Modelling the most catastrophic events in the universe: a journey into Einstein's theory of gravity

Luciano Rezzolla

Institute for Theoretical Physics, Frankfurt, Germany Frankfurt Institute for Advanced Studies, Frankfurt, Germany





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Plan of the talk

* a brief introduction of gravity
* Einstein's view of gravity
* black holes, neutron stars, and gravitational waves
* numerical relativity
* simulating catastrophic events

Our experience of gravity Our experience of gravity

* Instinctive notion



Our experience of gravity

* Instinctive notion

* Intuitive notion



Our experience of gravity

* Instinctive notion

* Intuitive notion

* Imaginative notion

DON'T LET GO

FROM DIRECTOR

GRAVITY

SANDRA BULLOCK GEORGE CLOONEY

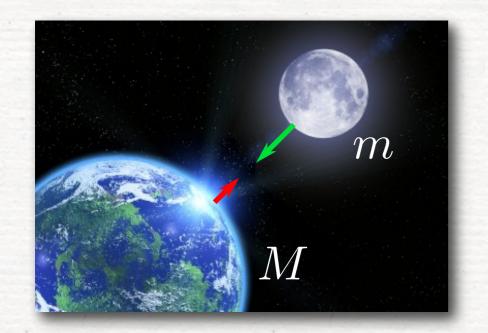
FIRSTSHOWINGNET

The fathers of gravity

In **1679** Newton publishes his theory of gravitation.

Gravity is an instantaneous **force** between two masses proportional to the masses and inversely proportional to the square of the distance.





 $\vec{F} = -\frac{G}{c^2} \frac{Mm}{r^2} \vec{e_r}$

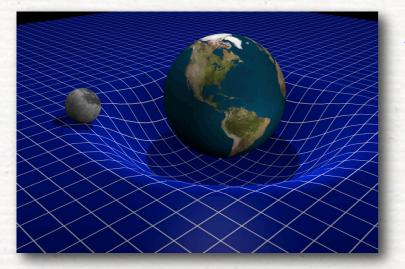
With this theory he could explain essentially **all astronomical** observations of his time.

The fathers of gravity

In **1915** Einstein publishes his theory of gravitation (Allgemeine Relativitätstheorie) changing our understanding of gravity.

According to Einstein, gravity is the manifestation of spacetime **curvature**.





Any form of mass/energy curves the spacetime.

Implications of this view are: black holes, neutron stars, gravitational waves.

$G_{\mu\nu} = 8\pi T_{\mu\nu}$

Einstein tensor

stress-energy tensor

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spacetime curvature mass and energy in the spacetime

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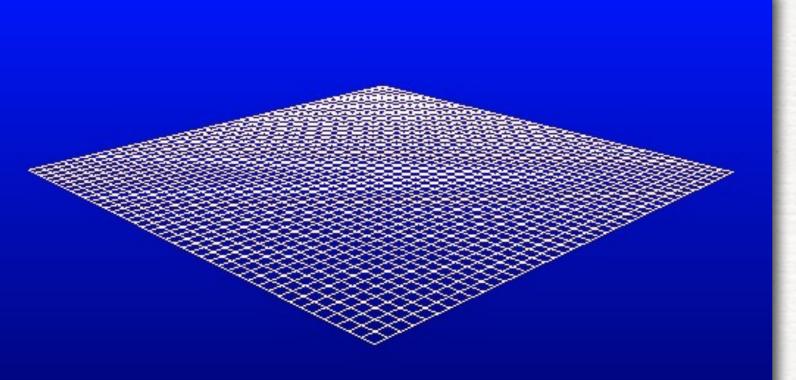
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spacetime curvature mass and energy in the spacetime

There is a relation between the curvature and mass/energy. gravity is the manifestation of spacetime curvature

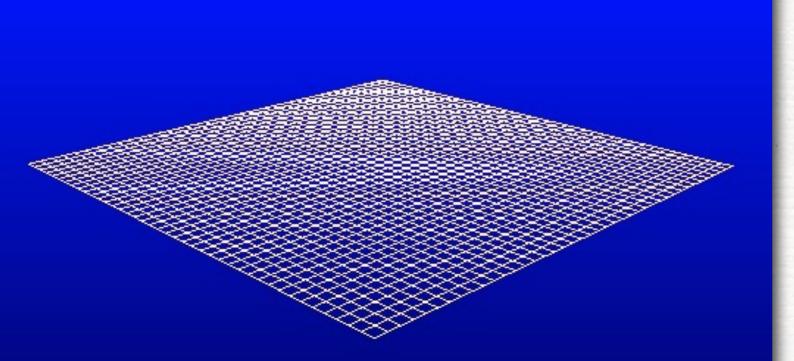
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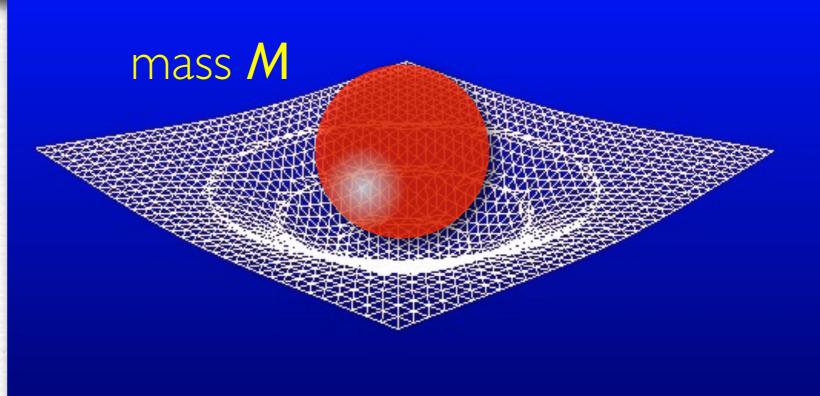
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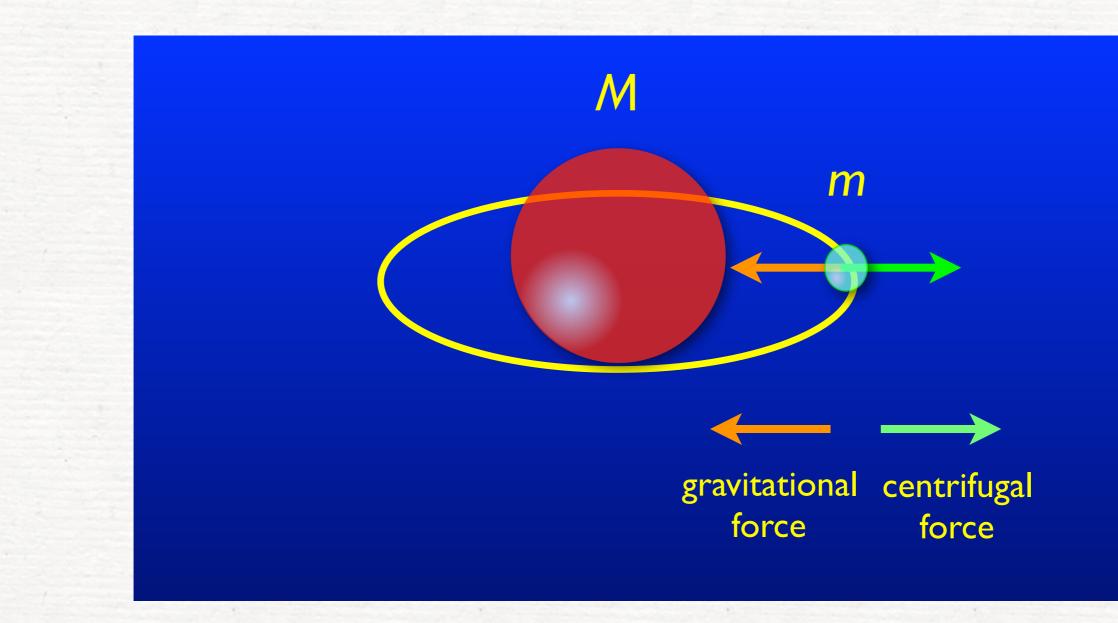
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If instead it contains a mass *M*, it will also have a nonzero curvature and will therefore be a *curved spacetime*



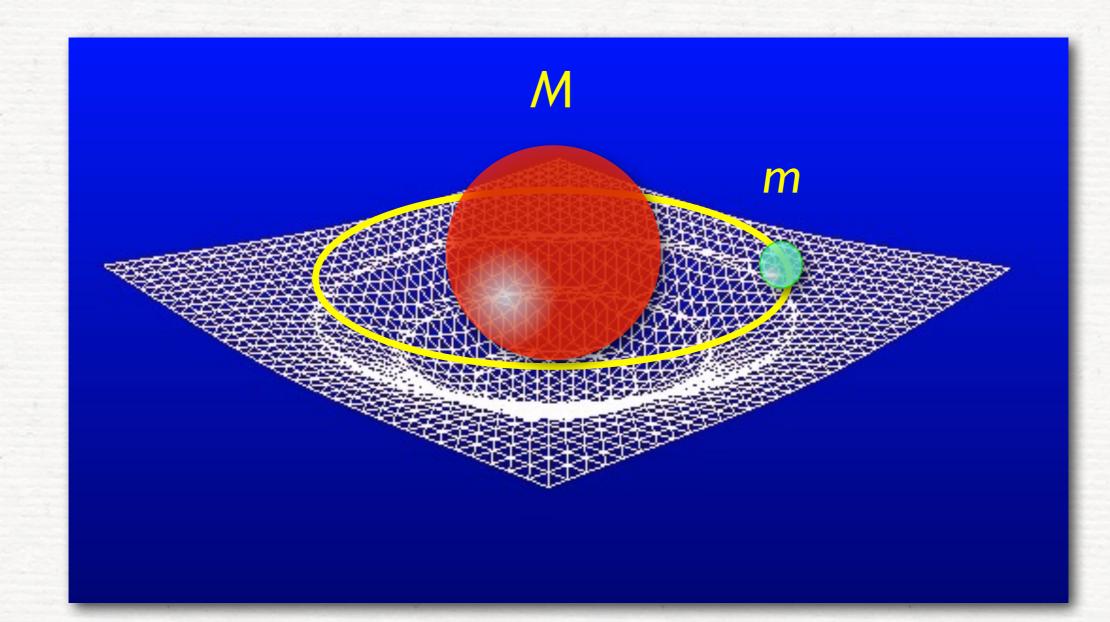
Gravity à la Newton

Let's consider the orbital motion of an object of small mass *m* around an object of large mass *M*: (e.g. Earth around the Sun) *Newton*: the orbit is the result of the balance between the gravitational force and the centrifugal one



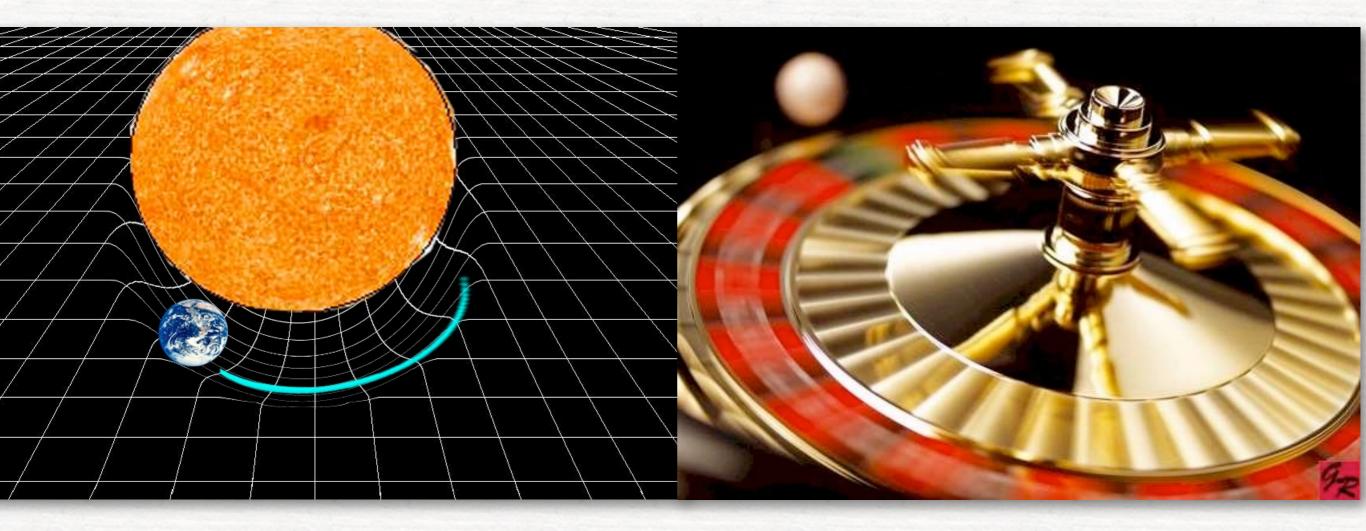
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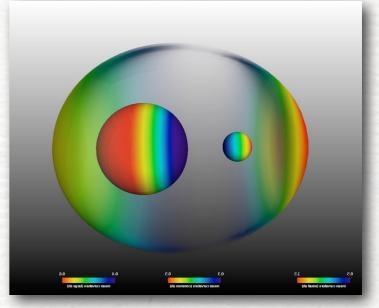
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 $\frac{M_{\oplus}}{R_{\oplus}} \simeq \frac{5.97 \times 10^{24} \text{ kg}}{6372 \text{ km}} \simeq (3 \times 10^{-9}) \simeq 0.0000000003$ In our neighbourhood, largest curvature is in the Sun $\frac{M_{\odot}}{R_{\odot}} \simeq \frac{1.98 \times 10^{30} \text{ kg}}{6.95 \times 10^5 \text{ km}} \simeq (2 \times 10^{-6}) \simeq 0.000002$ In other words: spacetime is **very hard to curve!**

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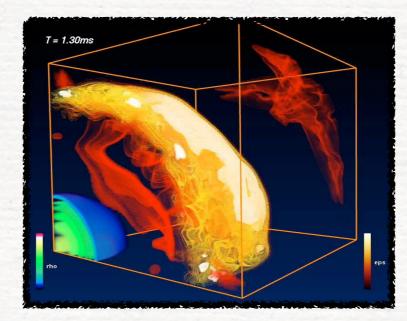
Black holes

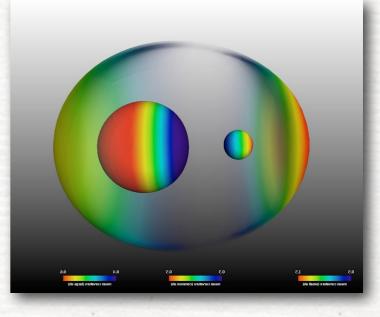


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Black holes

Neutron Stars

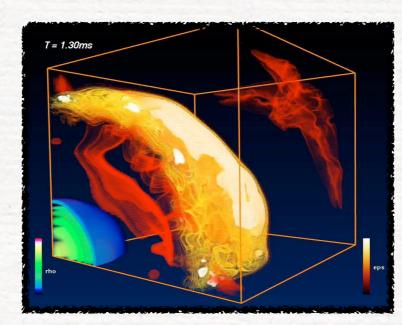


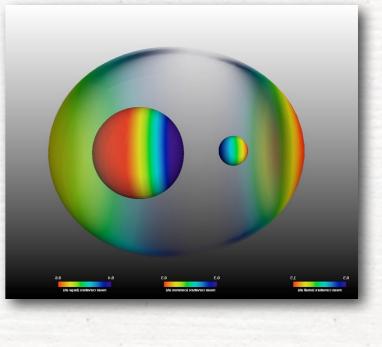


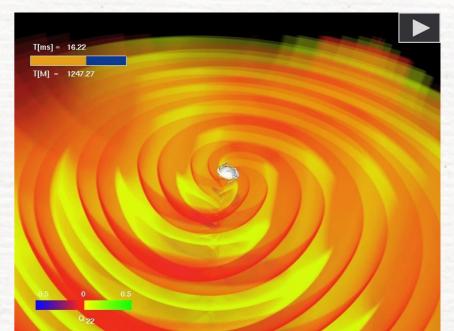
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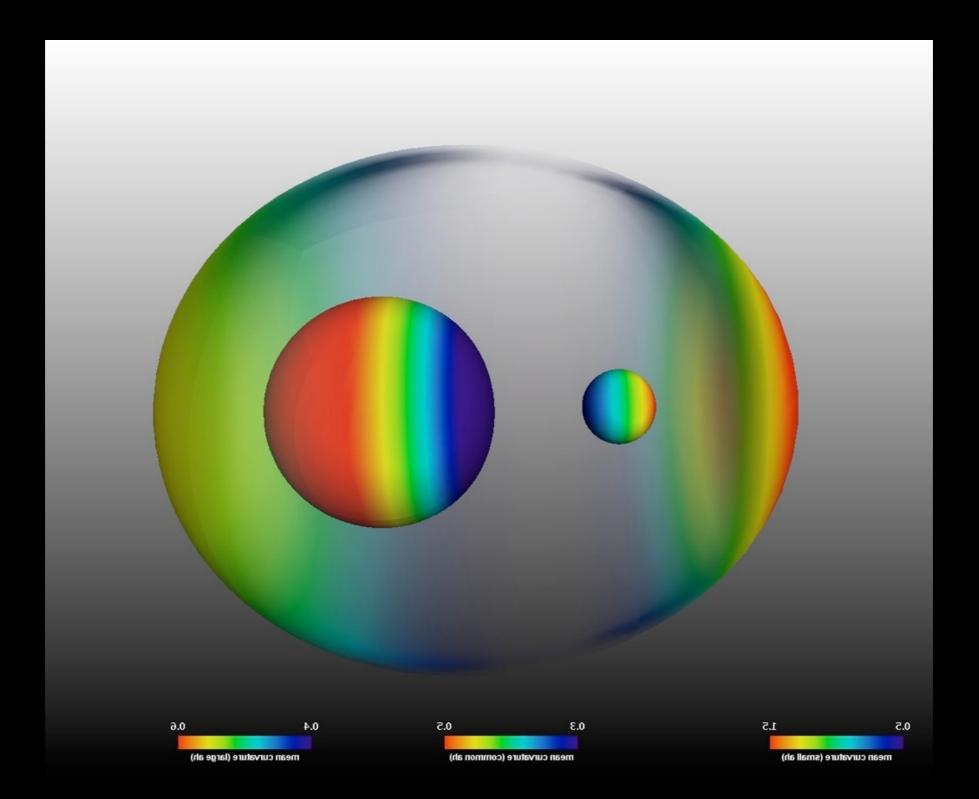


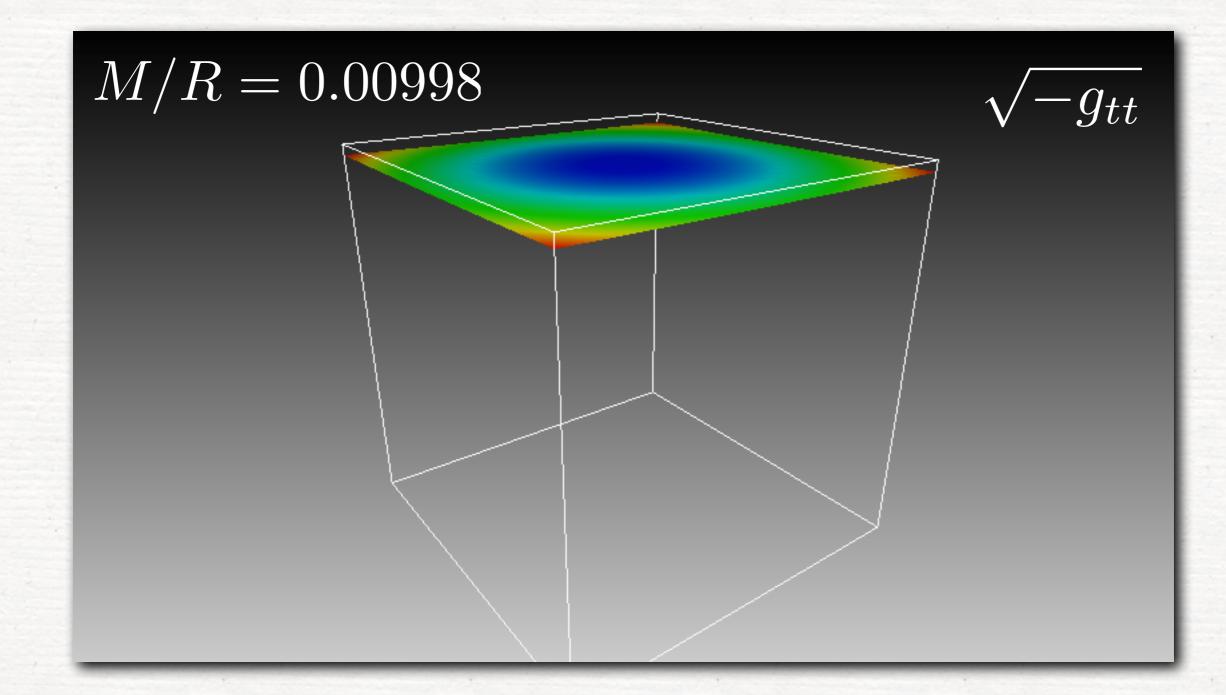


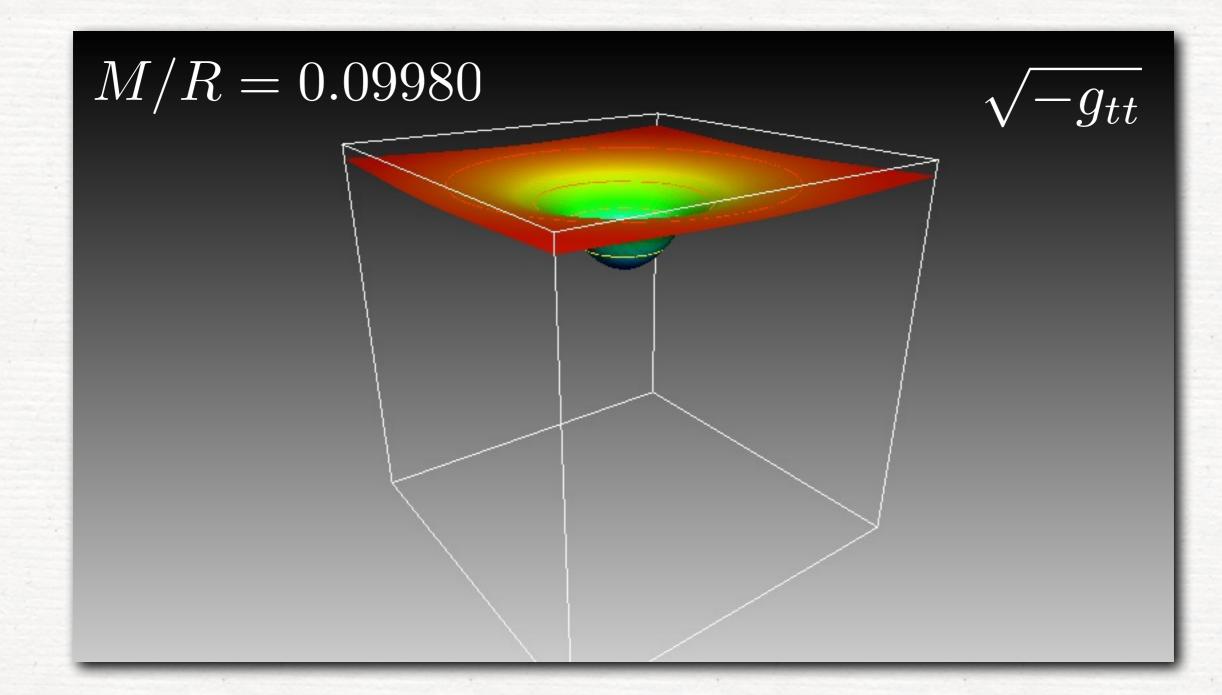


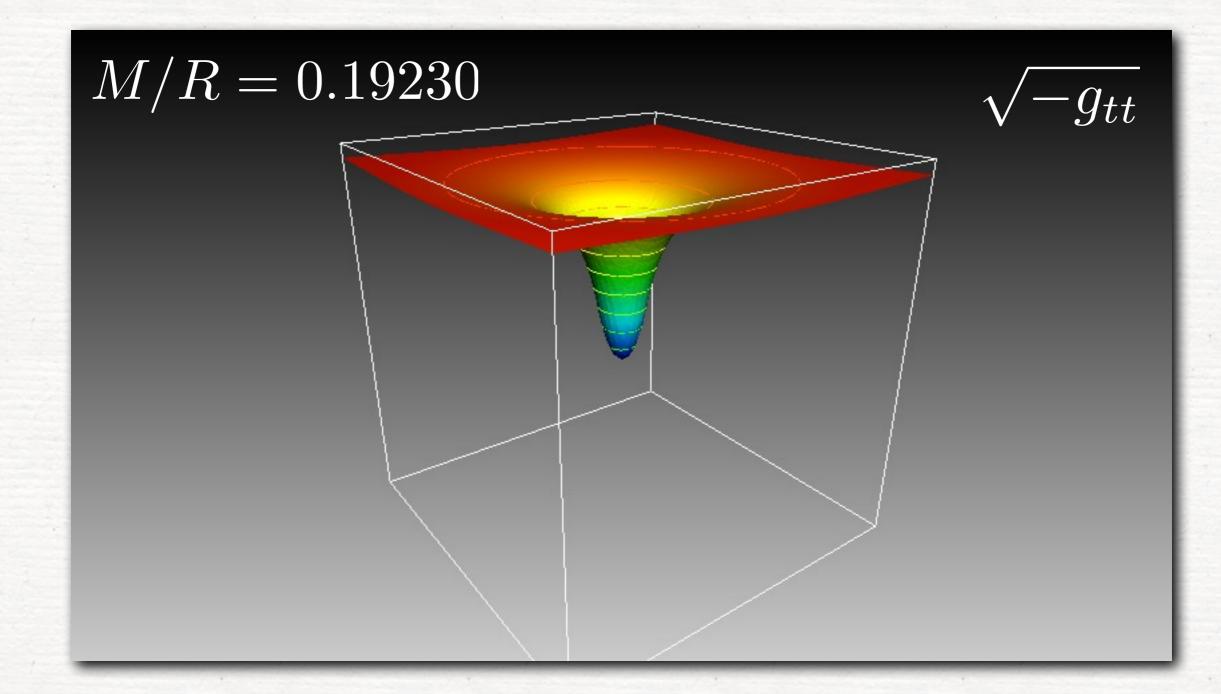
Gravitational Waves

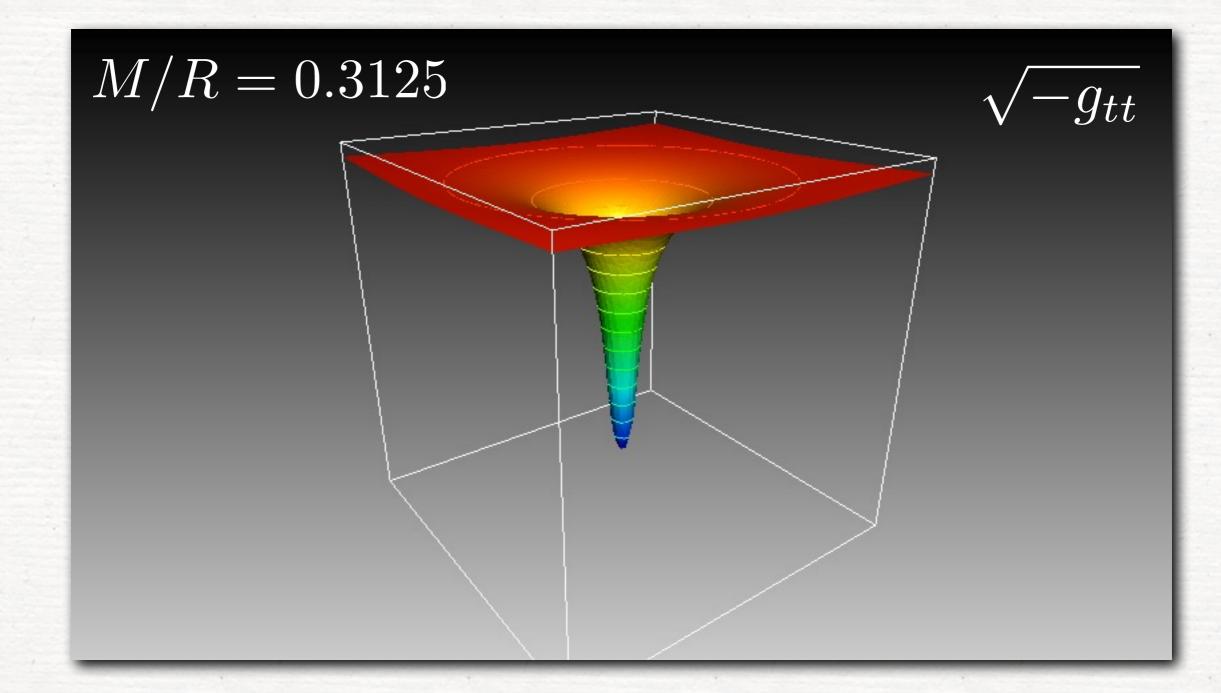
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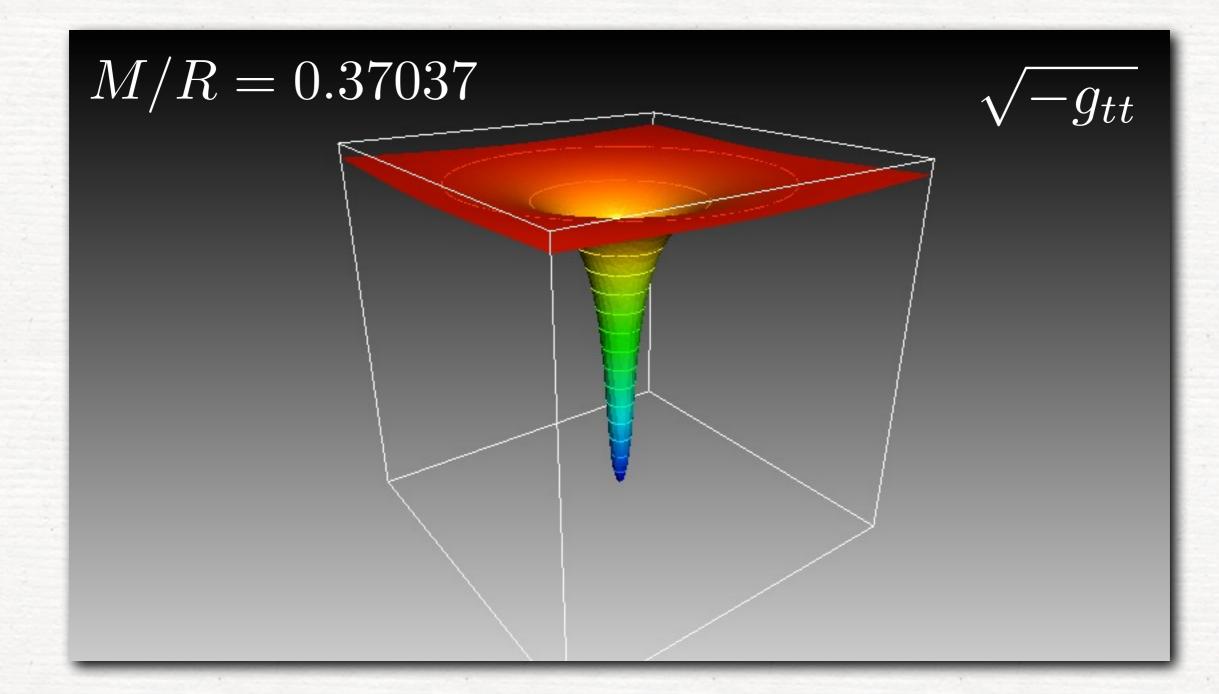


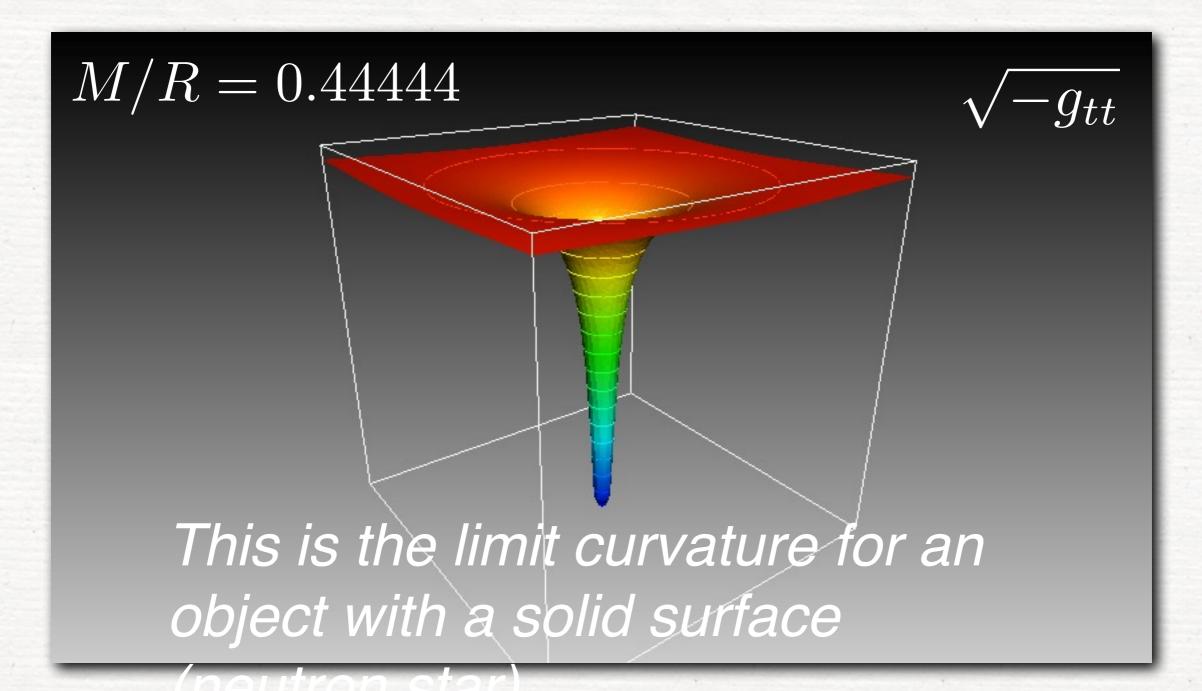


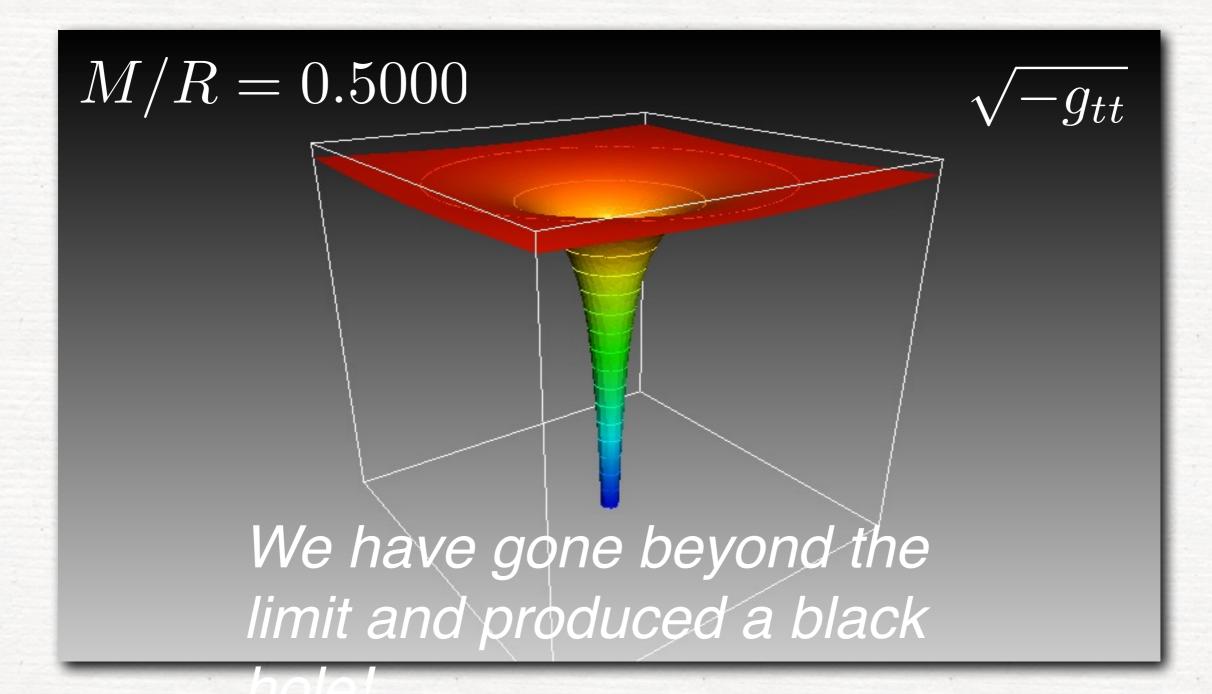






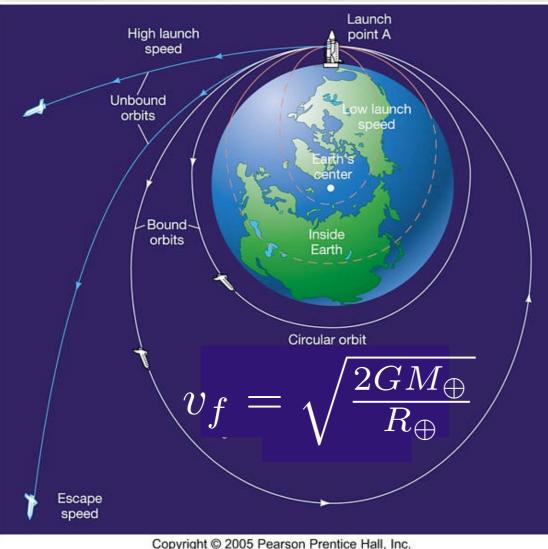






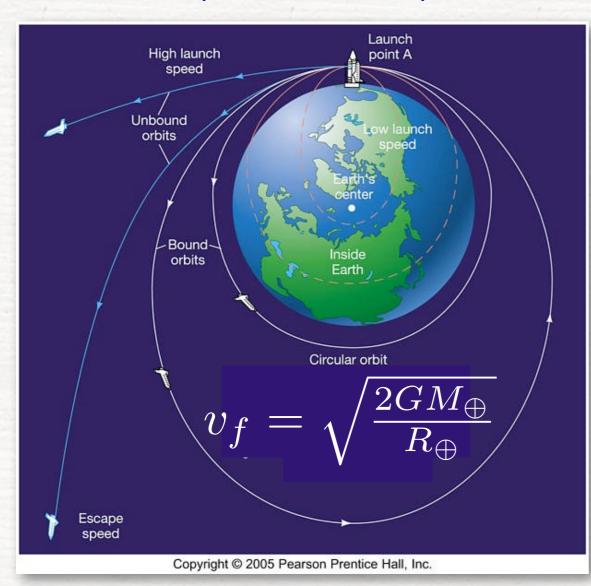
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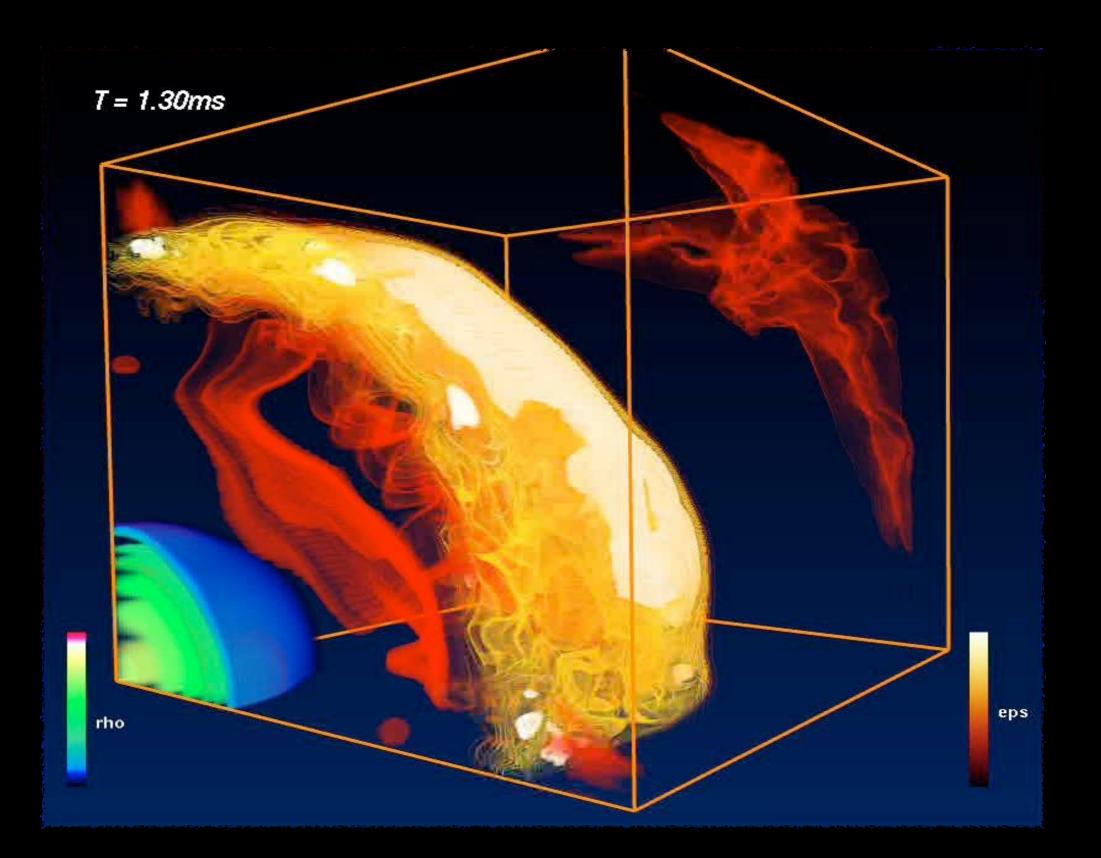
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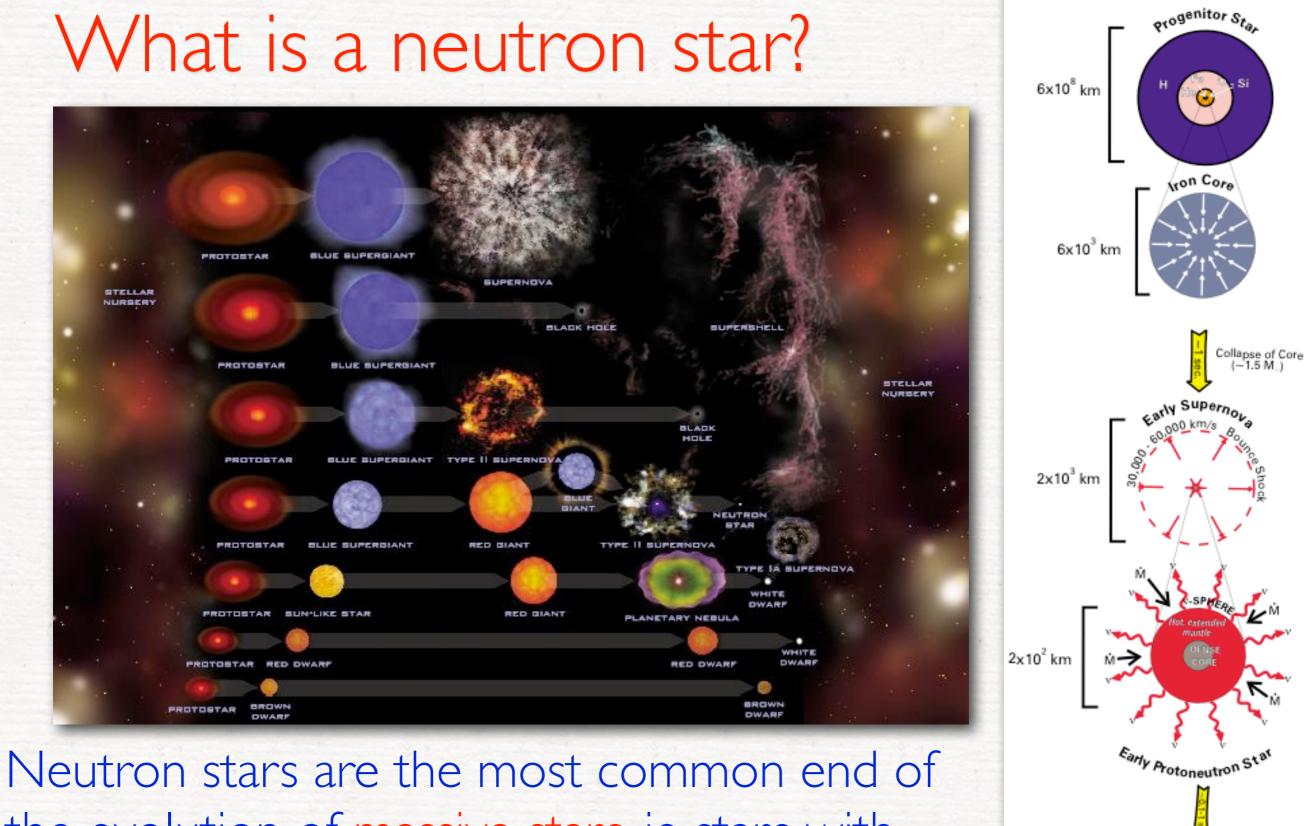


 $M_{\rm BH}$: black hole mass $R_{\rm EH}$: radius of event horizon

It's **impossible** to escape the black hole surface even for light

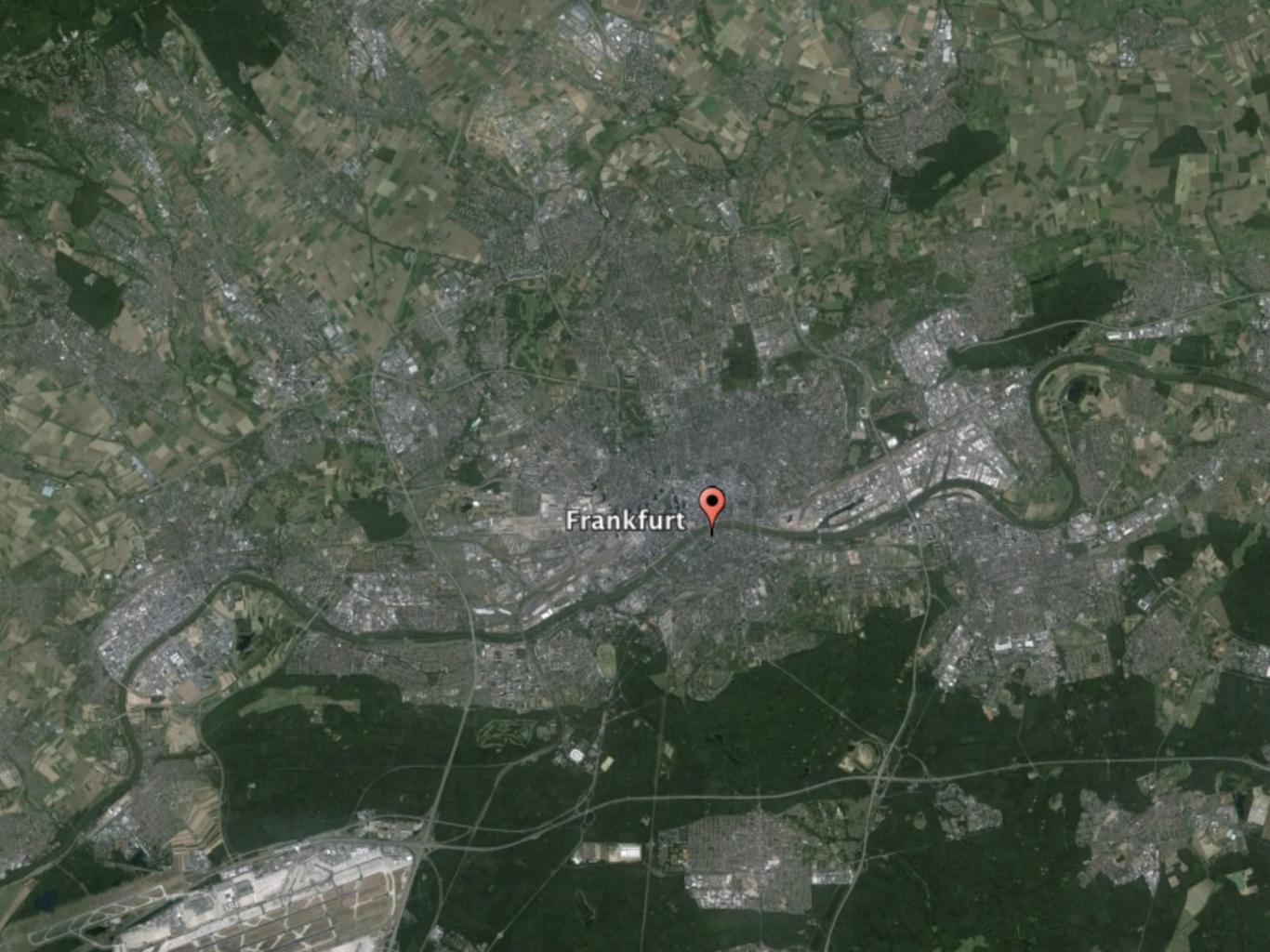
Neutron Stars





the evolution of massive stars, ie stars with $10M_\odot \lesssim M \lesssim 100M_\odot$ Such stars end their evolution as supernovae

Burrows

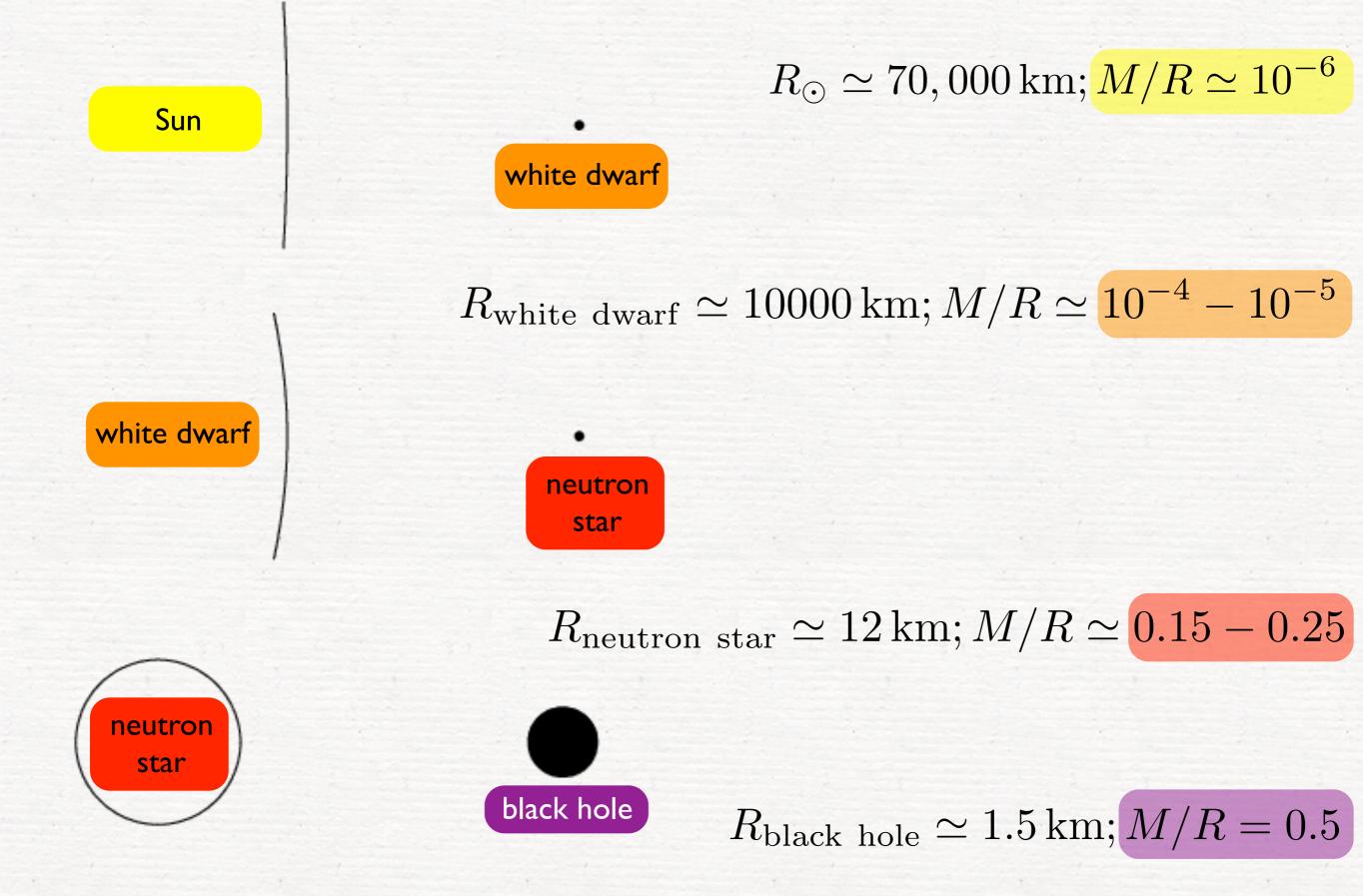




 $M \simeq 1.3 - 2.0 M_{\odot}$ $R \simeq 12 - 15 \,\mathrm{km}$ $\rho_c \simeq 10^{15} \,\mathrm{g/cm}^3$

A spoon of this matter is as heavy as the Mont Blanc

Let's compare again sizes and curvatures

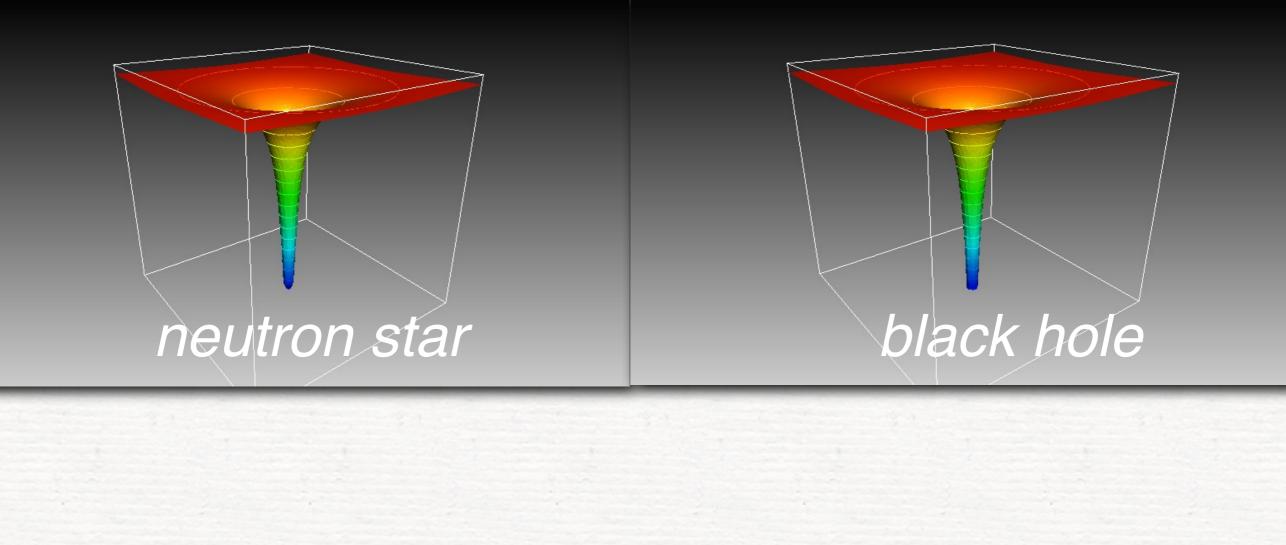


Compact Star vs Black Hole

When it comes to compactness, black holes and neutron stars are very similar and extreme!

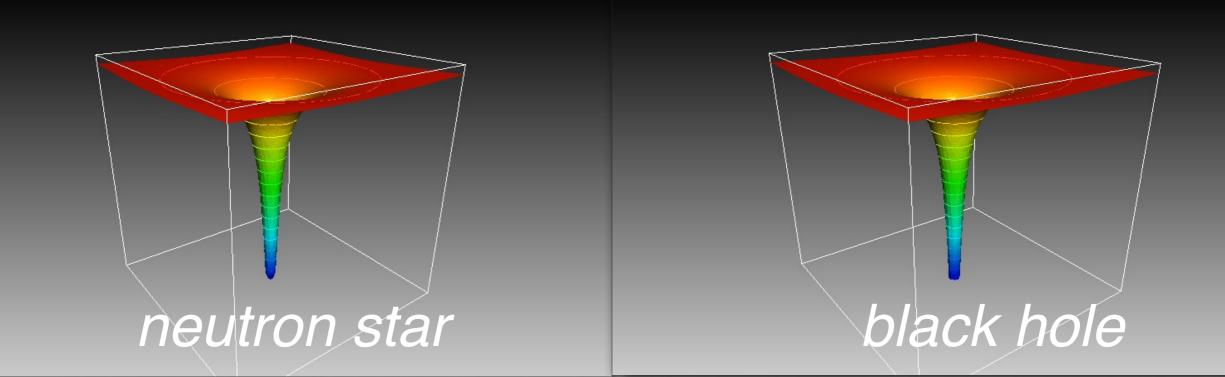
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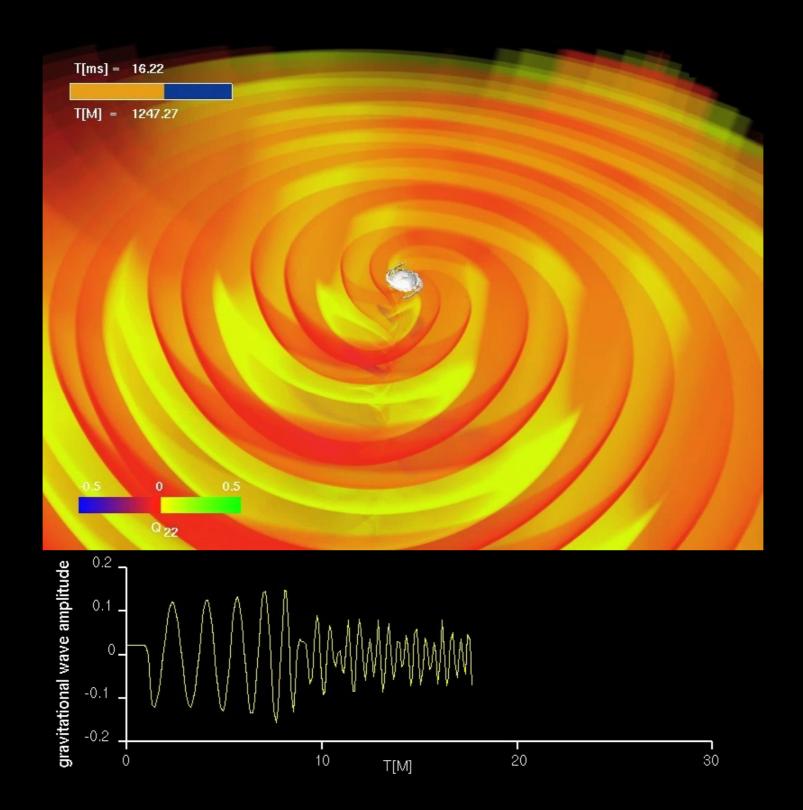
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Two aspects are different: a compact star has a *hard surface* and the curvature is large but *finite*; a black hole has *no surface* and the curvature is *infinite* at the centre

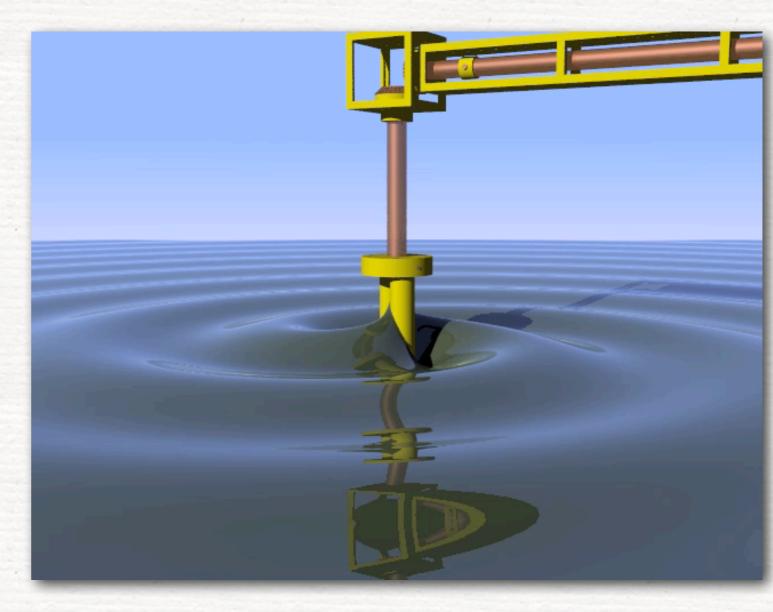
Gravitational waves



Gravitational waves: ripples in spacetime We have seen that compact objects like black holes and neutron stars curve the spacetime near them. Gravitational waves: ripples in spacetime We have seen that compact objects like black holes and neutron stars curve the spacetime near them.

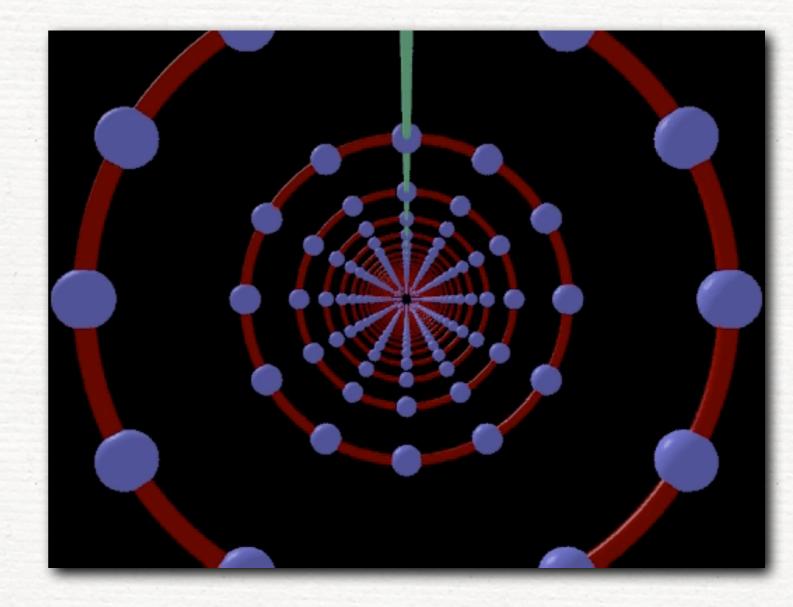
• What happens to the curvature when they move?

• What happens if they orbit around the same center of mass?



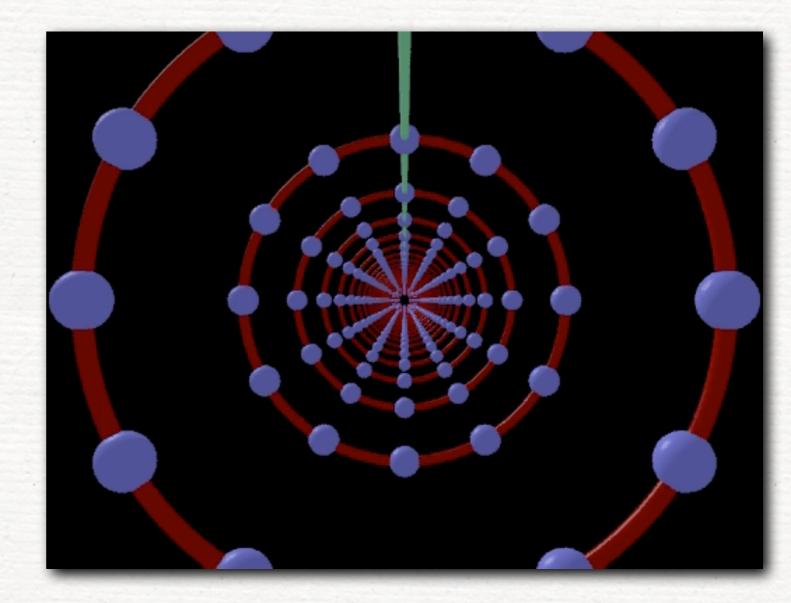
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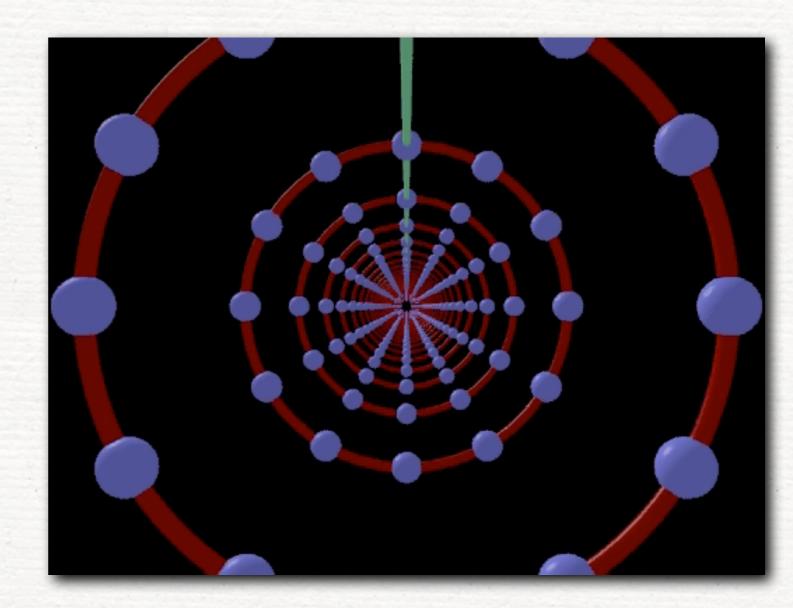
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• They are transverse waves moving at the speed of light: i.e. they produce changes in the direction orthogonal to the propagation one

• They distort space and time in a quadrupolar manner; squeeze in one direction and stretch in the other one.



Comparing EM and GWs

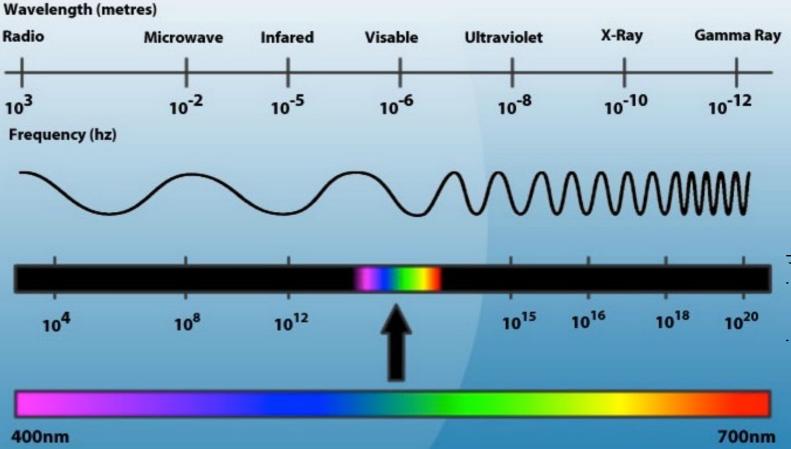
Electromagnetic and gravitational waves provide information which is complementary.

- EM waves tell us of the thermodynamical properties of matter.
- GWs tell us of the dynamical properties of compact objects

gravitational-wave spectrum

electromagnetic spectrum

I 0-4	I 0 -2	(Hz)	10 ²	1 0 ³
superma	ssive BHs	stellar	BHs	NSs
07	105		10 ²	I
M⊙	M⊙		M⊙	M⊙



How do you produce gravitational waves?

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However, spacetime is hard to curve and GWs are also hard to produce (not possible in laboratories).

What is needed is:

compact objects, i.e. large masses in small volumes
velocities close to that of light

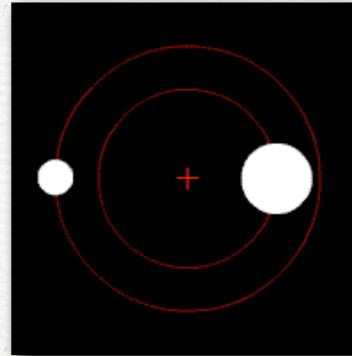
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Clearly, **black holes** and **neutron stars** are ideal sources, especially if in **binary systems**.



How do you detect gravitational waves?

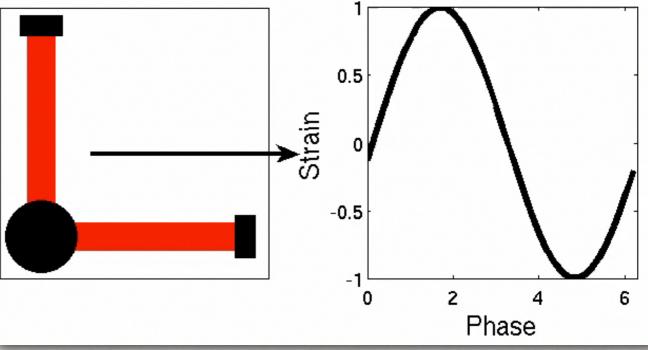
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LIGO, USA







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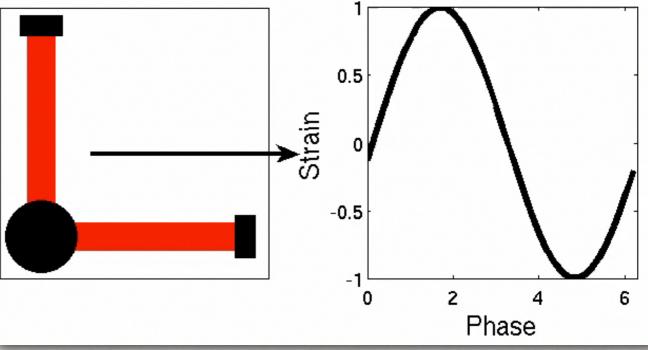
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Catastrophic events

Back-of-the-envelope calculation (Newtonian quadrupole approx.) shows the energy emitted in GWs per unit time is

$$L_{\rm GW} \simeq \left(\frac{G}{c^5}\right) \left(\frac{M\langle v^2 \rangle}{\tau}\right)^2 \simeq \left(\frac{c^5}{G}\right) \left(\frac{R_{\rm Schw.}}{R}\right)^2 \left(\frac{\langle v \rangle}{c}\right)^6$$

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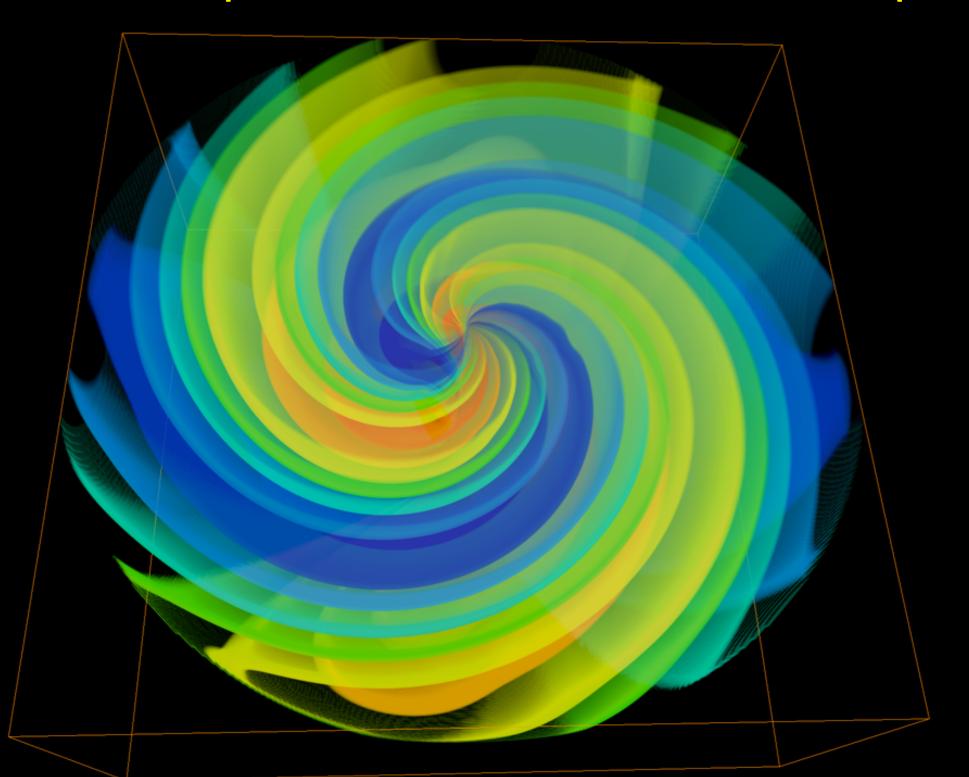
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As a result, the GW luminosity is:

$$L_{\rm GW} \simeq 10^{-8} \left(\frac{c^5}{G}\right) \simeq 10^{50} \,\mathrm{erg} \,\mathrm{s}^{-1} \simeq 10^{17} \,L_{\odot}$$

This is roughly the combined luminosity of I million galaxies!

Numerical Relativity: solving Einstein equations on a computer



Numerical relativity

Einstein's theory is as beautiful as intractable analytically

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Numerical relativity solves Einstein/HD/MHD eqs. in regimes in which no approximation is expected to hold. To do this we build codes: our "theoretical laboratories".





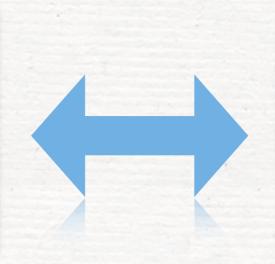












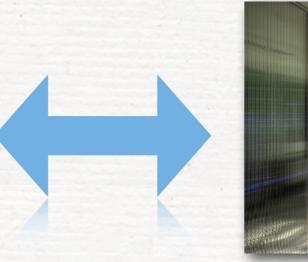


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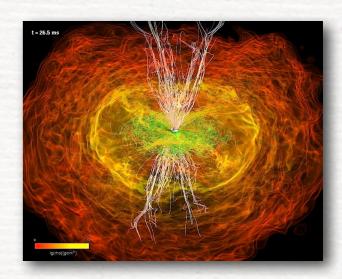








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Theoretical laboratories?

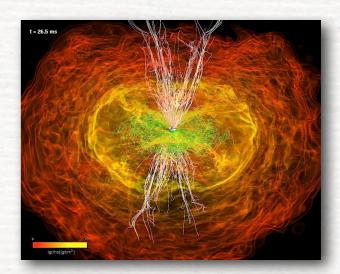








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Think of them as a factory of "gedanken experiments"

Theoretical laboratories?

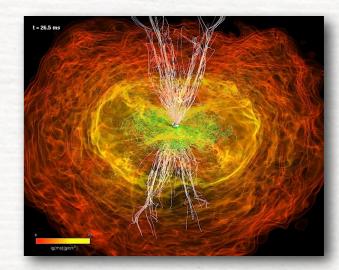




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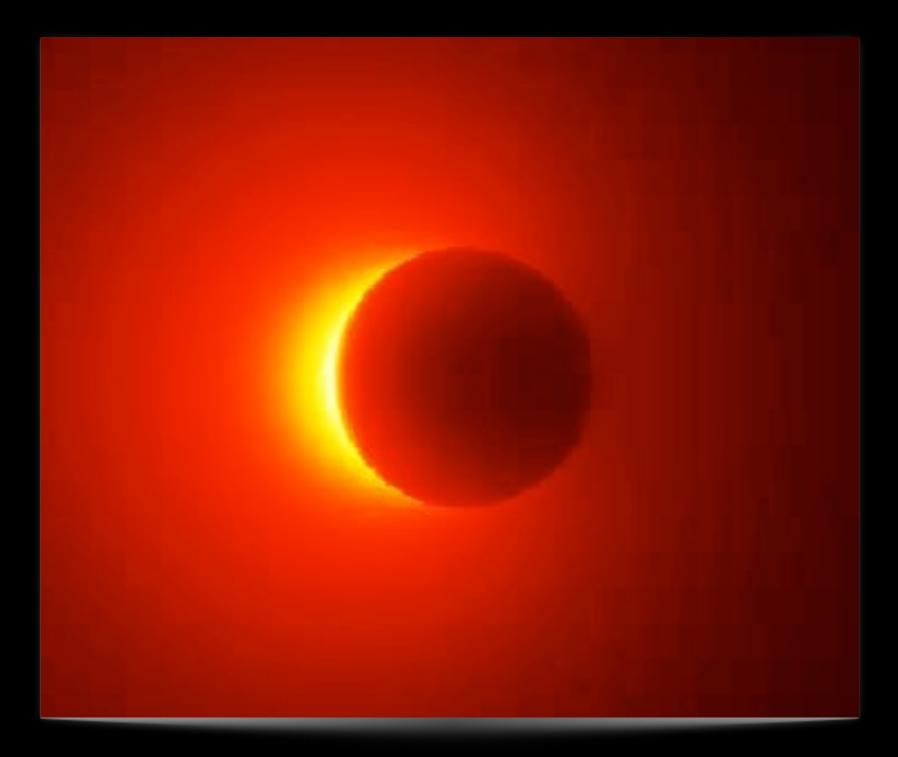






Think of them as a factory of "gedanken experiments" Einstein would have loved them...

Do black holes really exist?...





The Milky Way

View of the full sky (north and south) in the optical.



The Milky Way

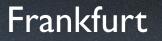


The Milky Way





The Milky Way



Sgr A* (Center of Milky Way)



The Milky Way



Sgr A* (Center of Milky Way)

Frankfurt

8.3 kpc ~ 10¹⁶ km ~ 10,000 light years

The Milky Way



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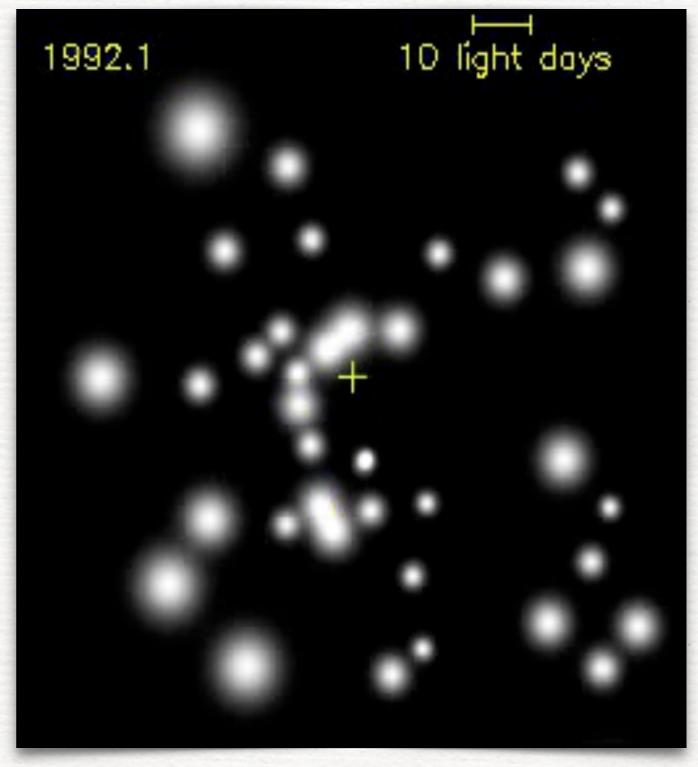
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- Black hole size is proportional to its mass: $R_S = 2GM/c^2$
- Biggest and largest BHs are at centers of galaxies
- The BH with largest diameter is at center of Milky Way

Sgr A*: the "dark object" in the Galactic Center

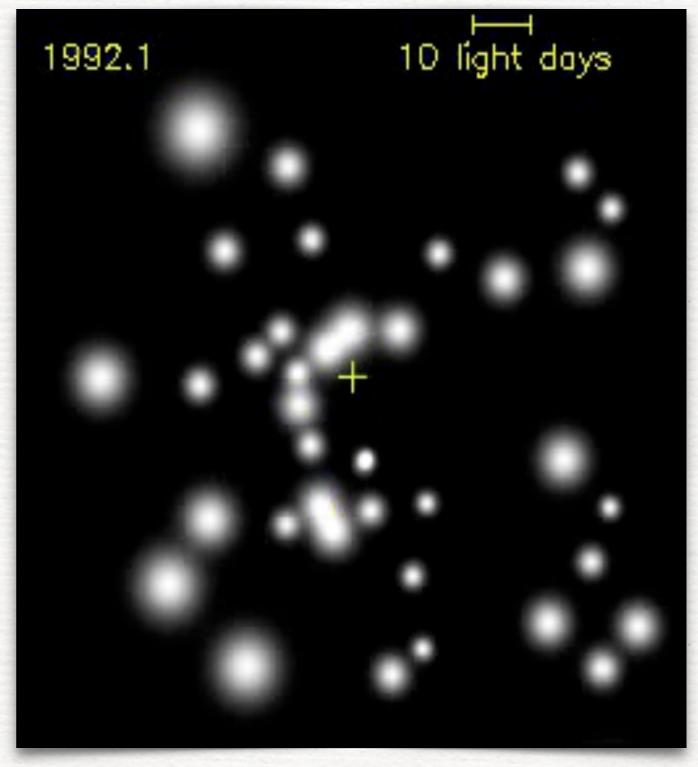
- Near-infrared telescopes (ESO) have measured orbits of individual stars.
- The stars orbit a dark object: the compact radio source Sgr A*.
- Study of orbits reveals a mass of 4.3 million times the mass of the Sun.



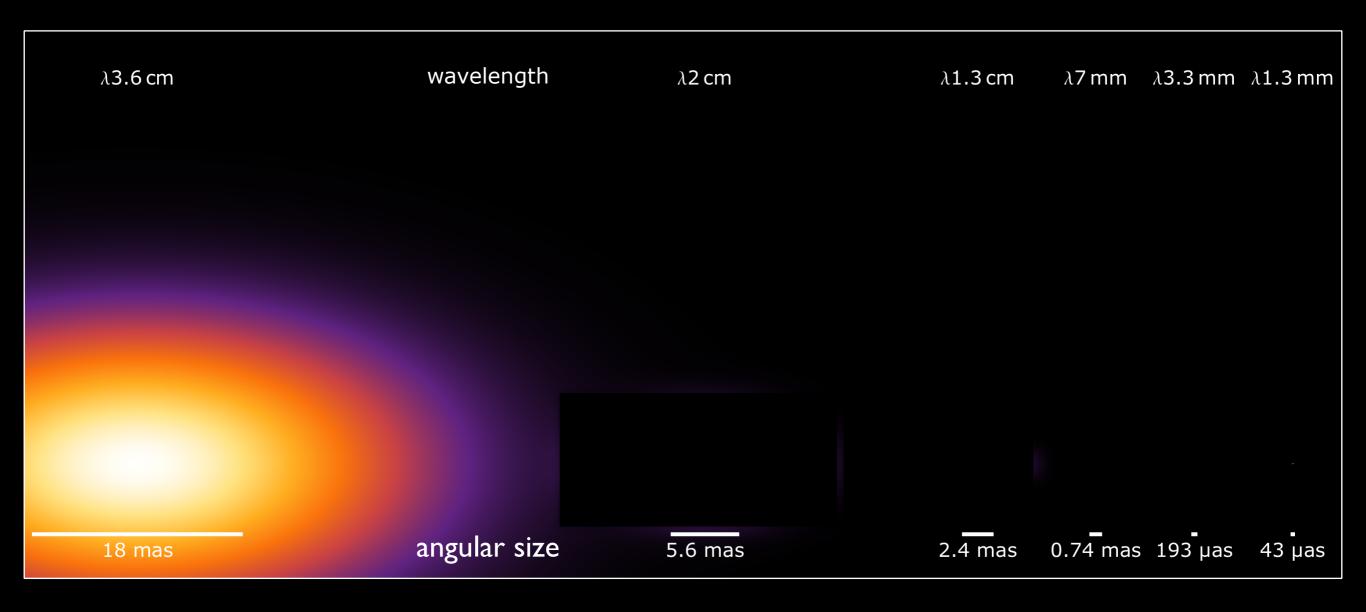
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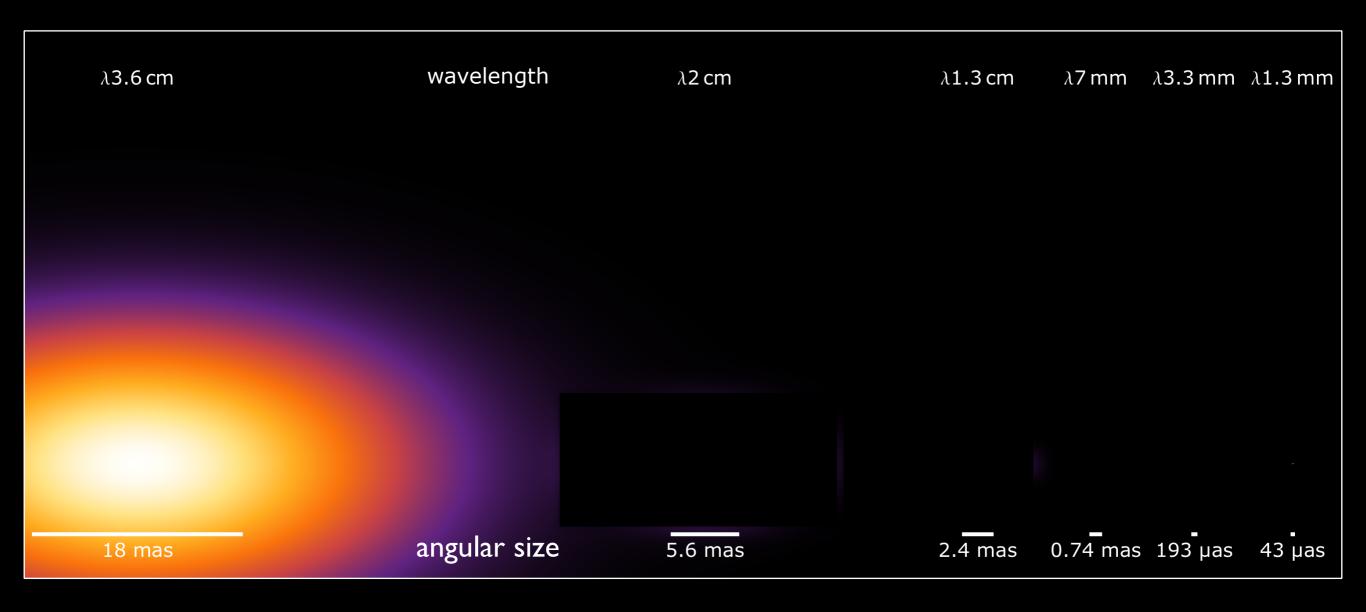


• The shorter the wavelength, the smaller the radio source.

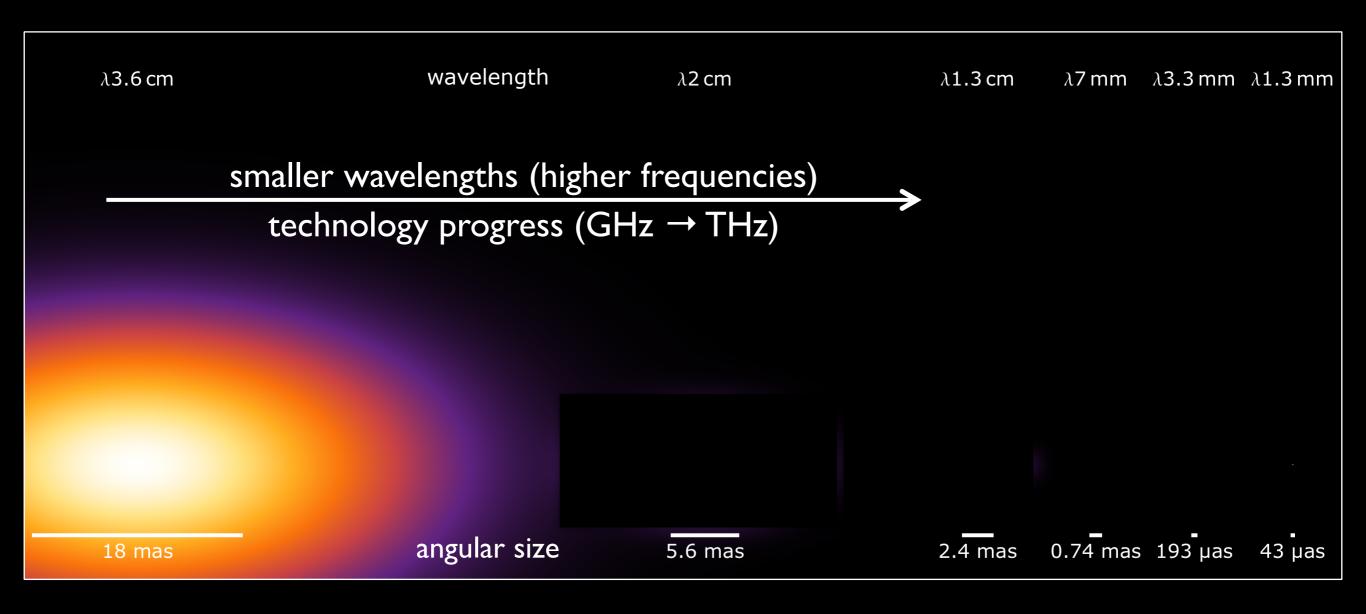
• At $\lambda = 1.3$ mm the radio source becomes the size of the event horizon.

mas = milli-arcsecond = 5×10^{-9} rad

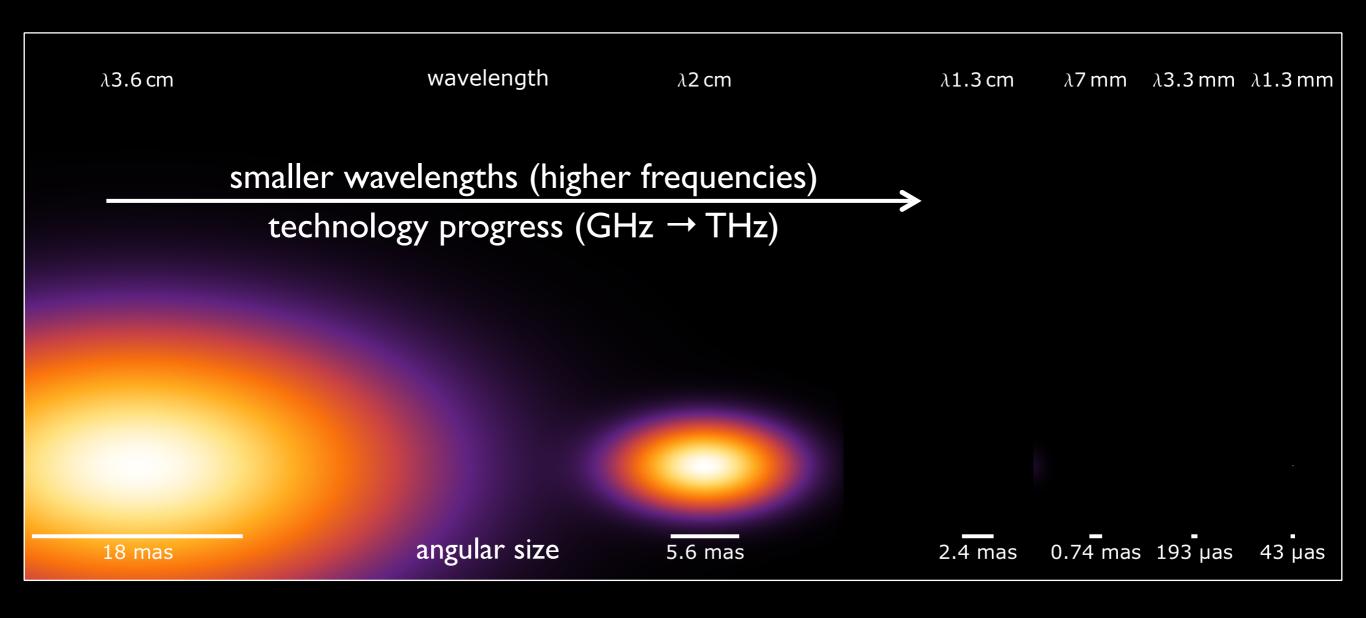
 μ as = micro-arcsecond = 5 × 10⁻¹² rad



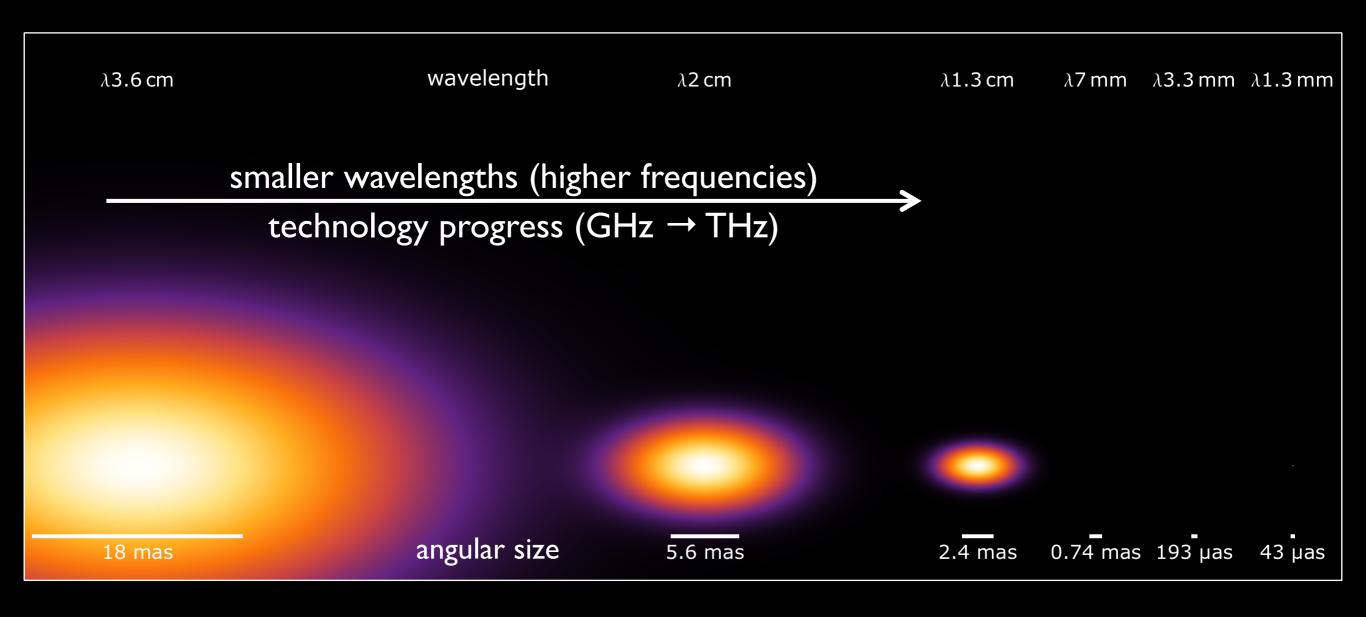
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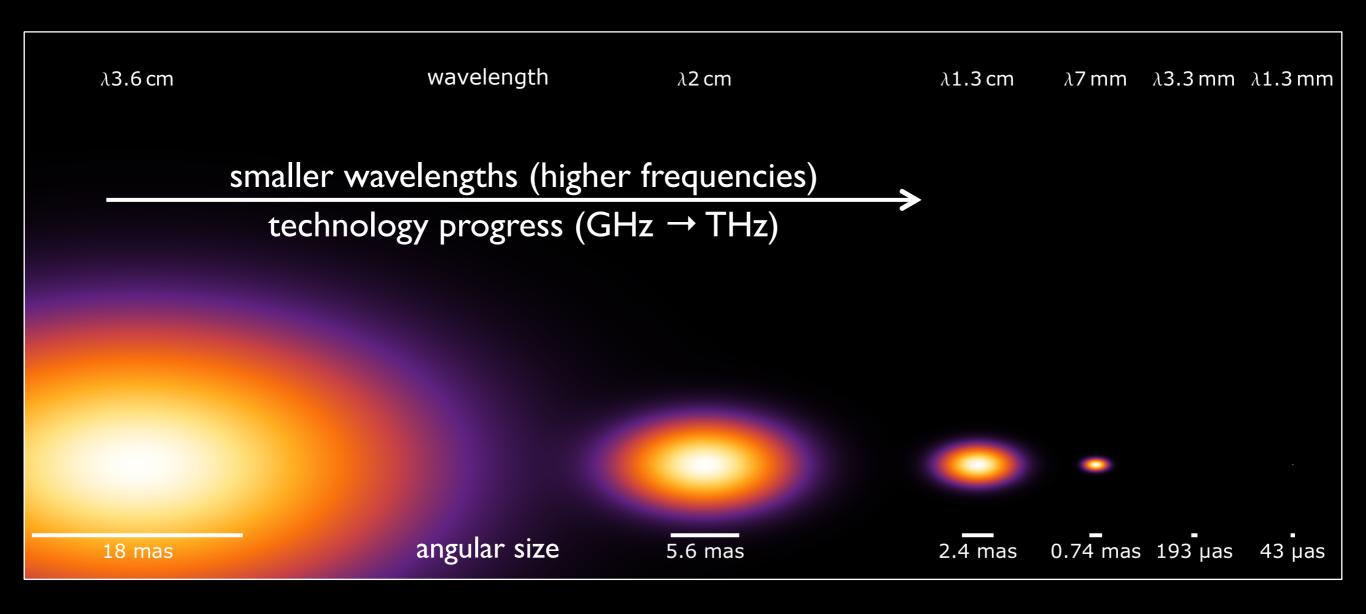
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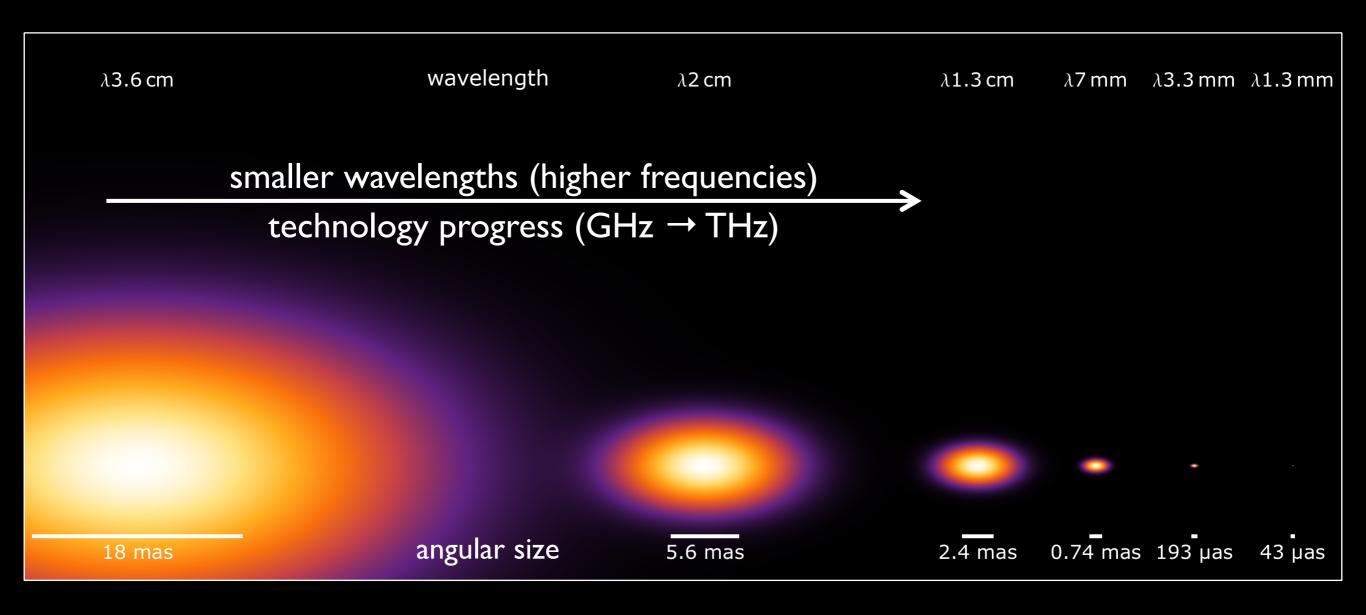
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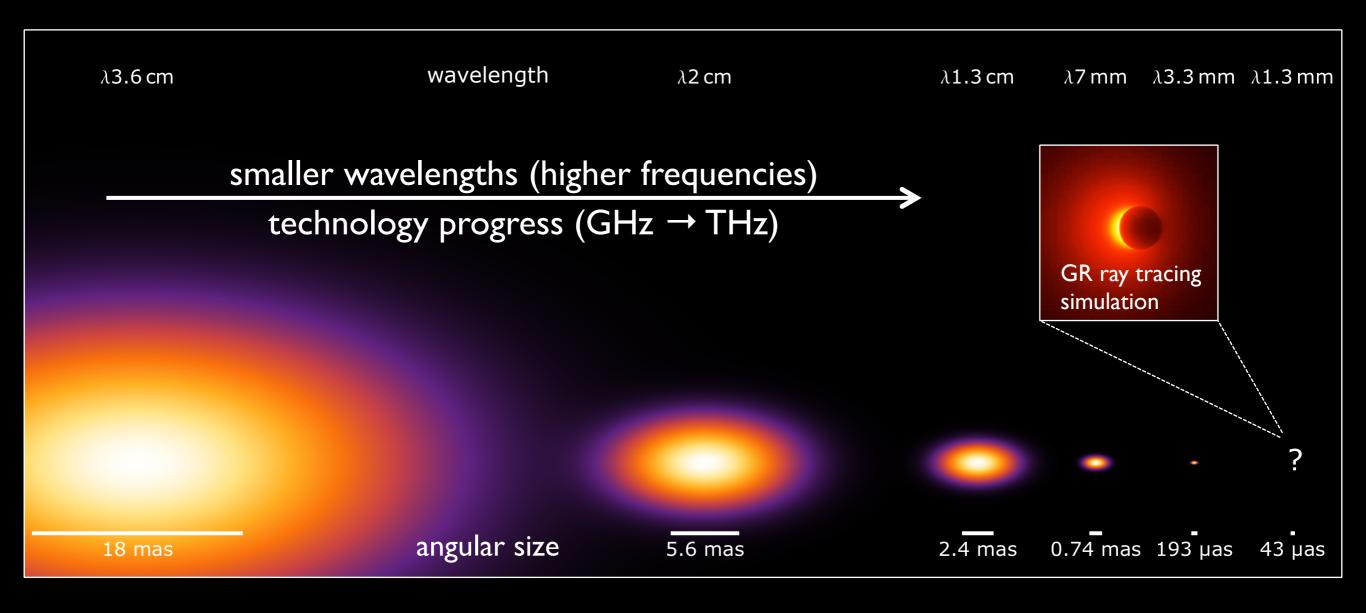
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• The shorter the wavelength, the smaller the radio source.

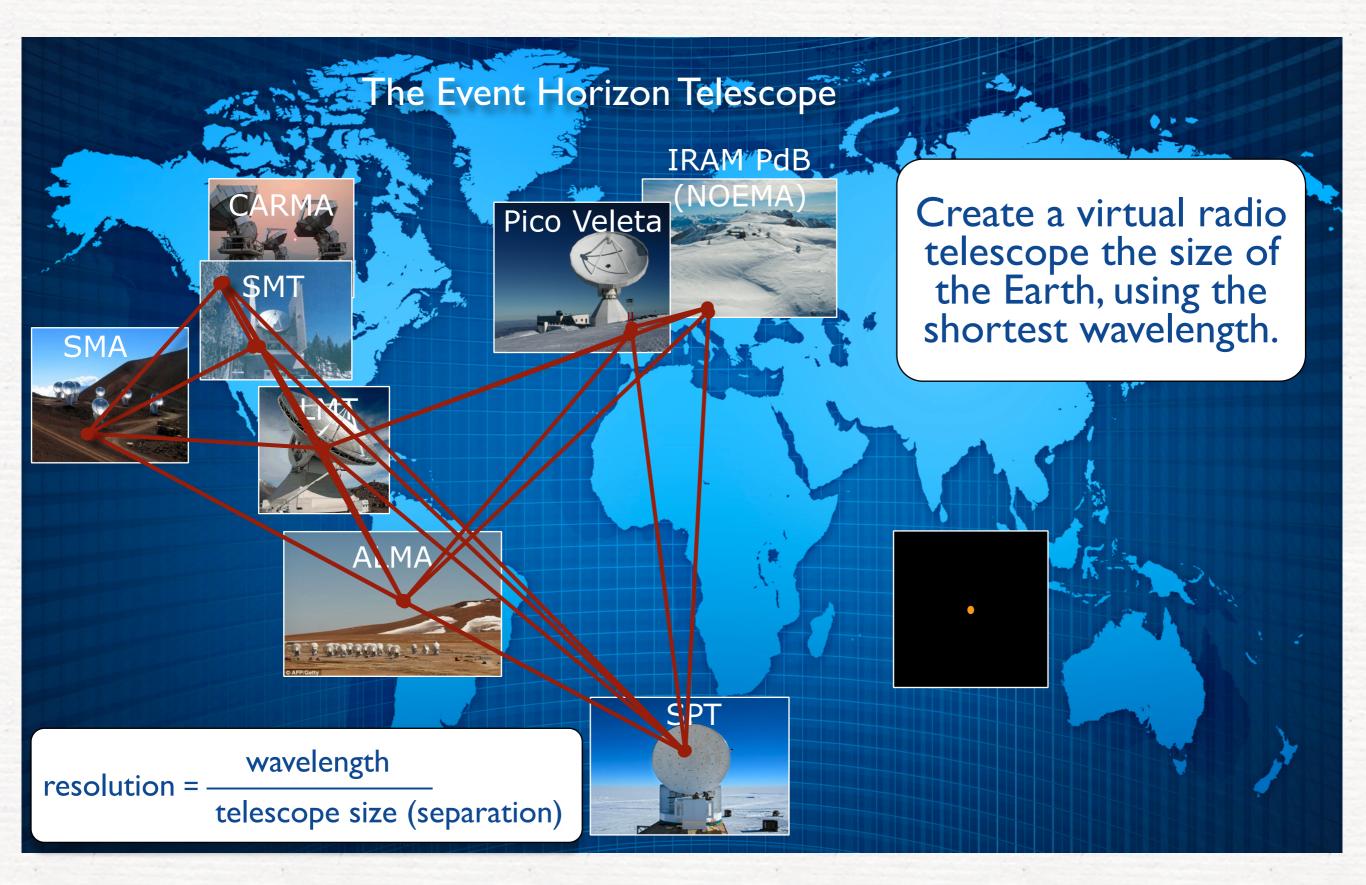


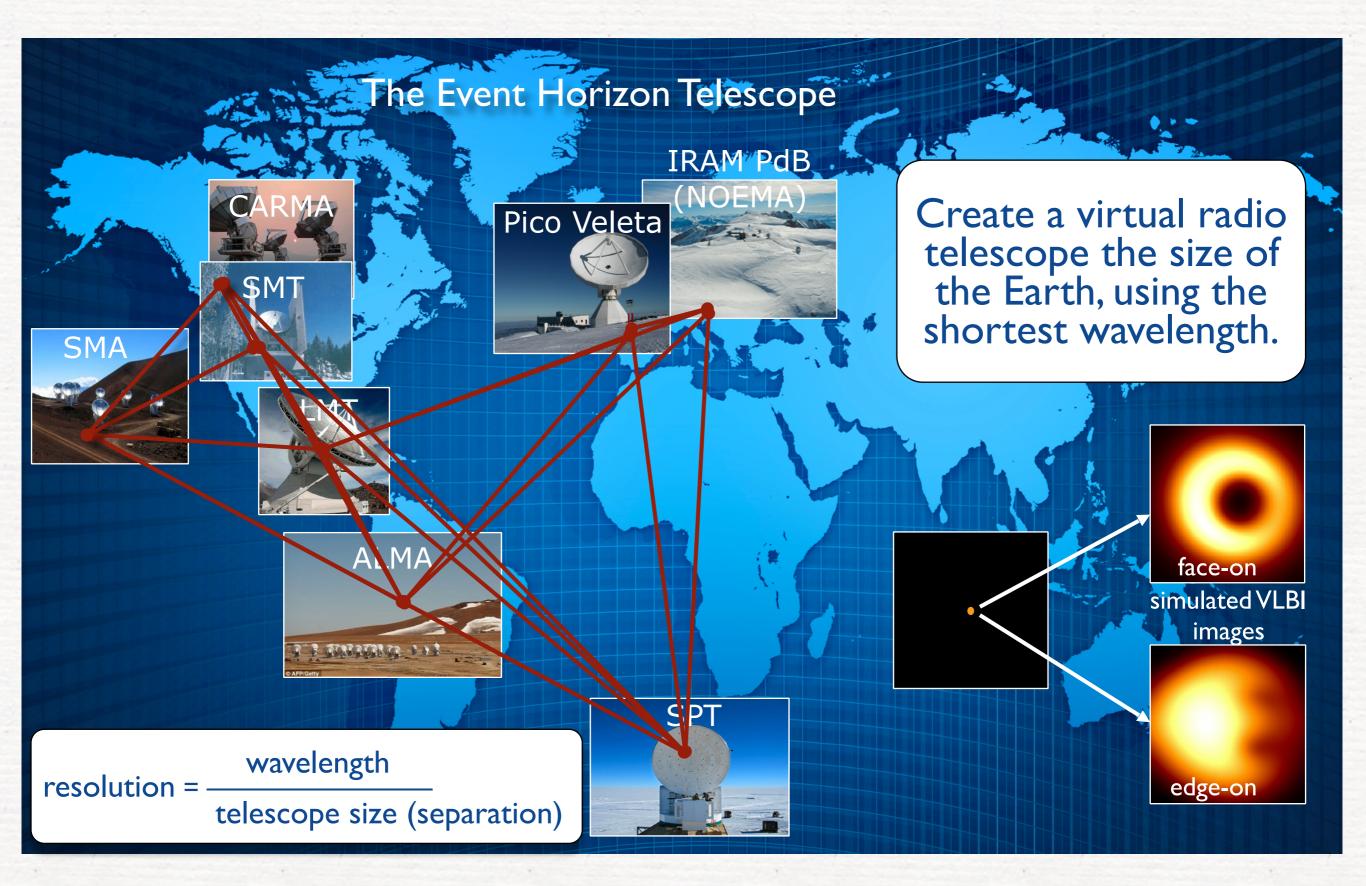
Create a virtual radio telescope the size of the Earth, using the shortest wavelength.







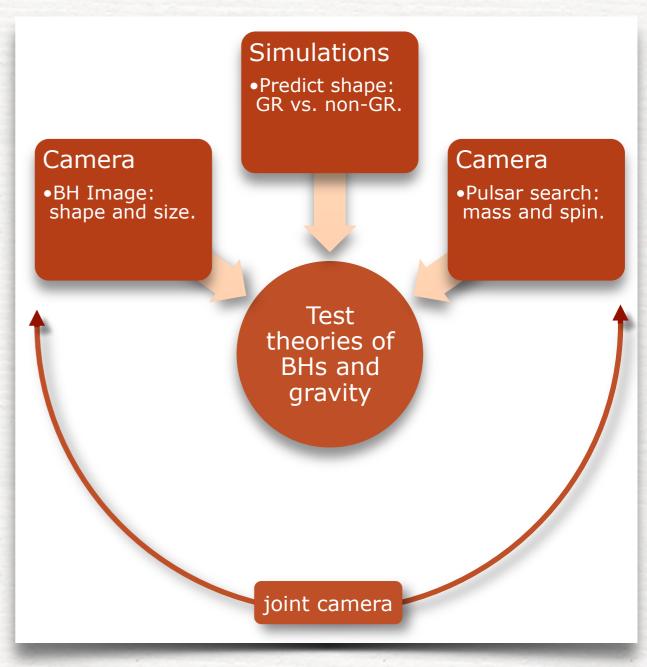




What do we want to do?

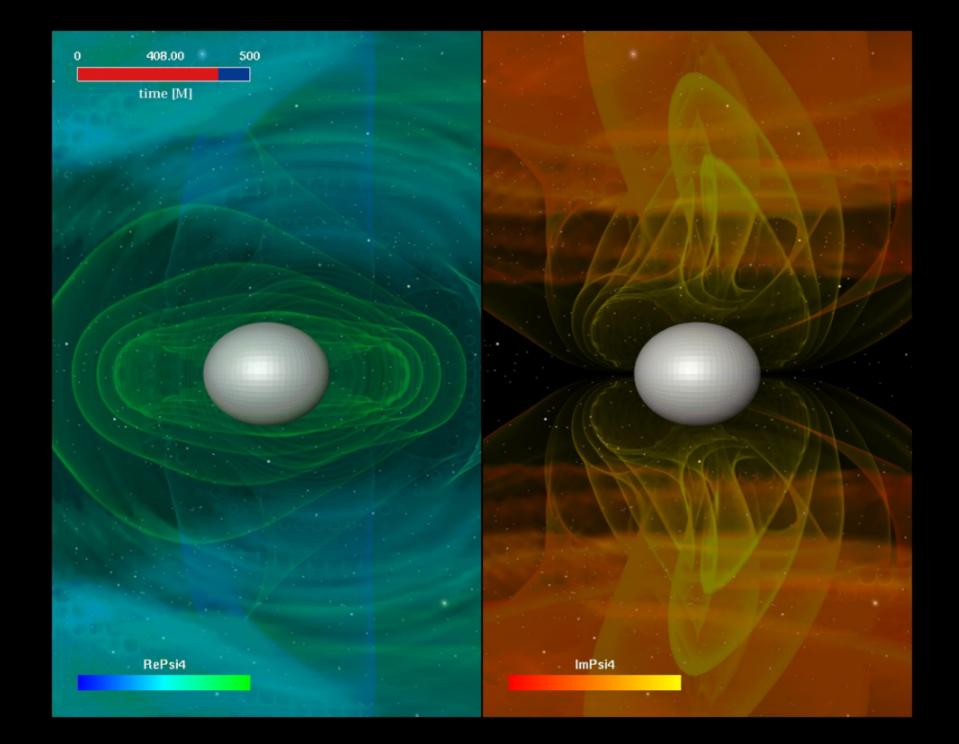
- Build a joint black hole camera
 * image event horizon to the best of present VLBI technology
- Hunt for pulsars near Sag-A*
 ★ detection of pulsars will provide unprecedented accuracy

 Make theoretical predictions/ interpretations
 * use numerical simulations to produce synthetic images
 * interpret observations to constrain theories of gravity



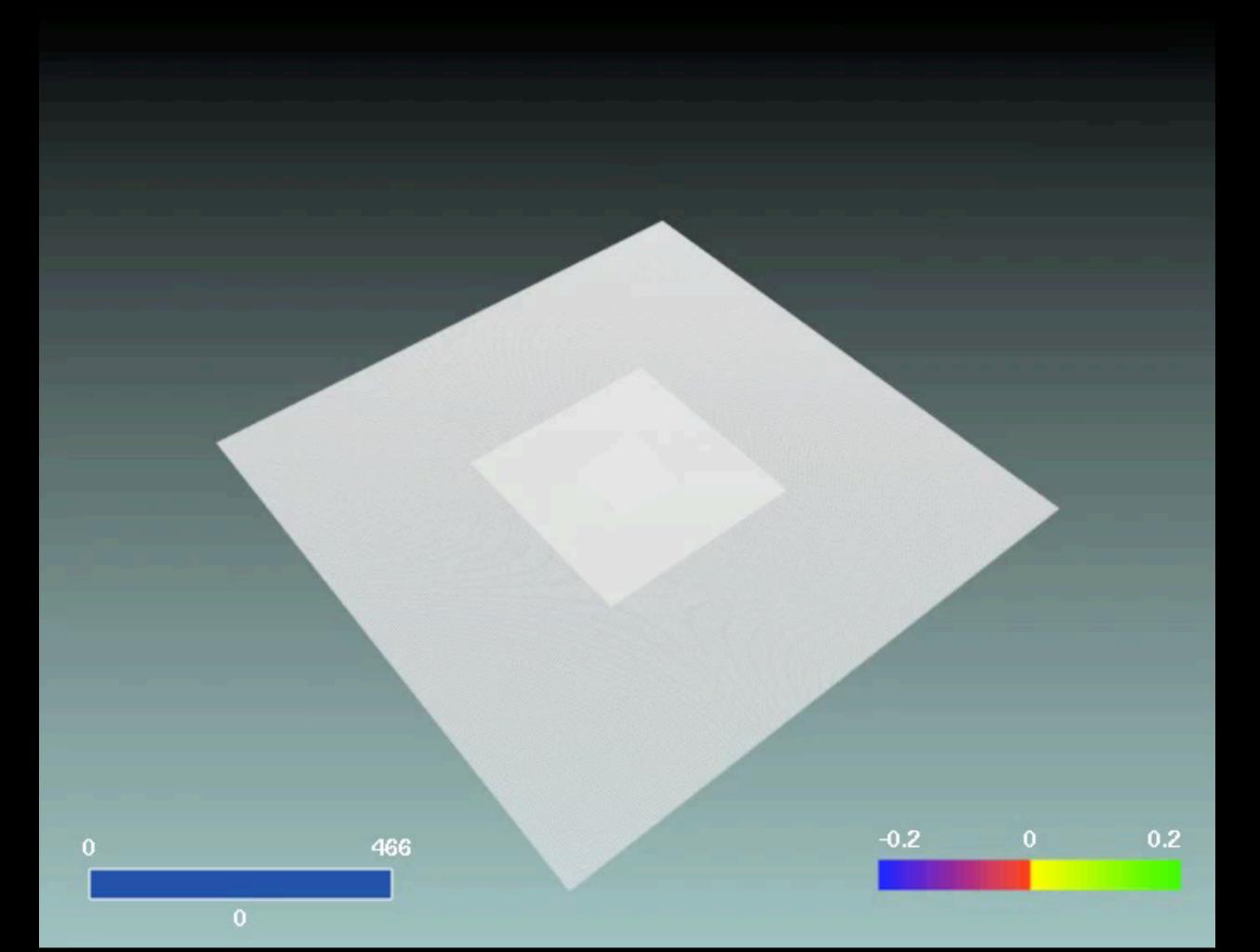
Not easy, but another milestone of modern physics

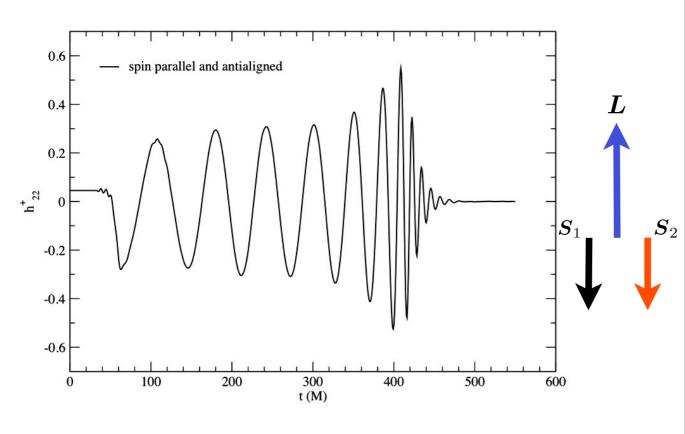
black-hole binaries

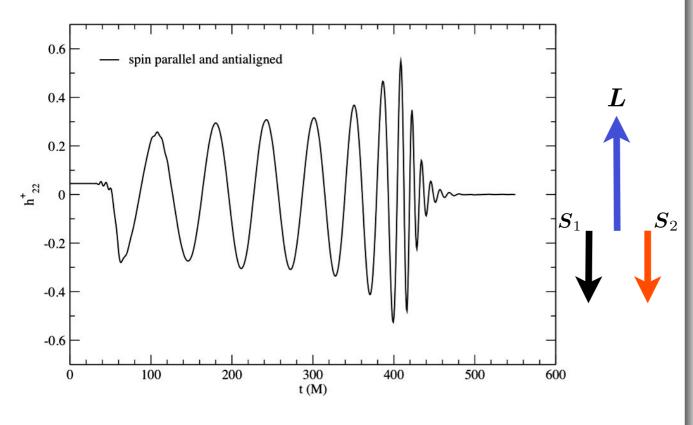


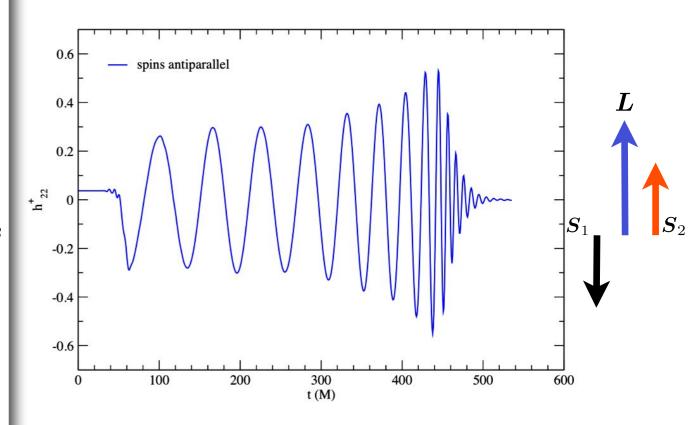


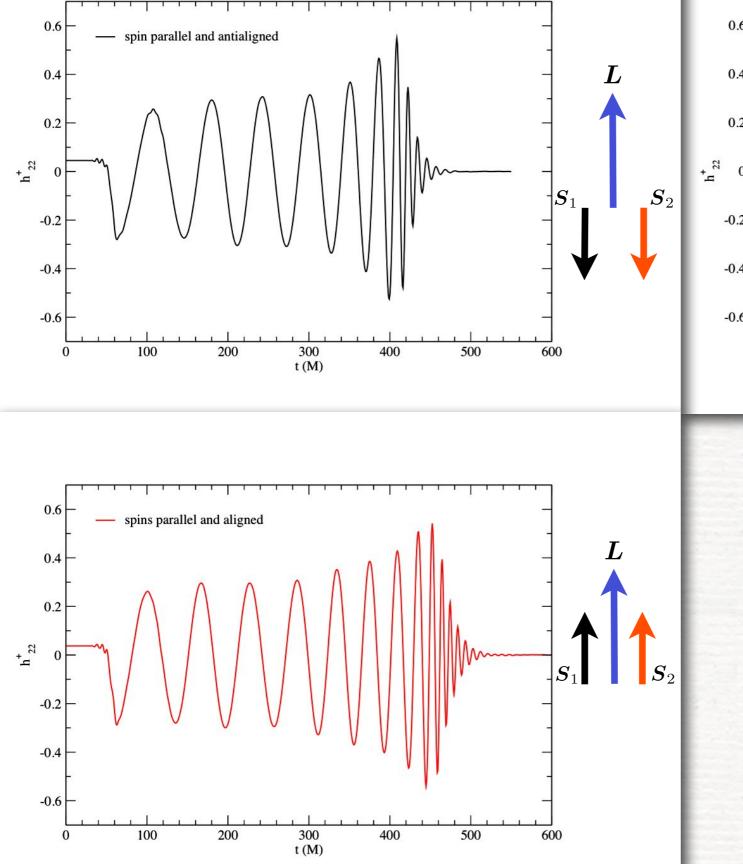
0

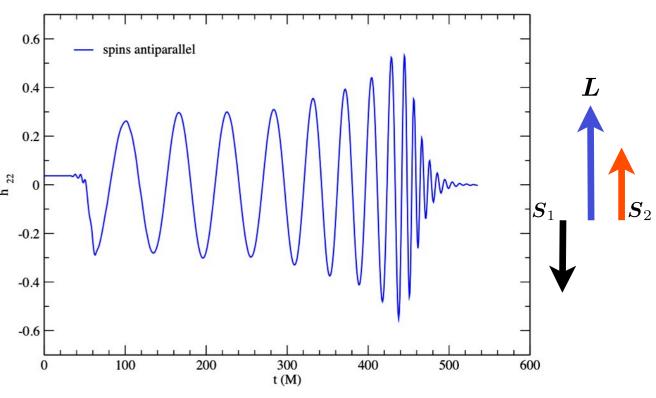




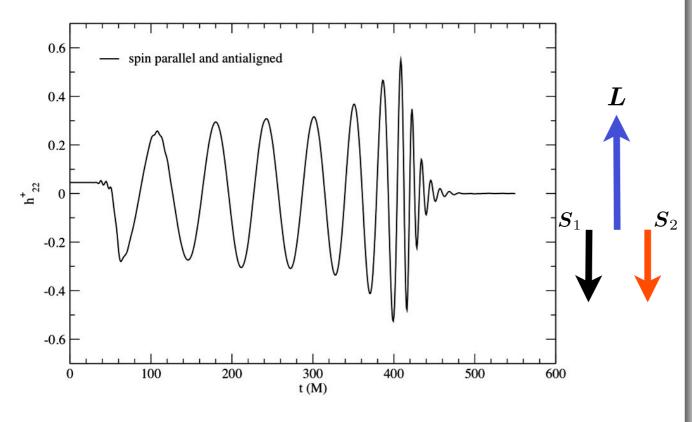


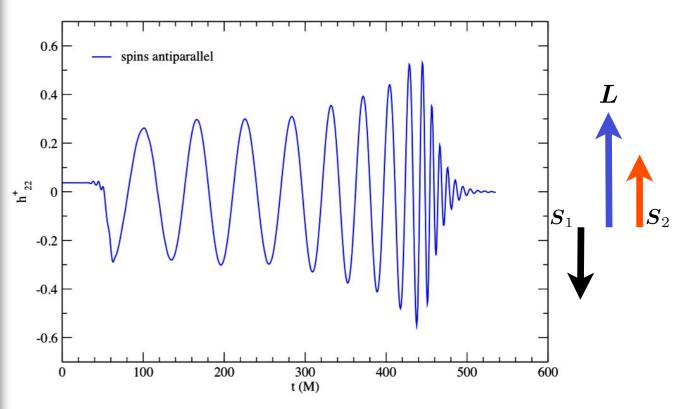


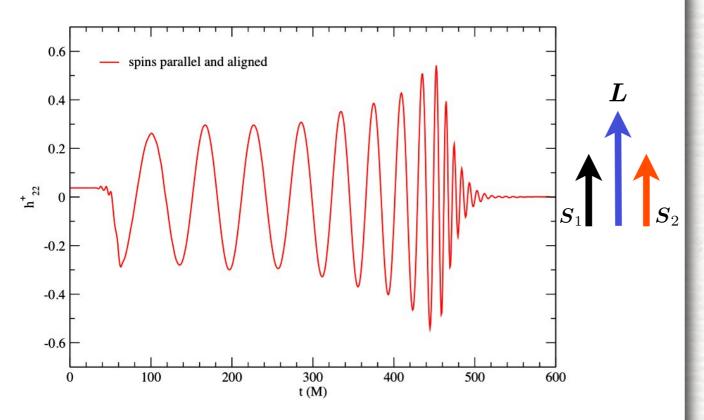






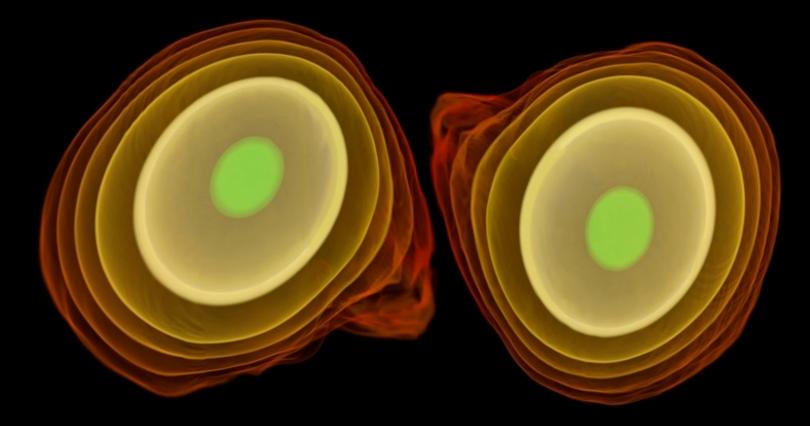


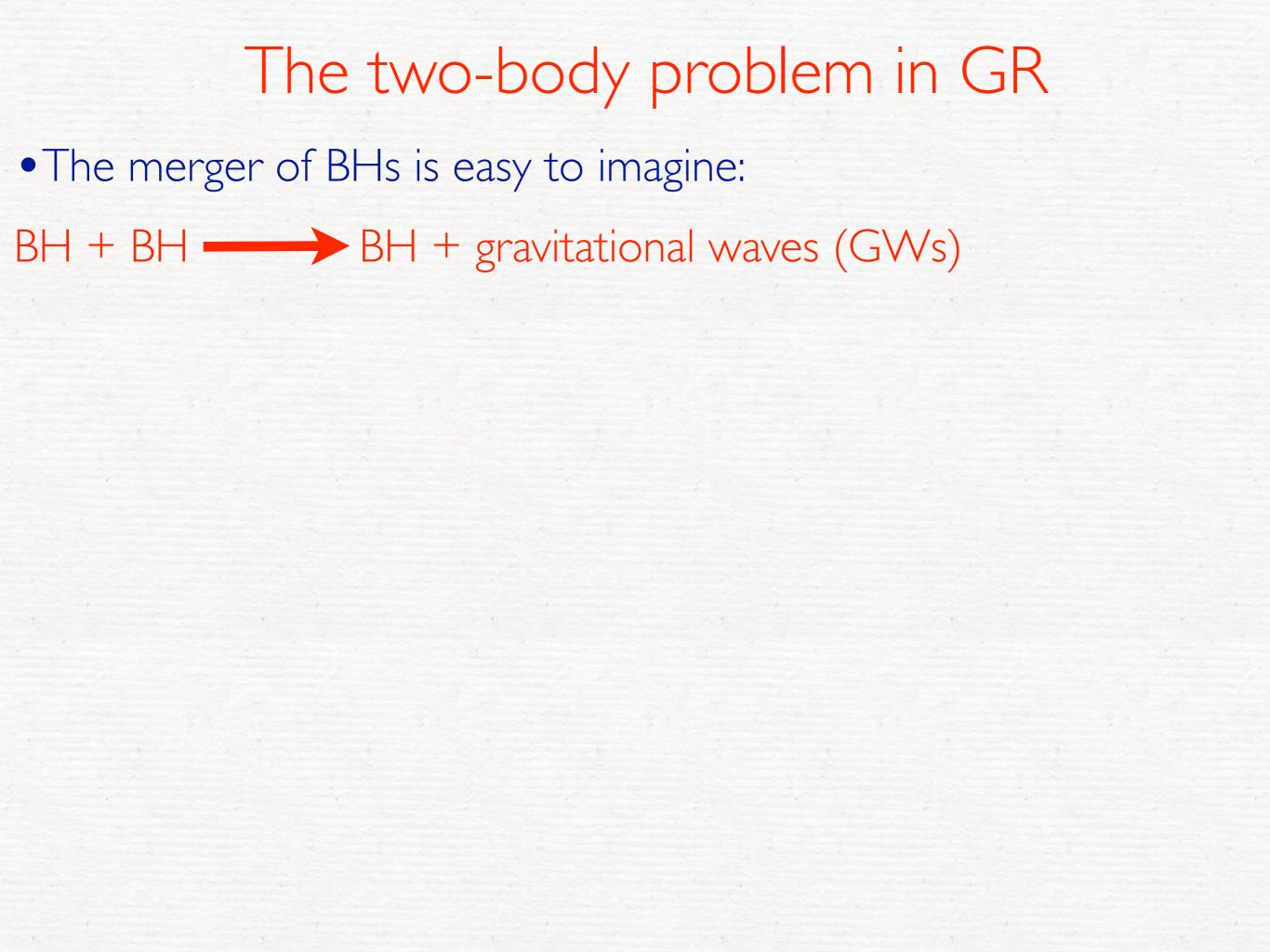




This is the signal that needs to be measured by modern gravitational detectors.

merging neutron-star binaries





The two-body problem in GR • The merger of BHs is easy to imagine: BH + BH -----> BH + gravitational waves (GWs) • For NSs the question is subtle: the merger leads to a star that is very massive (HMNS) but survive for some time (ms) $NS + NS \longrightarrow HMNS + \dots ? \longrightarrow BH + torus + \dots ? \longrightarrow BH$

The two-body problem in GR • The merger of BHs is easy to imagine: BH + BH ------> BH + gravitational waves (GWs) • For NSs the question is subtle: the merger leads to a star that is very massive (HMNS) but survive for some time (ms) $NS + NS \longrightarrow HMNS + \dots ? \longrightarrow BH + torus + \dots ? \longrightarrow BH$ All complications are in the intermediate stages:

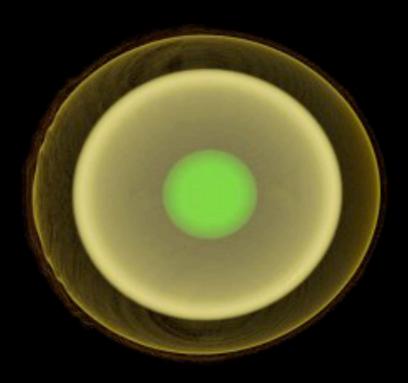
studying the HMNS we can learn how neutron stars are made,
 i.e. the equation of state (EOS) of nuclear matter.

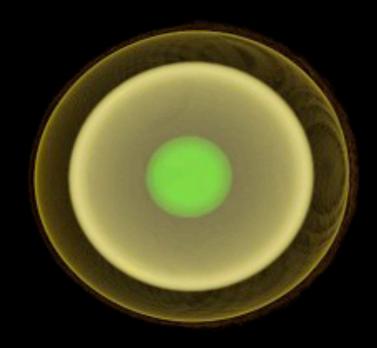
• studying the BH+torus we can possibly understand catastrophic events such as short gamma-ray bursts.

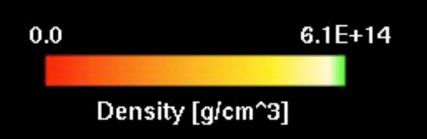
Animations: Kaehler, Giacomazzo, Rezzolla

T[ms] = 0.00

T[M] = 0.05



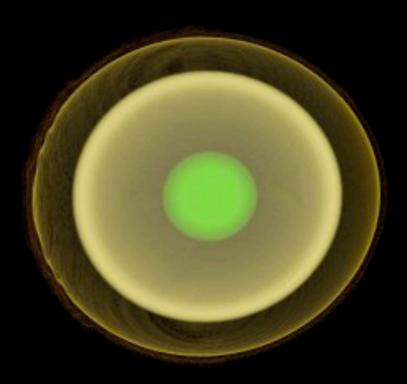


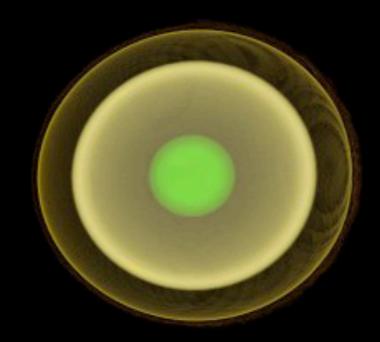


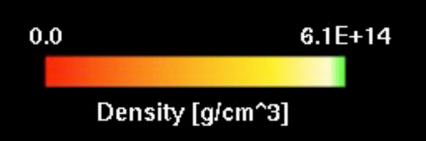


T[ms] = 0.00

T[M] = 0.05

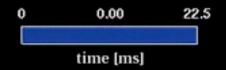






"merger HMNS BH + torus" Quantitative differences are produced by: - differences induced by the gravitational MASS: a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time - differences induced by the EOS: a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

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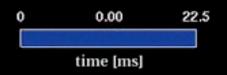


Animations: Giacomazzo, Koppitz, LR

Total mass : $3.37 M_{\odot}$; mass ratio :0.80;

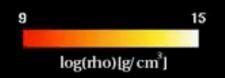










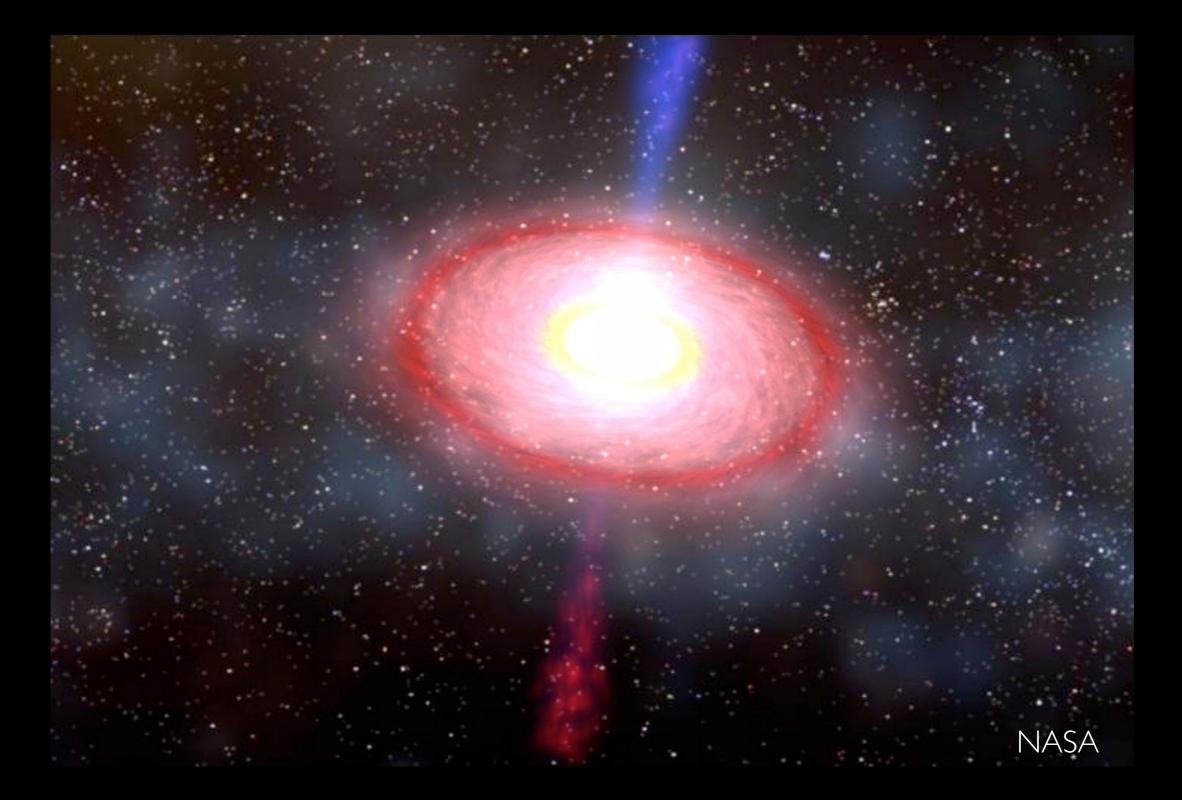


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 - radiative losses will alter the equilibrium of the HMNS

Short Gamma-ray Burst



The most energetic explosions

 Short Gamma-ray bursts (SGRBs) have been observed for 40 years and we see essentially a few per week.

• Energies released are huge: 1048-50 erg.

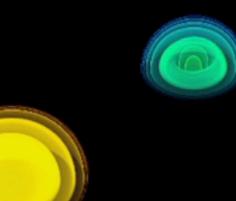
• The merger of two neutron stars can release sufficient energy over the correct timescale.

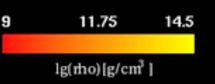
 No self-consistent model has yet been produced to explain them but a relativistic jet seems necessary.

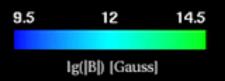
 Theoretical modelling has now reached level of maturity to shed light short SGRBs.

B-fields during inspiral phase

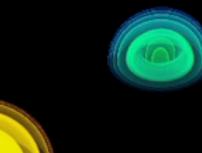
Typical evolution for a magnetized binary (hot EOS) $M = 1.5 M_{\odot}, B_0 = 10^{12} \text{ G}$

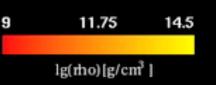


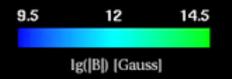


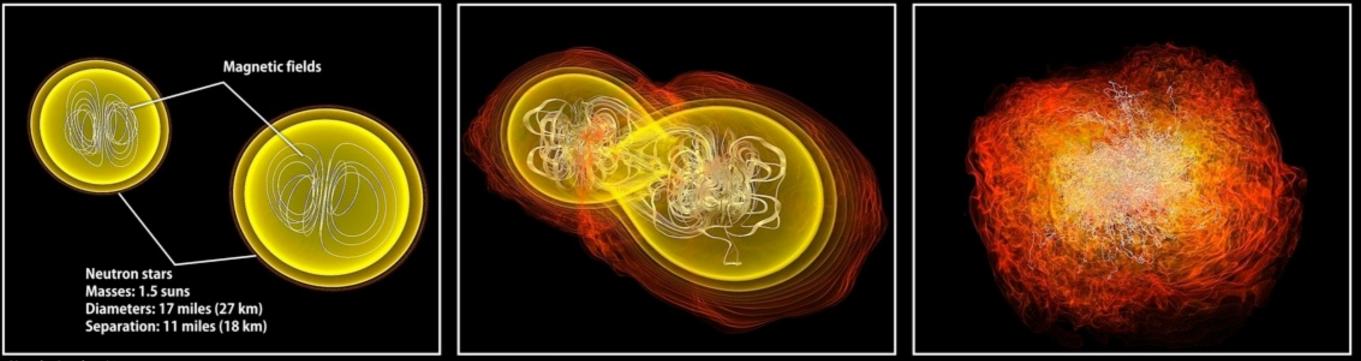


Animations:, LR, Koppitz









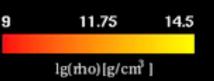
Simulation begins

7.4 milliseconds

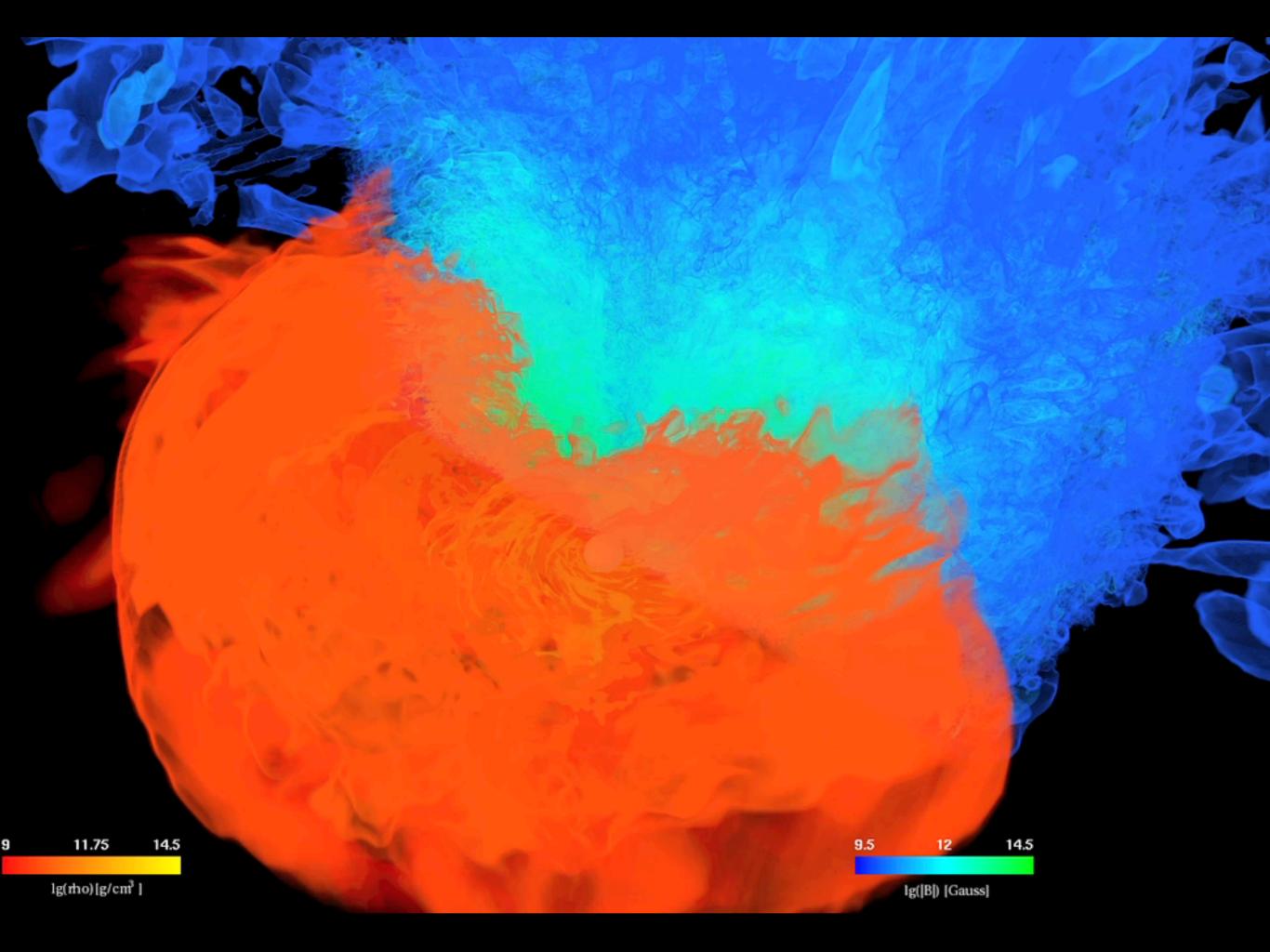
13.8 milliseconds

Magnetic fields in the HMNS have complex topology: dipolar fields are destroyed.

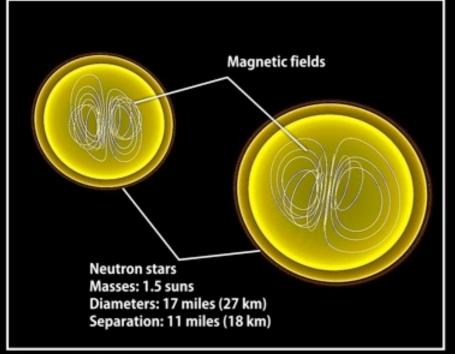
Animations:, LR, Koppitz



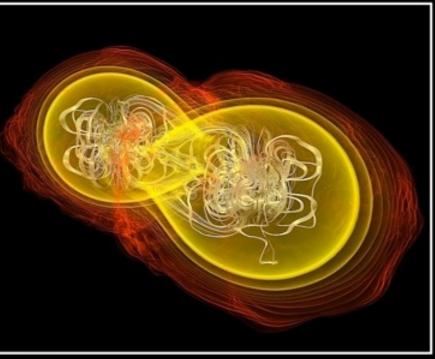




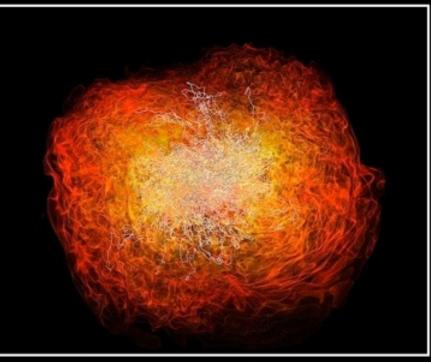
Crashing neutron stars can make gamma-ray burst jets



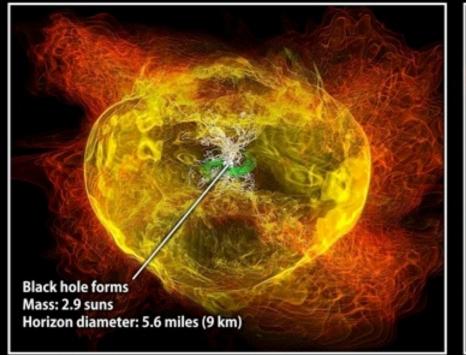
Simulation begins



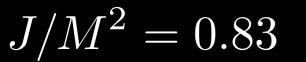
7.4 milliseconds



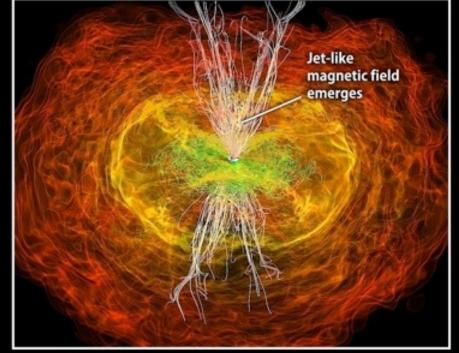
13.8 milliseconds



15.3 milliseconds



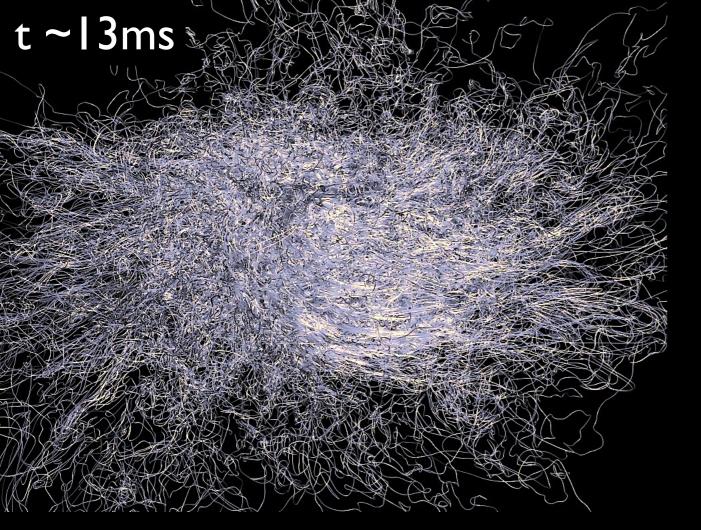
21.2 milliseconds

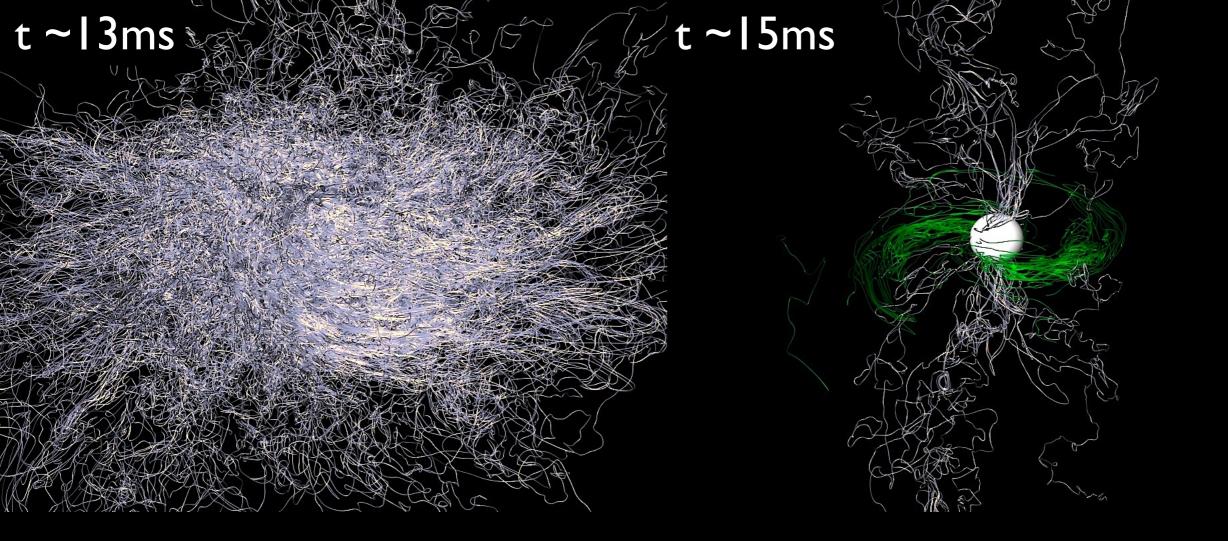


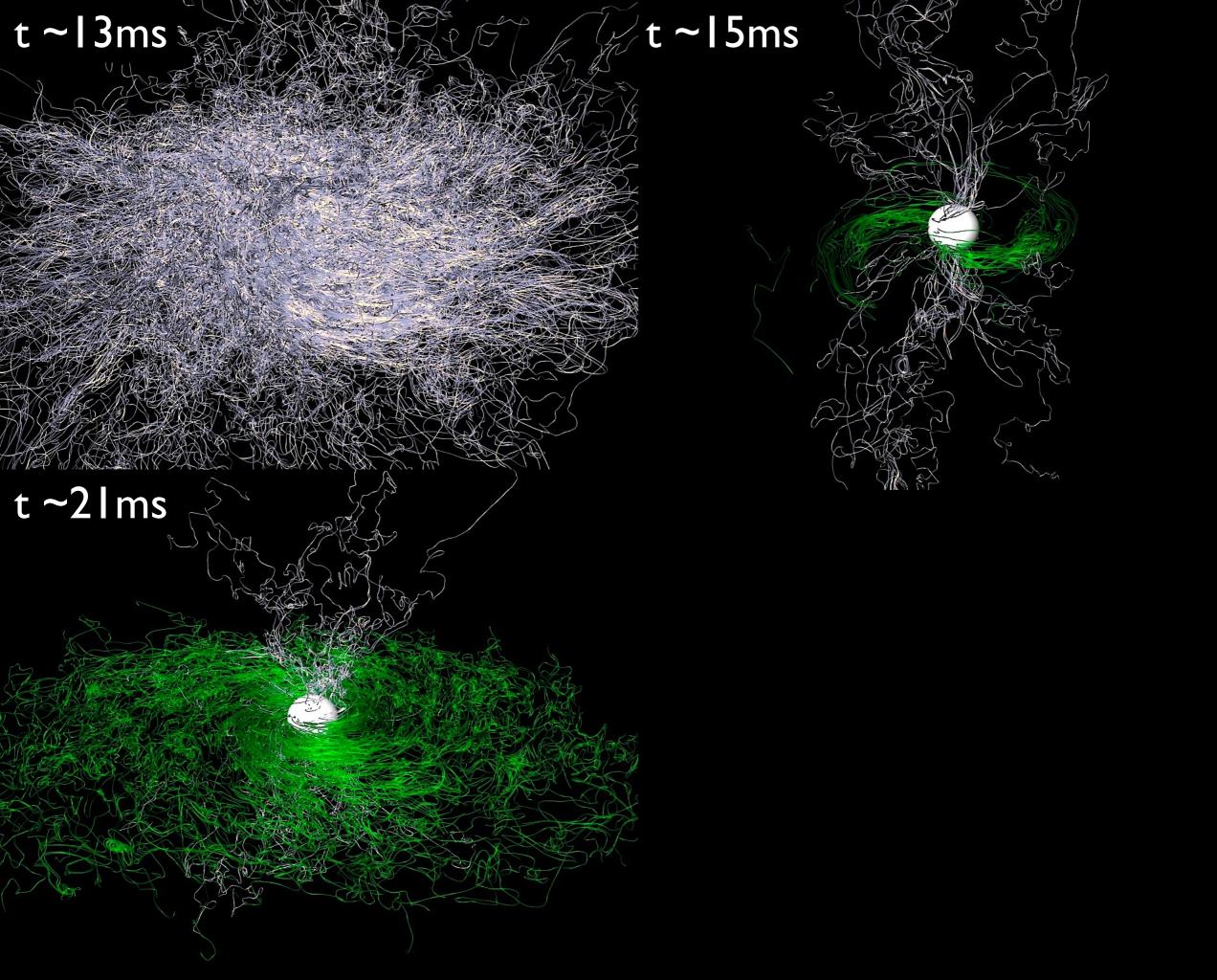
26.5 milliseconds

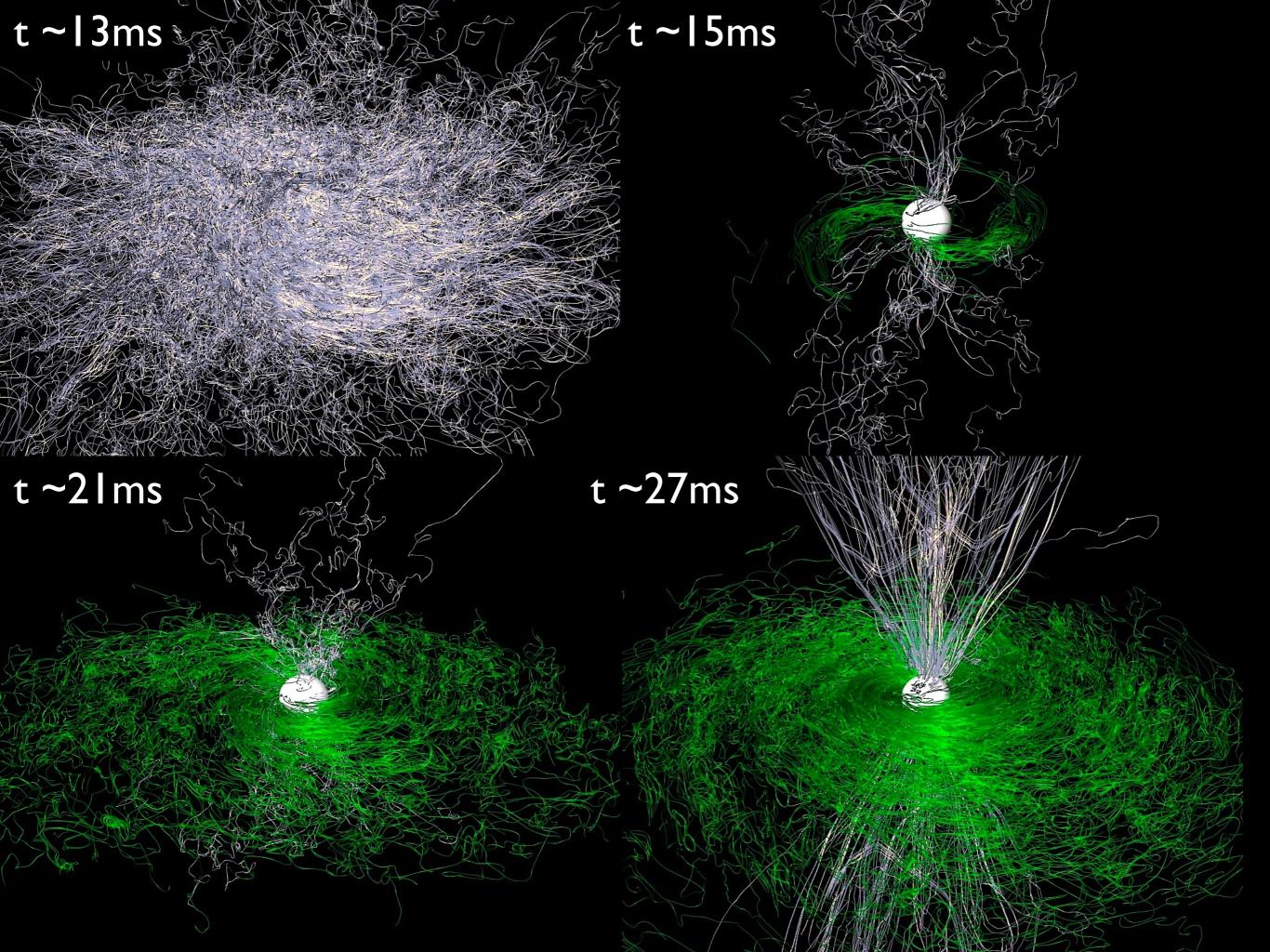
Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

 $M_{tor} = 0.063 M_{\odot}$ $t_{accr} \simeq M_{tor}/M \simeq 0.3 s$

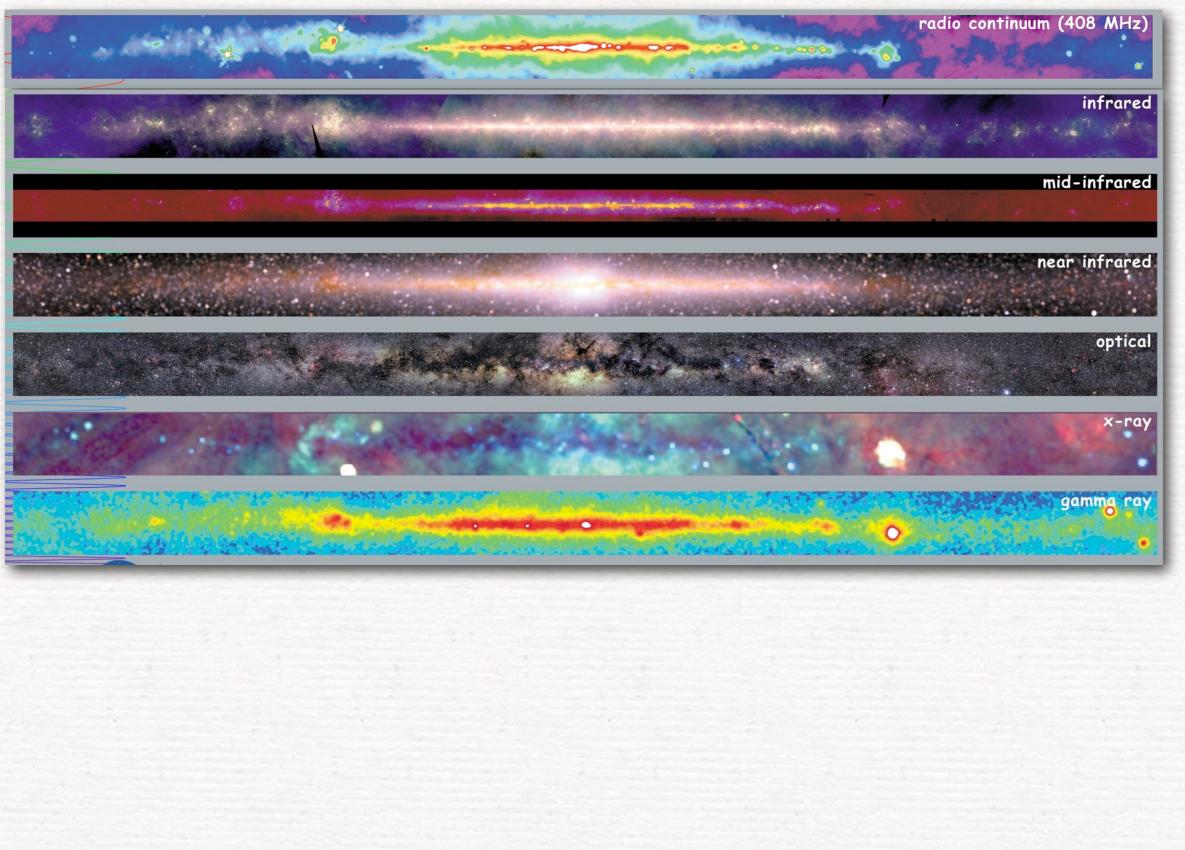




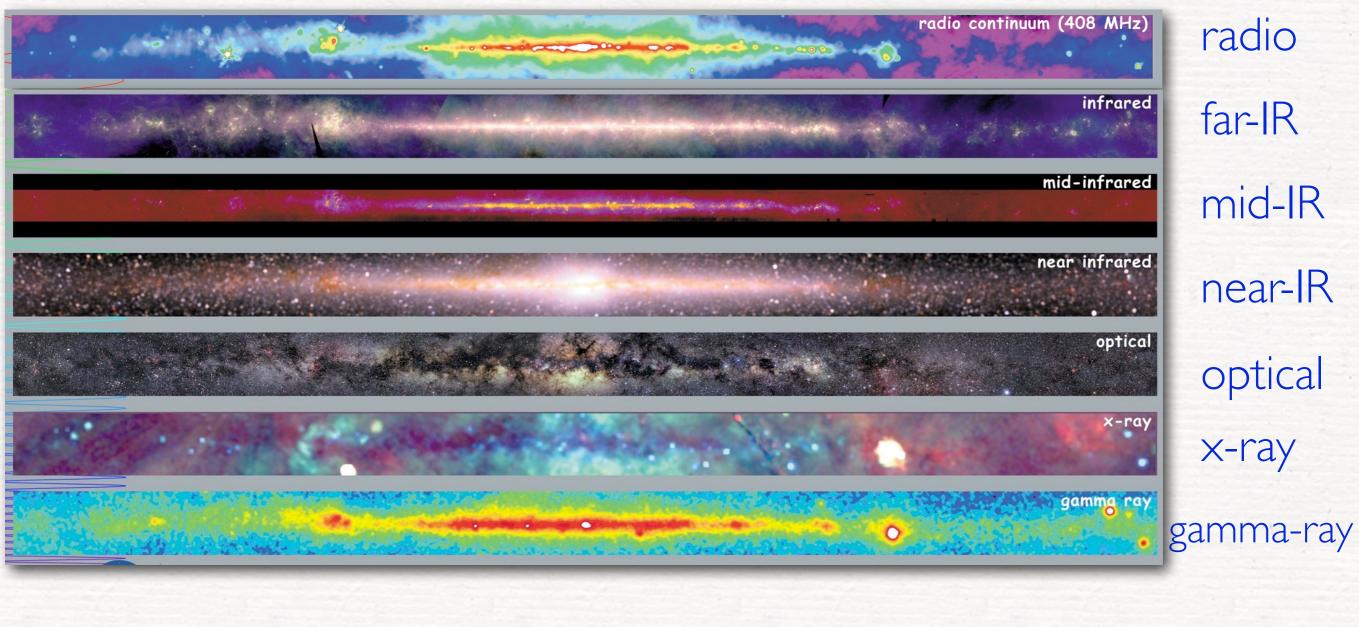




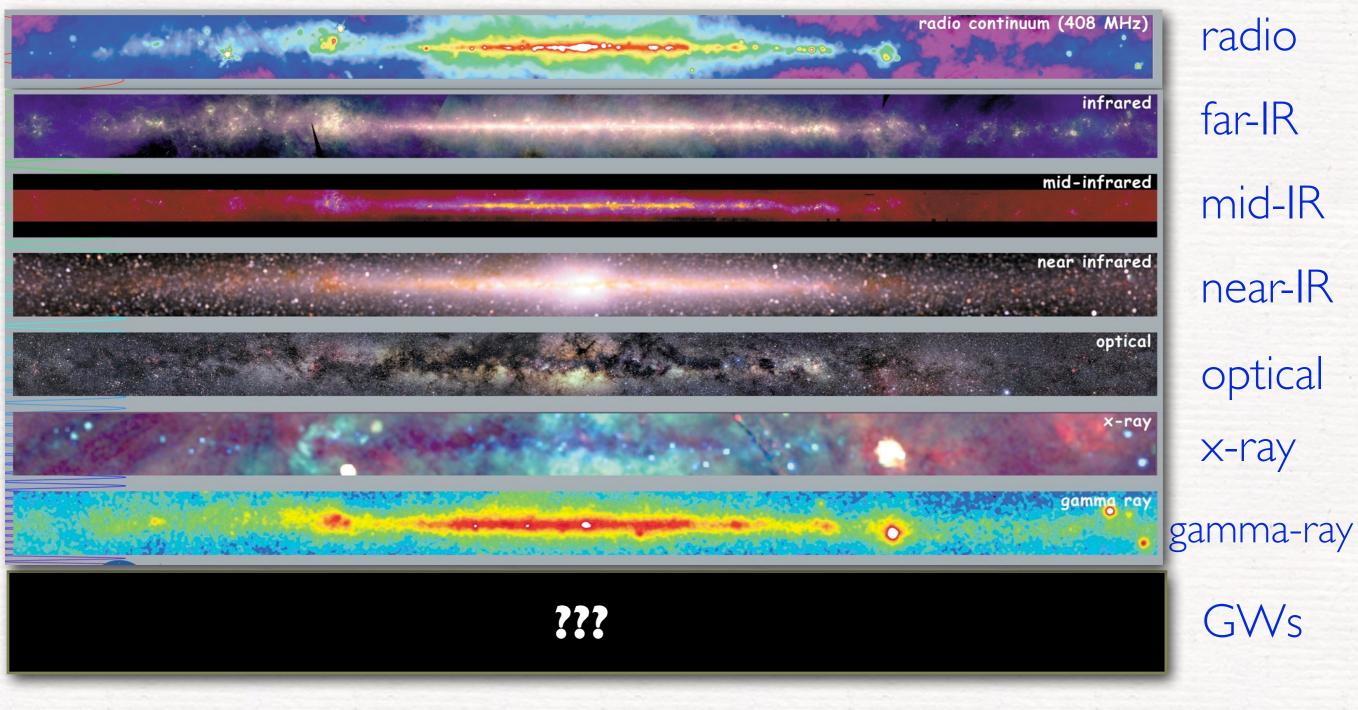
GSFC/NASA



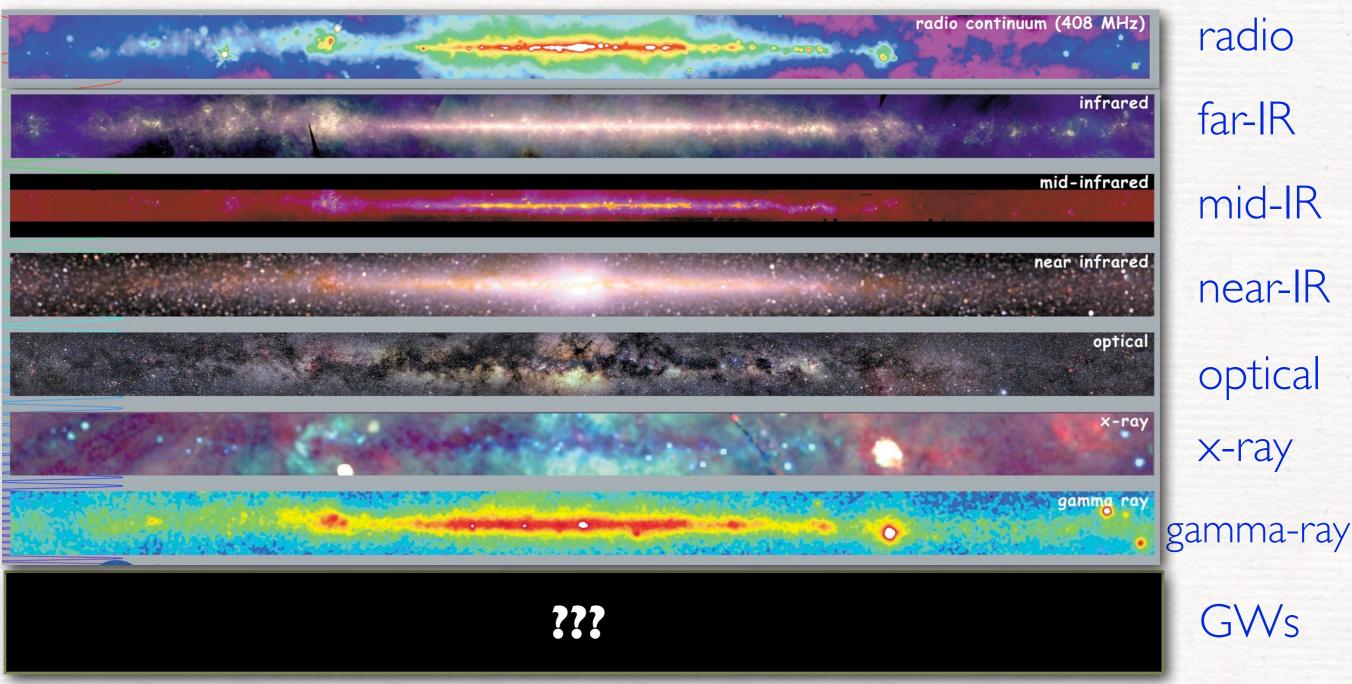
GSFC/NASA



GSFC/NASA



GSFC/NASA



It has happened over and over in the history of astronomy: as a new "window" has been opened, a "new", universe has been revealed. GWs will reveal Einstein's universe of black holes and neutron stars