} - \vec{v})^2 f_0 \, d^3u = \frac{m}{3k_B} \langle (\vec{u} - \vec{v})^2 \rangle. \]

• Page 84, last paragraph of Sect. 2.2.4. Change

“(Problem 1 is dedicated to showing...)”

\[ \rightarrow \]

“(Problem 4 is dedicated to showing...)”

• Page 86, after Eq. (2.70). Change

“...represents the flux of energy per unit surface and unit time, i.e., the Newtonian energy flux density vector.”

\[ \rightarrow \]

“...represents the flux of energy per unit time and unit area, i.e., the Newtonian energy-density flux vector.”

• Page 90, after Eq. (2.82). Change

“...and recalling that \( p_x' = 0 \) in the local Lorentz rest frame...”

\[ \rightarrow \]

“...and recalling that \( p_{\nu'} = 0 \) in the local Lorentz rest frame...”

• Page 90, last line. Change

“The relativistic Maxwell–Boltzmann equation can then be obtained...”

\[ \rightarrow \]

“The relativistic Boltzmann equation can then be obtained...”

• Page 91, Eq. (2.88). Change

\[ K := \sqrt{(p_1)^\alpha (p_2)_\alpha} - m^4c^4. \]

\[ \rightarrow \]

\[ K := \sqrt{(p_1)^\alpha (p_2)_\alpha} - m^2c^2. \]

• Page 91, after Eq. (2.88). Change

“Note that the collisionless Maxwell-Boltzmann equation, namely (2.86) with...”

\[ \rightarrow \]

“Note that the relativistic collisionless Boltzmann equation, namely (2.86) with...”
• Page 95, second line of Sec. 2.3.4. Change
“...we multiply the relativistic Maxwell–Boltzmann equation (2.86) by...”
→
“...we multiply the relativistic Boltzmann equation (2.86) by...”

• Page 95, after Eq. (2.110). Change
“...can be transformed into a volume integral in momentum space...”
→
“...can be transformed into a surface integral in momentum space...”

• Page 97, second line of Sec. 2.3.6. Change
“...is a solution of the relativistic Maxwell–Boltzmann equation (2.86)....”
→
“...is a solution of the relativistic Boltzmann equation (2.86)....”

• Page 106, Eq. (2.161). Change
\[
c_p - c_v = -T \left( \frac{\partial p}{\partial T} \right)_p / \left( \frac{\partial p}{\partial V} \right)_T > 0 ,
\]
→
\[
c_p - c_v = -T \left( \frac{\partial p}{\partial T} \right)_v / \left( \frac{\partial p}{\partial V} \right)_T > 0 ,
\]

• Page 107, Eq. (2.168). Change
\[
c_s^2 := \left( \frac{\partial p}{\partial e} \right)_s ,
\]
→
\[
c_s^2 := c^2 \left( \frac{\partial p}{\partial e} \right)_s ,
\]

• Page 108, Eq. (2.171). Change
\[
c_s^2 = \frac{1}{k} (c_s^2)_N .
\]
→
\[
c_s^2 = \frac{c^2}{k} (c_s^2)_N .
\]

• Page 108, Eq. (2.175). Change
\[
\mathcal{G} > \frac{3}{2} c_s^2 , \quad \left( \mathcal{G} < \frac{3}{2} c_s^2 \right).
\]
\[ G > \frac{3}{2} c^2, \quad \left( G < \frac{3}{2} c^2 \right). \]

• Page 108, Eq. (2.172) and (2.173). Change

\[ c^2_s = \frac{1}{h} \left( \frac{dp}{d\rho} \right)_s = \frac{1}{h} \left( \frac{d\ln h}{d\ln \rho} \right)_s = \frac{1}{h} \left( \frac{\partial p}{\partial \rho} \right)_s + \frac{d\epsilon}{d\rho} \left( \frac{\partial p}{\partial \epsilon} \right)_\rho. \]

• Page 116, Eq. (2.232). Change

\[ c^2_s = \frac{p(5\rho_h - 8p)}{3\rho_h(\rho_h - p)}. \]

• Page 117, Eq. (2.234). Change

\[ c^2_s = \frac{\gamma(\gamma - 1)}{c^2 + \gamma \epsilon} = \frac{1}{h} \left( \frac{h - c^2}{h} \right) (\gamma - 1) = \frac{\gamma p}{\rho_h}. \]

“A fluid obeying the ideal-fluid equation of state with \( \epsilon = 0 \) would also have a zero temperature and could provide a reasonable model for a cold and old neutron star.”

“A fluid obeying a general polytropic equation of state can have, at least mathematically, \( \epsilon = 0 \), although such a choice would be difficult to justify from a physical point of view. However, if the polytropic transformation is isentropic, then the specific internal energy is fully determined and is proportional to the rest-mass density [cf. Eq. (2.248) and discussion around it]. A polytropic and isentropic equation of state is often used to obtain a reasonable approximation of the description of matter of a cold and old neutron star.
Put differently, a polytropic equation of state is equivalent to an ideal-fluid equation of state only under those isentropic transformations for which the adiabatic index of the fluid $\gamma$ is the same as the adiabatic index of the polytrope $\Gamma$.

“Put differently, if a fluid obeys the ideal-fluid equation of state and is isentropic, then its equation of state can also be written in a polytropic form [cf., Eq. (2.242)], with polytropic exponent $\Gamma = \gamma$; in this case, the polytropic exponent is also the adiabatic index. On the other hand, if a fluid obeys the polytropic equation of state and is isentropic, then it is at least formally possible to express the pressure as $p = \rho \epsilon (\Gamma - 1)$ [cf., Eq. (2.228)]. However, this does not necessarily mean that such a fluid obeys an ideal-fluid equation of state. For this to be the case, $\Gamma$ must be the ratio of the specific heats $\frac{c_p}{c_v}$ and the specific internal energy must be a function of the temperature only.”

Page 119, Eq. (2.249). Change

$\frac{c_s^2}{\rho h} = \frac{\Gamma p}{\rho (\Gamma - 1) + \Gamma p} = \left( \frac{1}{\Gamma K \rho^2 - 1} + \frac{1}{\Gamma - 1} \right)^{-1}.$

$\Rightarrow$

$c_s^2 = c_s^2 \frac{\Gamma p}{\rho h} = c_s^2 \frac{\Gamma (\Gamma - 1) p}{\rho (\Gamma - 1) + \Gamma p} = c_s^2 \left( \frac{1}{\Gamma K \rho^2 - 1} + \frac{1}{\Gamma - 1} \right)^{-1}.$
3

Relativistic Perfect Fluids

- Page 139, before Eq. (3.28). Change The simplest quantity to determine is the *rest-mass density current*, namely → The simplest quantity to determine is the *rest-mass current*, namely.

- Page 139, before Eq. (3.28). Change

$$J^{\mu} : \text{flux of rest-mass current density in the } \hat{\mu}\text{-direction},$$

→

$$J^{\mu} : \text{flux of rest-mass in the } \hat{\mu}\text{-direction},$$

- Page 139, before Eq. (3.29). Change

$$T^{\hat{0}\hat{0}} : \text{total energy density,}$$

$$T^{\hat{i}\hat{0}} : \text{flux of energy density in } \hat{i}\text{-th direction,}$$

$$T^{\hat{i}\hat{0}} : \text{flux of } \hat{i}\text{-momentum in } \hat{0}\text{-th direction (} \hat{i}\text{-momentum density),}$$

$$T^{\hat{j}\hat{i}} : \text{flux of } \hat{j}\text{-component of momentum density in } \hat{i}\text{-th direction.}$$

→

$$T^{\hat{0}\hat{0}} : \text{total energy density,}$$

$$T^{\hat{i}\hat{0}} : \text{flux of energy in } \hat{i}\text{-th direction,}$$

$$T^{\hat{i}\hat{0}} : \text{flux of } \hat{i}\text{-momentum in } \hat{0}\text{-th direction,}$$

$$T^{\hat{j}\hat{i}} : \text{flux of } \hat{j}\text{-momentum in } \hat{i}\text{-th direction.}$$

- Page 139, (3.29). Change the second line as follows:

$$T^{\hat{0}\hat{i}} = T^{\hat{i}\hat{0}} = 0,$$

→

$$T^{\hat{0}\hat{i}} = T^{\hat{i}\hat{0}} = 0,$$

- Page 146, Eq. (3.69). Change

$$\mathcal{L}_u(h u_\mu) = -\frac{1}{\rho} \nabla_\mu p - \nabla_\mu h.$$  

→

$$\mathcal{L}_u(h u_\mu) = -\frac{1}{\rho} \nabla_\mu p - \nabla_\mu h.$$
• Page 146, Eq. (3.71). Change
\[ \mathcal{L}_u(hu_\mu \xi^\mu) = -\frac{\xi^\mu \nabla_\mu p}{\rho} - \xi^\mu \nabla_\mu h = -\frac{1}{\rho} \mathcal{L}_\xi p - \mathcal{L}_\xi h , \]
→
\[ \mathcal{L}_u(hu_\mu \xi^\mu) = -\frac{\xi^\mu \nabla_\mu p}{\rho} = -\frac{1}{\rho} \mathcal{L}_\xi p = -\mathcal{L}_\xi h , \]

• Page 146, before Eq. (3.72). Change
"and thus use the condition (3.65) with \( \mathcal{L}_u p = 0 \) and \( \mathcal{L}_u h = 0 \), to finally obtain"
→
"and thus use the condition (3.65) with \( \mathcal{L}_\xi p = 0 \) to finally obtain"

• Page 146, after Eq. (3.72). Change
Note the similarity between expression (3.72) and the corresponding equation (1.185) along geodesic trajectories, \( \text{i.e.}, \mathcal{L}_u(u_\mu \xi^\mu) \).
→
Note the similarity between expression (3.72) and the corresponding equation (1.185) along geodesic trajectories, \( \text{i.e.}, \mathcal{L}_u(u_\mu \xi^\mu) = 0 \).

• Page 155, caption of Fig. 3.4. Change
... Show with blue solid lines
→
... Shown with blue solid lines

• Page 177, Eq. (3.253). Change
\[ P_{\alpha\beta}^R := \int I^\nu N^\alpha N^\beta d\nu d\Omega . \]
→
\[ P_{\alpha\beta}^R := h^\alpha_\gamma h^\beta_\delta I^R_{\gamma\delta} = h^\alpha_\gamma h^\beta_\delta \int I^\nu N^\gamma N^\delta d\nu d\Omega . \]

• Page 179, footnote 26. Change
“Multifluids of this type as sometimes also referred to as...”
→
“Multifluids of this type are sometimes also referred to as...”

• Page 190, Exercise 1. should read:
Use the definitions (3.11)–(3.13) to show that
\[ \sigma_{\alpha\beta} \sigma^{\alpha\beta} = \frac{1}{2} \left( \nabla_\mu a^\mu - 2u_\nu \nabla_\mu \nabla^\nu (a_\mu) + a^2 - \frac{2}{3} \Theta^2 \right) , \quad (3.1) \]
\begin{equation}
\omega_\alpha \omega^\alpha \omega^\beta = \frac{1}{2} \left( a^2 - \nabla_\mu a^\mu - 2 u_\nu \nabla_\mu \nabla^\nu u^\mu \right),
\tag{3.2}
\end{equation}

thus concluding that the term \(2(\omega^2 - \sigma^2)\) entering the Raychaudhuri equation (3.27) can be written as

\begin{equation}
2(\omega^2 - \sigma^2) = \frac{1}{3} \Theta^2 - \nabla_\mu a^\mu + u_\nu \nabla_\mu \nabla^\nu u^\mu.
\tag{3.3}
\end{equation}
4

Linear and Nonlinear Hydrodynamic Waves

- Page 200, Eq. (4.54). Change
  \[ \det(A^t - \lambda A^x) = 0, \]
  \[ \rightarrow \]
  \[ \det(A^x - \lambda A^t) = 0, \]
- Page 213, Fig. 4.5. The tangents to the fluidlines on either side of the rarefaction tail should be exactly the same and not as shown.
- Page 216, below Eq. (4.114). Change
  “It is also convenient to rewrite the continuity equation (4.112) and the conservation of energy (4.113)”
  \[ \rightarrow \]
  “It is also convenient to rewrite the continuity equation (4.112) and the conservation of momentum (4.113)”
- Page 216. Change Eq. (4.117) as follows
  \[ (h_b W_b v_b)^2 - (h_a W_a v_a)^2 = \frac{h_a}{\rho_a} + \frac{h_b}{\rho_b} [p]. \]
  \[ \rightarrow \]
  \[ (h_b W_b v_b)^2 - (h_a W_a v_a)^2 = \frac{h_a}{\rho_a} + \frac{h_b}{\rho_b} [p]. \]
- Page 217, first paragraph. Change
  “The classical Hugoniot adiabat is readily obtained from (4.118) after recalling that in the Newtonian limit \( h_N = 1 + \epsilon + p/\rho \approx 1 \)”
  \[ \rightarrow \]
  “The classical Hugoniot adiabat is readily obtained from (4.118) after recalling that in the Newtonian limit \( h = 1 + \epsilon + p/\rho \approx 1 \)”
- Page 221, Eq. (4.138). Change
  \[ W_{ab}^2 = \frac{(3e_a + e_b)(3e_b + e_a)}{16e_1e_2} = \frac{4}{9} W_a^2 W_b^2, \]
  \[ \rightarrow \]
  \[ W_{ab}^2 = \frac{(3e_a + e_b)(3e_b + e_a)}{16e_a e_b} = \frac{4}{9} W_a^2 W_b^2, \]
• Page 222, after Eq. (4.141), the expression for the shock velocity should be modified as follows:

\[ V^\pm_s = \frac{\rho_b W_b v_b}{\rho_b W_b \pm \rho_a} \]

→

\[ V^\pm_s = \frac{\rho_b W_b v_b}{\rho_b W_b \mp \rho_a} \]

• Page 225, Fig. 4.11, panel on bottom right. The labels in the spacetime diagram should be corrected as follows:

\( R^+ \rightarrow R^- \) and \( P^+ \rightarrow P^- \).

• Page 228, last sentence in the first paragraph should be modified as follows:

These values mark the transition from one wave pattern to another one, and that are directly computed from the initial conditions (Rezzolla and Zanotti, 2001).

→

These values mark the transition from one wave pattern to another one, and are directly computed from the initial conditions (Rezzolla and Zanotti, 2001).
5

Reaction Fronts: Detonations and Deflagrations

- Page 284, problem 2. Change
  “Derive the inequalities (5.4)–(5.6) across a reaction front [Hint: start from the laws of conservation of momentum and energy (4.114)–(4.113)].
  →
  “Derive the inequalities (5.4)–(5.6) across a reaction front [Hint: start from the laws of conservation of momentum and energy (4.113)–(4.114)].
Relativistic Non-Perfect Fluids

- Page 297, Eq. (6.73). Change: $\chi \rightarrow \chi t$
- Page 301, Eq. (6.85). Change:

$$q_\nu = -\kappa \left[ D_\nu \ln T + a_\nu \beta_1 \dot{q}_\nu + \frac{1}{2} T \nabla_\mu \left( \frac{\beta_1}{T} u^\mu \right) q_\nu \right] ,$$

\rightarrow

$$q_\nu = -\kappa T \left[ D_\nu \ln T + a_\nu \beta_1 \dot{q}_\nu + \frac{1}{2} T \nabla_\mu \left( \frac{\beta_1}{T} u^\mu \right) q_\nu \right] ,$$

- Page 306: invert the inequalities in Eqs. (6.115) and (6.116).
7
Formulations of the Einstein–Euler Equations

- Page 321, below Eq. (7.9). Change “into a timelike component $n \alpha$” → “into a timelike component $\alpha n$”.
- Page 337, Eq. (7.100). Change

$$\tilde{\Gamma}^i_{jk} = \Gamma^i_{jk} - \frac{1}{3}(\delta^i_j \Gamma^m_{km} + \delta^i_k \Gamma^m_{jm} - \gamma_{jk} \gamma^{il} \Gamma^m_{lm}) = \Gamma^i_{jk} + 2(\delta^i_j \partial_k \ln \phi + \delta^i_k \partial_j \ln \phi - \gamma_{jk} \gamma^{il} \partial_l \ln \phi),$$

→

$$\tilde{\Gamma}^i_{jk} = \Gamma^i_{jk} - \frac{1}{3}(\delta^i_j \Gamma^m_{km} + \delta^i_k \Gamma^m_{jm} - \gamma_{jk} \gamma^{il} \Gamma^m_{lm}) = \Gamma^i_{jk} + \delta^i_j \partial_k \ln \phi + \delta^i_k \partial_j \ln \phi - \gamma_{jk} \gamma^{il} \partial_l \ln \phi.$$  

- Page 339, Eq. (7.108). Change

$$(3) R + K^2 = K^{ij} K_{ij} + 4\pi E = \tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} K^2 + 4\pi E,$$

→

$$(3) R + K^2 = K^{ij} K_{ij} + 4\pi E = \tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} K^2 + 16\pi E,$$

- Page 339, last two lines:
  (note that $\tilde{\gamma}_{ij}$ and $\tilde{A}_{ij}$, have only five independent components each since they are traceless)

→

(note that $\tilde{\gamma}_{ij}$ and $\tilde{A}_{ij}$, have only five independent components each since they have traces that are equal to three or zero, respectively)

- Page 342, correct sign in third term of Eq. (7.113). Change

$$R_{\mu\nu} + 2\nabla_{(\mu} Z_{\nu)} + \kappa_1 [2n_{(\mu} Z_{\nu)} - (1 + \kappa_2) g_{\mu\nu} n_{(\alpha} Z^{\alpha)}] = 8\pi \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right),$$

→

$$R_{\mu\nu} + 2\nabla_{(\mu} Z_{\nu)} + \kappa_1 [2n_{(\mu} Z_{\nu)} + (1 + \kappa_2) g_{\mu\nu} n_{\alpha} Z^{\alpha}] = 8\pi \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right),$$
**Formulations of the Einstein--Euler Equations**

- Page 343, first term on the right-hand-side of Eq. (7.116). Change
  \[ D_j \alpha_j \]
  \[ \rightarrow \]
  \[ D_i D_j \alpha_j \]

- Page 345, first term in Eq (7.131). Change
  \[ D_j \Theta_{ij} = 0 \]
  \[ \rightarrow \]
  \[ D_i \Theta_{ij} = 0 \]

- Page 345, Eq (7.132). Change
  \[ \Sigma_{ij} := \Theta_{ij} - \frac{1}{3} \gamma_{ij} \Theta^{kl} \gamma^{kl} = \frac{1}{2} \gamma^{1/3} L \tilde{\gamma}_{ij} \]
  \[ \rightarrow \]
  \[ \Sigma_{ij} := \Theta_{ij} - \frac{1}{3} \gamma_{ij} \Theta^{kl} \gamma^{kl} = \frac{1}{2} \gamma^{1/3} L \tilde{\gamma}_{ij} \]

- Page 345, after Eq (7.132). Change
  where \( \tilde{\gamma}_{ij} := \gamma^{-1/3} \tilde{\gamma}_{ij} \) is the conformal metric.
  \[ \rightarrow \]
  where \( \tilde{\gamma}_{ij} := \gamma^{-1/3} \tilde{\gamma}_{ij} \) is the conformal metric.

- Page 345, first term in Eq (7.133). Change
  \[ D_j \Sigma_{ij} = 0 \]
  \[ \rightarrow \]
  \[ D_i \Sigma_{ij} = 0 \]

- Page 354, first line. Change
  “constraint decouples from the Hamiltonian constraint and it is possible to solve the latter to obtain the three vectors \( \bar{V}^{\alpha} \)”
  \[ \rightarrow \]
  “constraint decouples from the Hamiltonian constraint and it is possible to solve the former to obtain the three vectors \( \bar{V}^{\alpha} \)”

- Page 354, fourth line. Change
  “The calculation of initial data via the solution of the constrains simplifies considerably if”
  \[ \rightarrow \]
  “The calculation of initial data via the solution of the constraints simplifies considerably if”

- Page 356, first sentence before Eq. (7.180). Change
  “we further introduce the conformal metric \( \tilde{\gamma} \) [cf., Eq. (7.152)], such that”
  \[ \rightarrow \]
  “we further introduce the conformal metric \( \tilde{\gamma} \) [cf., Eq. (7.152), although we here use a tilde rather than a bar to be closer to the notation of Bonazzola et al. (2004)], such that”
• Page 356, Eq. (7.180). Change

\[ f := \det(f_{ij}) = \tilde{\gamma} := \det(\tilde{\gamma}_{ij}), \]

\[ \rightarrow \]

\[ f := \det(f_{ij}) = \hat{\gamma} := \det(\hat{\gamma}_{ij}), \]

• Page 356, Eq. (7.181). Change

\[ \psi = (\gamma/\tilde{\gamma})^{1/12} = (\gamma/f)^{1/12}. \]

\[ \rightarrow \]

\[ \psi = (\gamma/\hat{\gamma})^{1/12} = (\gamma/f)^{1/12}. \]

• Page 384, second equation in Exercise 7. Change

\[ D_i D_j \phi = -\frac{1}{\phi} \tilde{D}_i \tilde{D}_j \phi + \frac{1}{\phi^2} \partial_i \phi \partial_j \phi. \]

\[ \rightarrow \]

\[ D_i D_j \phi = \tilde{D}_i \tilde{D}_j \phi + \frac{2}{\phi} \partial_i \phi \partial_j \phi - \frac{1}{\phi} \gamma_{ij} \partial_k \phi \partial_k \phi. \]
• Page 393, Eq. (8.16). Change

\[ \epsilon_j^{(h)} = \tilde{C} h^{p_j} + O(h^{p_j+1}) , \]

\[ \rightarrow \]

\[ \epsilon_j^{(h)} = C h^{p_j} + O(h^{p_j+1}) , \]

• Page 393, below Eq. (8.16). Change “with \( \tilde{C} \) a constant” to “with \( C \) a constant”.

• Page 393, Eq. (8.30). Change

\[ \tilde{p} := \frac{\log R(h, k)}{\log(h/k)} , \]

\[ \rightarrow \]

\[ \tilde{p} := \frac{\log |R(h, k)|}{\log(h/k)} , \]

• Page 395, the 7th line before Eq. (8.37). Change

“...its application across a time interval \( \Delta t \) introduces an associate truncation error \( \epsilon_j(h) \).”

\[ \rightarrow \]

“...its application across a time interval \( \Delta t \) introduces an associated truncation error \( \epsilon(h) \).”

• Page 406, Eq. (8.87). Change

\[ \tilde{u}(x, t) = e^{-\epsilon k^2 t} e^{i k [x - (\nu + \beta k^2) t]} , \]

\[ \rightarrow \]

\[ \tilde{u}(x, t) = e^{-\epsilon k^2 t} e^{i k [x - (\lambda + \beta k^2) t]} , \]
• Page 464, Eq. (10.14). Change

\[ A_{ik} := \int_0^1 \Psi_k(\xi) d\xi, \quad \forall I_i \in S_j. \]

→

\[ A_{ik} := \int_{I_i} \Psi_k(\xi) d\xi, \quad \forall I_i \in S_j. \]
11

Relativistic Hydrodynamics of Non-Selfgravitating Fluids

- Page 518, Eq. (11.84). Change

\[
\frac{dW}{\mathcal{W}} = \frac{M}{r^2 \mathcal{W}} \, dr + \frac{u}{\mathcal{W}} \, du.
\]

\[\rightarrow\]

\[
\frac{d\mathcal{W}}{\mathcal{W}} = \frac{M}{r^2 \mathcal{W}^2} \, dr + \frac{u}{\mathcal{W}^2} \, du.
\]
Relativistic Hydrodynamics of Selfgravitating Fluids

- Page 596, 12th line after Eq. (12.13)
  \[ M = 2.01 \pm 0.4 M_\odot \]  \rightarrow  \[ M = 2.01 \pm 0.04 M_\odot \]

- Page 597, caption of Fig. 12.1
  \[ M = 2.01 \pm 0.4 M_\odot \]  \rightarrow  \[ M = 2.01 \pm 0.04 M_\odot \]

- Page 601, second term in Eq. (12.31). Change
  \[ H_0^2 = \frac{1}{R_i^2} \left[ 1 - \frac{\varepsilon(1 + w_i)^2}{w_i} \right] . \]
  \[ \rightarrow \]
  \[ H_0^2 = \frac{1}{R_i^2} \left[ 1 - \frac{\varepsilon(1 + w_i)^2}{w_i} \right] . \]

- Page 601, second paragraph, change:
  ... the energy density \( \rho(r) \) and the pressure \( p(r) \) ...
  \[ \rightarrow \]
  ... the energy density \( e(r) \) and the pressure \( p(r) \) ...

- Page 606, caption of Fig. 12.4:
  \[ K = 100 \]  \rightarrow  \[ K = 164 \]

- Page 606, second but last line:
  \[ K = 100 \]  \rightarrow  \[ K = 164 \]