

# Advanced General Relativity

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## Abstract

This is a course on advanced general relativity and provides an introduction to the study of the solutions of the Einstein equations, either when they lead to static/stationary spacetimes, or, more interestingly, when they involve spacetimes that are dynamical. The course also provides an introduction to the mathematical and numerical techniques presently employed for the accurate solution of the Einstein equations together with those of relativistic hydrodynamics. The first part of the course will concentrate on the mathematical aspects of the solutions of the Einstein equations for compact objects (i.e., black holes, neutron stars), either as static/stationary solutions or when evolved (perturbation theory, gravitational collapse). A second part of the course will provide an introduction to numerical relativity, reviewing the 3+1 formulation of the equations, be it the field equations or those of relativistic hydrodynamics, the definition of hyperbolic system of partial differential equations and the development of nonlinear waves in hydrodynamics. A final part of the course, will concentrate on the numerical aspects and the most advanced techniques for the numerical solution of these equations. The students are expected to be familiar with the theory of General Relativity and to be proficient in differential geometry and tensor calculus. A series of exercises parallels the course. A detailed syllabus and a list of references can be found here. A series of exercises will parallel the course. The content of the lectures can be found in a series of books [1, 2, 3, 4, 5].

## Syllabus and plan of the lectures

Below is an approximate split of the contents of the Course. While most of the time a single topic will be covered in a given week, overlap is possible.

1. Review of the curvature tensor  
Review of the geodesic equations
2. Review of the geodesic-deviation equations  
Review of the Einstein equations
3. Black hole solution: Schwarzschild  
Black hole solution: coordinate choices and conformal maps
4. Black hole solution: Schwarzschild  
Black hole solution: massive particle motion
5. Black hole solution: Schwarzschild  
Black hole solution: massless particle motion and shadows
6. Black hole solution: Kerr  
Black hole solution: particle motion around Kerr and energy extraction
7. Laws of BH dynamics  
Nonrotating relativistic stars
8. Rotating relativistic stars (slow rotation)  
Rotating relativistic stars (rapid rotation)
9. Gravitational collapse to black hole  
Apparent and event horizons
10. Perturbation theory: black holes  
Perturbation theory: relativistic stars
11. Numerical gravity  
3+1 decomposition of spacetime
12. Formulations of the Einstein equations. Lagrangian formulations  
The ADM formulation
13. Conformal traceless formulations  
Gauge conditions in 3+1 formulations
14. Hyperbolic systems of partial differential equations  
Quasi-linear formulation. Conservative formulation

## References

- [1] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (San Francisco: W.H. Freeman and Company)
- [2] Rezzolla L and Zanotti O 2013 *Relativistic Hydrodynamics* (Oxford University Press)
- [3] Alcubierre M 2008 *Introduction to 3+1 Numerical Relativity* (Oxford, UK: Oxford University Press)
- [4] Baumgarte T W and Shapiro S L 2010 *Numerical Relativity: Solving Einstein's Equations on the Computer* (Cambridge, UK: Cambridge University Press)
- [5]ourgoulhon E 2012 *3+1 Formalism in General Relativity (Lecture Notes in Physics, Berlin Springer Verlag vol 846)* (Springer)