Advanced General Relativity

Luciano Rezzolla

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Abstract

This is a course on advanced general relativity and provides an introduction to the study of the solutions of the Einstein equations, either when they lead to static/stationary spacetimes, or, more interestingly, when they involve spacetimes that are dynamical. The course also provides an introduction to the mathematical and numerical techniques presently employed for the accurate solution of the Einstein equations together with those of relativistic hydrodynamics. The first part of the course will concentrate on the mathematical aspects of the solutions of the Einstein equations for compact objects (i.e., black holes, neutron stars), either as static/stationary solutions or when evolved (perturbation theory, gravitational collapse). A second part of the course will provide an introduction to numerical relativity, reviewing the 3+1 formulation of the equations, be it the field equations or those of relativistic hydrodynamics, the definition of hyperbolic system of partial differential equations and the development of nonlinear waves in hydrodynamics. A final part of the course, will concentrate on the numerical aspects and the most advanced techniques for the numerical solution of these equations. The students are expected to be familiar with the theory of General Relativity and to be proficient in differential geometry and tensor calculus. A series of exercises parallels the course. A detailed syllabus and a list of references can be found here. A series of exercises will parallel the course. The content of the lectures can be found in a series of books [1, 2, 3, 4, 5].

Syllabus and plan of the lectures

Below is an approximate split of the contents of the Course. While most of the time a single topic will be covered in a given week, overlap is possible.

1. Review of the curvature tensor
   Review of the geodesic equations
2. Review of the geodesic-deviation equations
   Review of the Einstein equations
3. Black hole solution: Schwarzschild
   Black hole solution: particle motion around Schwarzchild
4. Black hole solution: Kerr
   Black hole solution: particle motion around Kerr
5. Nonrotating relativistic stars
   TOV equations
6. Rotating relativistic stars (slow rotation)
   Rotating relativistic stars (rapid rotation)
7. Gravitational collapse to black hole
   Apparent and event horizons
8. Perturbation theory: black holes
9. Perturbation theory: relativistic stars
10. The 3+1 decomposition of spacetime
11. Formulations of the Einstein equations. Lagrangian formulations
    The ADM formulation
12. Conformal traceless formulations
13. Gauge conditions in 3+1 formulations
    Constraint equations. initial data and constrained evolution
14. Hyperbolic systems of partial differential equations
    Quasi-linear formulation. Conservative formulation

References

[1] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (San Francisco: W. H. Freeman)

[2] Rezzolla L and Zanotti O 2013 *Relativistic Hydrodynamics* (Oxford, UK: Oxford University Press)

[3] Alcubierre M 2008 *Introduction to* $3+1$ *Numerical Relativity* (Oxford, UK: Oxford University Press)

[4] Baumgarte T W and Shapiro S L 2010 *Numerical Relativity: Solving Einstein's Equations on the Computer* (Cambridge University Press, Cambridge UK)

[5] Gourgoulhon E 2012 *3+1 Formalism in General Relativity* (*Lecture Notes in Physics, Berlin Springer Verlag* vol 846)