Screened perturbation theory for 3d YM theory

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- The Linde problem
- The QCD pressure with effective field theory methods
- Resummation and screened perturbation theory

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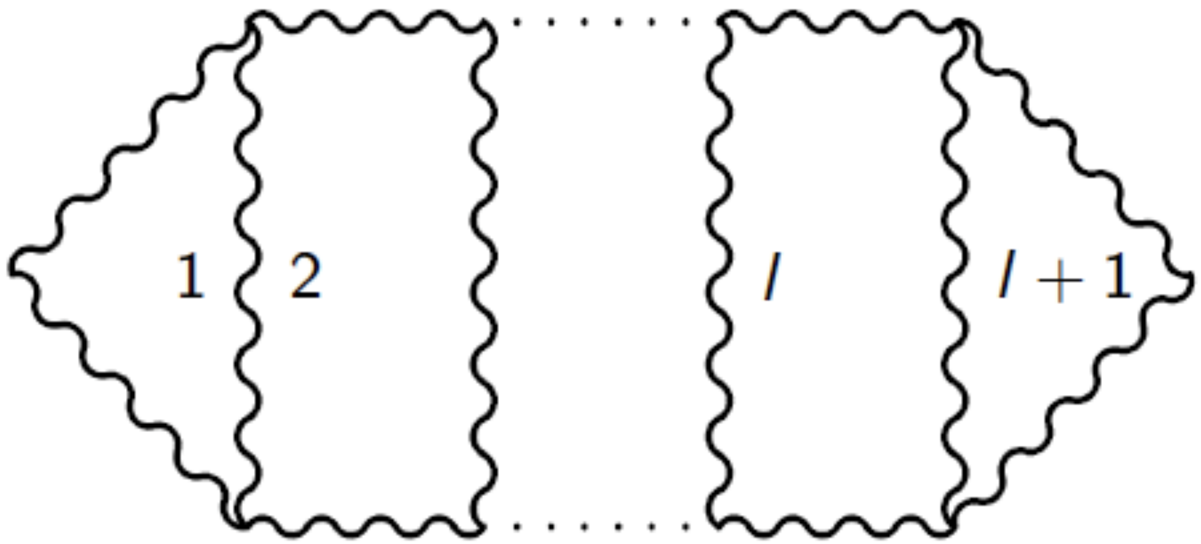
The Linde problem of finite T QCD / 3d YM

- $(l+1)$-loop diagram: contribution to pressure



contribution from Matsubara 0-mode:

$$P \sim g^{2l} \left(T \int d^3p\right)^{l+1} p^{2l}(p^2 + m^2)^{-3l}$$

$$\begin{aligned} g^{2l} \quad & \text{for} \quad l = 1,2 \\ g^6 T^4 \ln(T/m) \quad & \text{for} \quad l = 3 \\ g^6 T^4 (g^2 T/m)^{l-3} \quad & \text{for} \quad l > 3 \end{aligned}$$

magnetic mass $m_{mag} \sim g^2 T \Rightarrow$ **all loops** $(l > 3)$ contribute to g^6

even for weak coupling!

So why bother?

Lattice MC does not work at finite density or real time!

Momentum scales in QGP

- high T and small g $\Rightarrow$ hierarchy of 3 momentum scales:
  1. *hard scale:*
     $2\pi T$ of a typical momentum of a particle in the plasma
  2. *soft scale:*
     gT associated with the screening of the color-electric field A_0^a (EQCD)
  3. *ultra-soft scale:*
     g^2T associated with the screening of the color-magnetic field A_k^a (MQCD)

scale hierarchy

$$g^2 T < gT < 2\pi T$$

High T effective theory: dimensional reduction

$$\int_0^\beta d\tau \int d^d x \mathcal{L}_{QCD} \Rightarrow \sum_{\omega_n} \int d^d x \mathcal{L}_E$$

Step 1: Integrating out all modes with $n \neq 0$ (using dimensional regularisation)

$$p_{QCD}(T) = p_E(T) + \frac{T}{V} \ln \int \mathcal{D}A_k^a \mathcal{D}A_0^a \exp\left(-\int d^d x \mathcal{L}_E\right)$$

$$\mathcal{L}_E = \frac{1}{2} Tr F_{kl}^2 + Tr\,[D_k, A_0]^2 + m_e^2 Tr A_0^2 + \lambda_E^{(1)} (Tr A_0^2)^2 + \lambda_E^{(2)} Tr A_0^4 + \ldots$$

matching coefficients

$$p_E \sim T^4; \quad m_E^2 \sim g^2 T^2; \quad g_E^2 \sim g^2 T; \quad \lambda_E^{(1)} \sim g^4 T; \quad \lambda_E^{(2)} \sim g^4 T$$

- A_0^a becomes an adjoint Higgs field
- still two dynamical scales: $g^2 T, gT$

Step 2: Integrating out A_0

$$\frac{T}{V}\ln\int \mathcal{D}A_k^a\mathcal{D}A_0^a \exp\left(-\int d^dx\mathcal{L}_E\right) = p_M(T) + \frac{T}{V}\ln\int \mathcal{D}A_k^a \exp\left(-\int d^dx\mathcal{L}_M\right)$$

$$\mathcal{L}_M = \frac{1}{2}TrF_{kl}^2 + \ldots$$

- 2 matching coefficients: $p_M \sim m_E^3 T$ and $g_M^2 \sim g_E^2$

scale $g^2 T$ = contribution from 3d YM, starting at 4-loop

$$p_G(T) = \frac{T}{V}\ln\int \mathcal{D}A_k^a \exp\left(-S_M\right) \sim Tg^6$$

$$\frac{p_G(T)}{T\mu^{-2\epsilon}} = d_A C_A^3 \frac{g_M^6}{(4\pi)^4}\left[\alpha_G\left(\frac{1}{\epsilon} + 8\ln\frac{\bar{\mu}}{2m_G}\right) + \tilde{\beta_G}(\xi) + O(\epsilon)\right]$$

The complete result up to order g^6 = 4-loop!

Kajantie et al. 03

$$\frac{p_{QCD}(T)}{T^4\mu^{-2\epsilon}} = \frac{p_E(T)+p_M(T)+p_G(T)}{T^4\mu^{-2\epsilon}} = g^0\left[\alpha_{E1}\right] + g^2\left[\alpha_{E2}\right] + \frac{g^3}{(4\pi)}\left[\frac{d_A}{3}\alpha_{E4}{}^{3/2}\right]$$
$$+ \frac{g^4}{(4\pi)^2}\left[\alpha_{E3} - d_A C_A\left(\alpha_{E4}\left(\frac{1}{4\epsilon}+\frac{3}{4}+\ln\frac{\bar\mu}{2gT\alpha_{E4}{}^{1/2}}\right)+\frac{1}{4}\alpha_{E5}\right)\right]$$
$$+ \frac{g^5}{(4\pi)^3}\left[d_A\alpha_{E4}{}^{1/2}\left(\frac{1}{2}\alpha_{E6} - C_A^2\left(\frac{89}{24}+\frac{\pi^2}{6}-\frac{11}{6}\ln 2\right)\right)\right]$$
$$+ \frac{g^6}{(4\pi)^4}\left[\beta_{E1} - \frac{1}{4}d_A\alpha_{E4}\left((d_A-2)\beta_{E4}+\frac{2d_A-1}{N_c}\beta_{E5}\right)\right.$$
$$- d_A C_A\left(\frac{1}{4}(\alpha_{E6}+\alpha_{E5}\alpha_{E7}+3\alpha_{E4}\alpha_{E7}+\beta_{E2}+\alpha_{E4}\beta_{E3})\right.$$
$$\left. + (\alpha_{E6}+\alpha_{E4}\alpha_{E7})\left(\frac{1}{4\epsilon}+\ln\frac{\bar\mu}{2gT\alpha_{E4}{}^{1/2}}\right)\right)$$
$$+ d_A C_A^3\left(\beta_M+\beta_G+\alpha_M\left(\frac{1}{\epsilon}+8\ln\frac{\bar\mu}{2gT\alpha_{E4}{}^{1/2}}\right)\right.$$
$$\left.\left. +\alpha_G\left(\frac{1}{\epsilon}+8\ln\frac{\bar\mu}{2g^2TC_A}\right)\right)\right]$$

idea: p_E, p_M by (HTL resummed) perturbation theory, p_G non-perturbatively, e.g. 3d lattice

N.B.: coefficients N-independent!

Here: resummation schemes

effective Lagrangian

$$\mathcal{L}_{\text{eff.}} = \frac{1}{l}\left[\mathcal{L}_M(\sqrt{l}X) + \mathcal{L}_\phi(\sqrt{l}X) - l\mathcal{L}_\phi(\sqrt{l}X)\right]$$

- perturbative calculation: power series in l
- subtracted term enters perturbation theory one loop higher than added term
- free effective theory: contribution l^0 to the pressure
- 2-loop-diagrams: contribution l^1 to the pressure

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_0 + \mathcal{L}_m + \mathcal{L}_{int} - \mathcal{L}_m$$

in particular, a mass term for the gluon will regulate the IR divergences

How to do this in a gauge invariant way?

Also resum interactions, such as to maintain ST-identities!

e.g. using Higgs effect, non-linear sigma model:

$$\mathcal{L}_\phi = \quad \mathrm{Tr}[(D_i\phi)^\dagger D_i\phi]$$

$$\phi(x) = \frac{m}{g_M}\mathrm{e}^{i\pi^a(x)T^a}$$

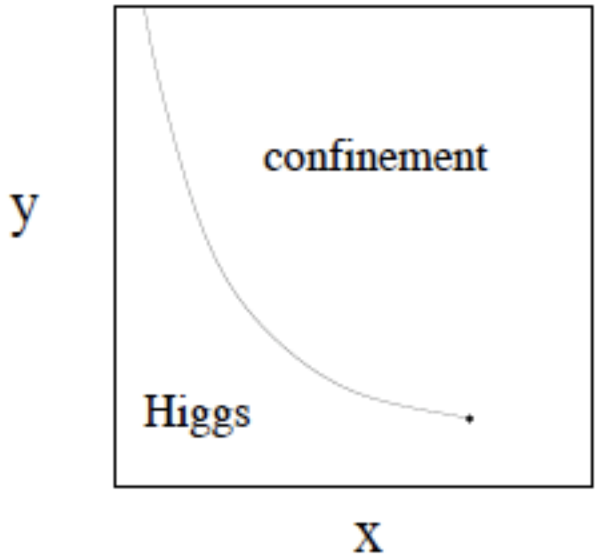
Buchmüller, O.P. 95

$D_i = \partial_i - ig_M A_i^a$ local action, but involves auxiliary field

=limiting case of linear model:

$$\begin{aligned}
\mathcal{L}_\sigma &= \mathrm{Tr}\,(\mathcal{D}_i\Phi)^\dagger(\mathcal{D}_i\Phi) + \mu^2\mathrm{Tr}\,(\Phi^\dagger\Phi) + 2\lambda\mathrm{Tr}\,(\Phi^\dagger\Phi)^2 \\
\phi &= \frac{1}{2}(\sigma + i\tau^a\pi^a) \quad \text{with} \quad T^a = \frac{\tau^a}{2} \\
\sigma^2 &= v^2 + (\pi^a)^2 \quad \text{with} \quad <\phi>= v \\
\mu, \lambda &\rightarrow \infty
\end{aligned}$$

$$x = \frac{\lambda}{g^2}, \quad y = \frac{m_0^2}{g^4}$$



Meaning of auxiliary field?

Jackiw, Pi 97

$$Z = \int DA\, D\phi\, \Delta_{FP} \exp -\frac{1}{l}(S_{YM} + S_\phi - lS_\phi + S_{gf}[A])$$

gauge transform with $A_i \to A_i^U, U = \exp i\pi^a T^a$ (unitary gauge)

integrate $\int D\phi\, \Delta_{FP} e^{-\frac{1}{l}S_{gf}} = 1$

$$\Rightarrow \quad Z = \int DA \exp -(\frac{1}{l}S_{YM} - m^2 \int \mathrm{Tr} A^2 + lm^2 \int \mathrm{Tr} A^2)$$

just YM with gauge-invariantly resummed mass term!

Effective Lagrangian not unique

$\mathcal{L}_\phi = m^2 \,\mathrm{Tr}\, F_\mu \frac{1}{D^2} F_\mu, \quad F_\mu = \frac{1}{2}\epsilon_{\mu\alpha\beta}F_{\alpha\beta}$ non-local Jackiw, Pi 97

Chern-Simons eikonal (HTL inspired) non-local Alexanian, Nair 95

....

- Different gauge invariant additions/subtractions = different resummations
- No small expansion parameter, expansion in dynamically generated number:

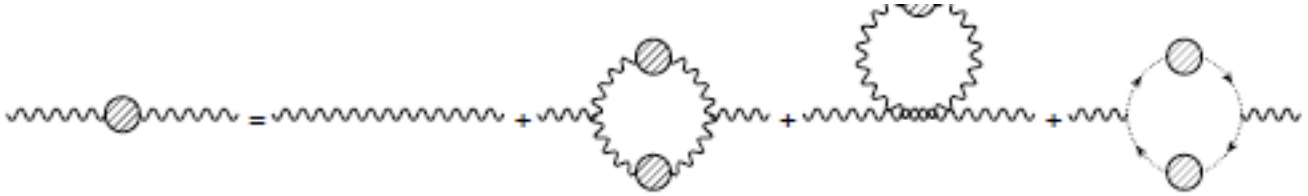
$$\frac{g_M^2}{m} = \frac{g_M^2}{C g_M^2} \quad \text{(times factors} \sim \frac{1}{4\pi}\text{)}$$

- Need to check convergence empirically!

determination of $m = Cg^2 T$ via gap equations ~ Dyson-Schwinger eq., self-consistent

$$D_{\text{trans.}}(p^2) = \frac{1}{p^2 + m^2 - \Pi_{\text{trans.}}(p^2)} \sim \frac{1}{p^2+m^2} \quad \text{for} \quad p^2 = -m^2$$

$$\Pi_{\text{trans.}}(p^2 = -m^2)\left(1 + \frac{\partial \Pi_{\text{trans.}}}{\partial p^2}(p^2 = -m^2)\right) = 0$$



transverse self-energy gauge independent on-shell = pole mass

Alternatively: gap equation from pinch technique, gauge inv. for all p Cornwall 97

Results for m, 1-loop SU(2):

$$\begin{aligned} m|_{BP} &= 0.28 g_M^2 \\ m|_{AN} &= 0.38 g_M^2 \\ m|_{C} &= 0.25 g_M^2 \end{aligned}$$

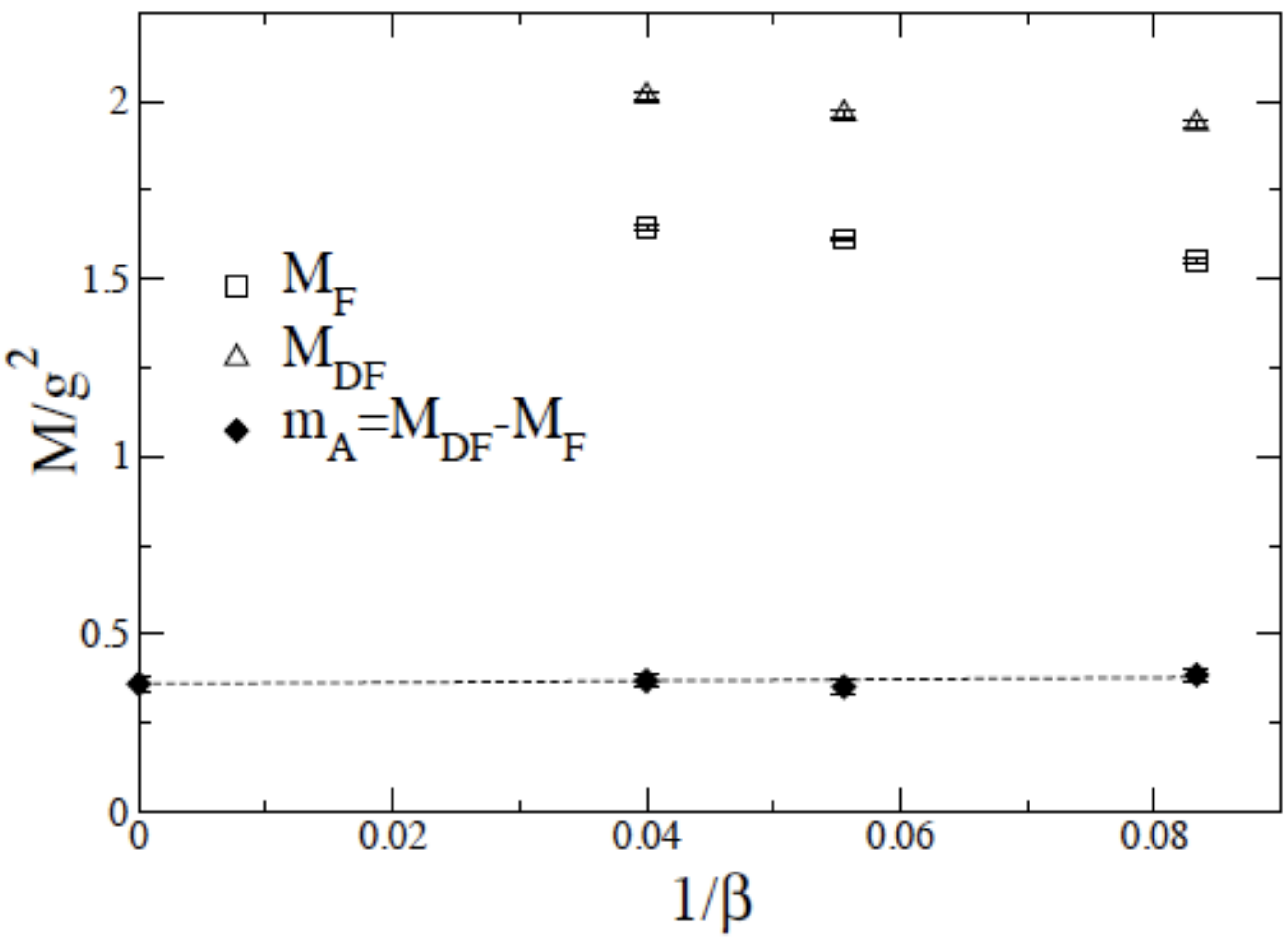
2-loop: ~15% corrections in NLSM scheme Eberlein 98

Lattice: gauge fixed propagators $m \sim 0.35 - 0.46 g_M^2$ Karsch et al.

gauge invariant correlators (cf. 'gluelumps') O.P. 02

$$\frac{\langle (D_i F_{ij})^a(x) U_{ab}^{Ad}(x,y)(D_k F_{lm})^b(y)\rangle}{\langle (F_{ij})^a(x) U_{ab}^{Ad}(x,y)(F_{lm})^b(y)\rangle} \sim \exp[-(3m-2m)|x-y|]$$

$$= 0.36(2) g_M^2$$



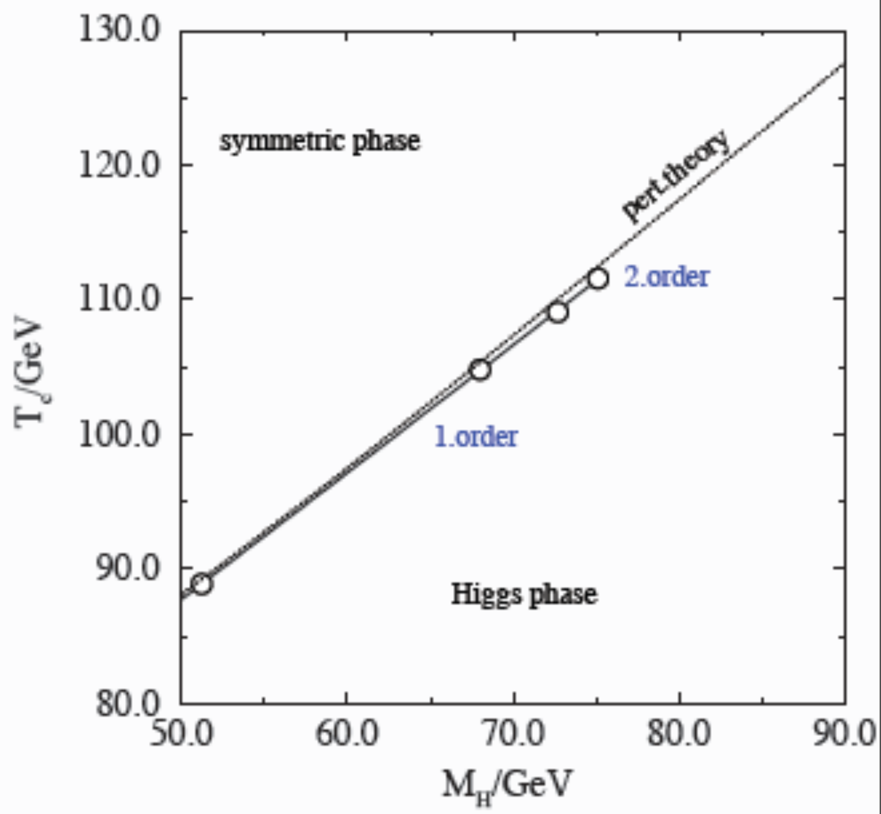
Application to electroweak phase transition

linear model (SU(2)-Higgs) predicts electroweak crossover,
critical Higgs mass ~10% accurate at 1-loop !

Buchmüller, O.P. 97

→ infrared divergent for $m = 0$,
e.g. QCD, symmetric phase electroweak theory

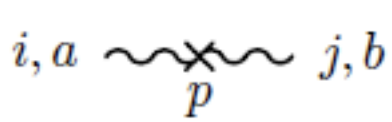
Buchmüller, O.P.
Kajantie et al., Fodor et al.



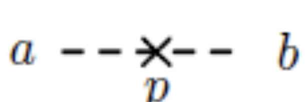
Application to pressure, general covariant gauge, SU(2)

$$\begin{aligned}
\mathcal{L}_{\text{eff.}} = {} & \frac{1}{4}(\partial_i A_j^a + \partial_j A_i^a)^2 + \frac{1}{2\xi}(\partial_i A_i^a)^2 + \frac{1}{2}m^2 A_i^a A_i^a \\
& + \frac{1}{2}(\partial_i \pi^a)^2 + \frac{1}{2}\xi m^2 \pi^a \pi^a + (\partial \bar{c}^a)(\partial c^a) + \xi m^2 \bar{c}^a c^a \\
& + g\sqrt{l} f^{abc} A_i^b A_j^c \partial_i A_j^a + \frac{1}{4} g^2 l f^{abe} f^{cde} A_i^a A_j^b A_i^c A_j^d \\
& + \frac{1}{2} g\sqrt{l} f^{abc} \partial_i \pi^a A_i^b \pi^c + \frac{1}{16}\frac{g^2 l}{m^2}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\pi^a \pi^b \partial_i \pi^c \partial_i \pi^d \\
& - g\sqrt{l} f^{abc} \partial_i \bar{c}^a c^b A_i^c - \frac{1}{2} g\sqrt{l} f^{abc} \xi m \bar{c}^a c^b \pi^c \pi^d \\
& - \frac{1}{8} g^2 l \xi \delta^{ab} \delta^{cd} \bar{c}^a c^b \pi^c \pi^d \\
& - \frac{1}{2\xi} l (\partial_i A_i^a)^2 - \frac{1}{2} m^2 l A_i^a A_i^a \\
& - \frac{1}{2} l (\partial_i \pi^a)^2 - \frac{1}{2}\xi m^2 l \pi^a \pi^a - l(\partial \bar{c}^a)(\partial c^a) - \xi m^2 l \bar{c}^a c^a
\end{aligned}$$

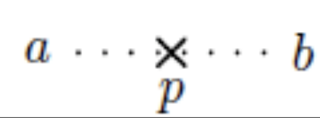
up to 2 loop, O(l)

Counter terms: 

$\Longrightarrow \quad \Gamma^{ab}_{ij,m}(A^2) = m^2 l \delta_{ij} \delta^{ab},$

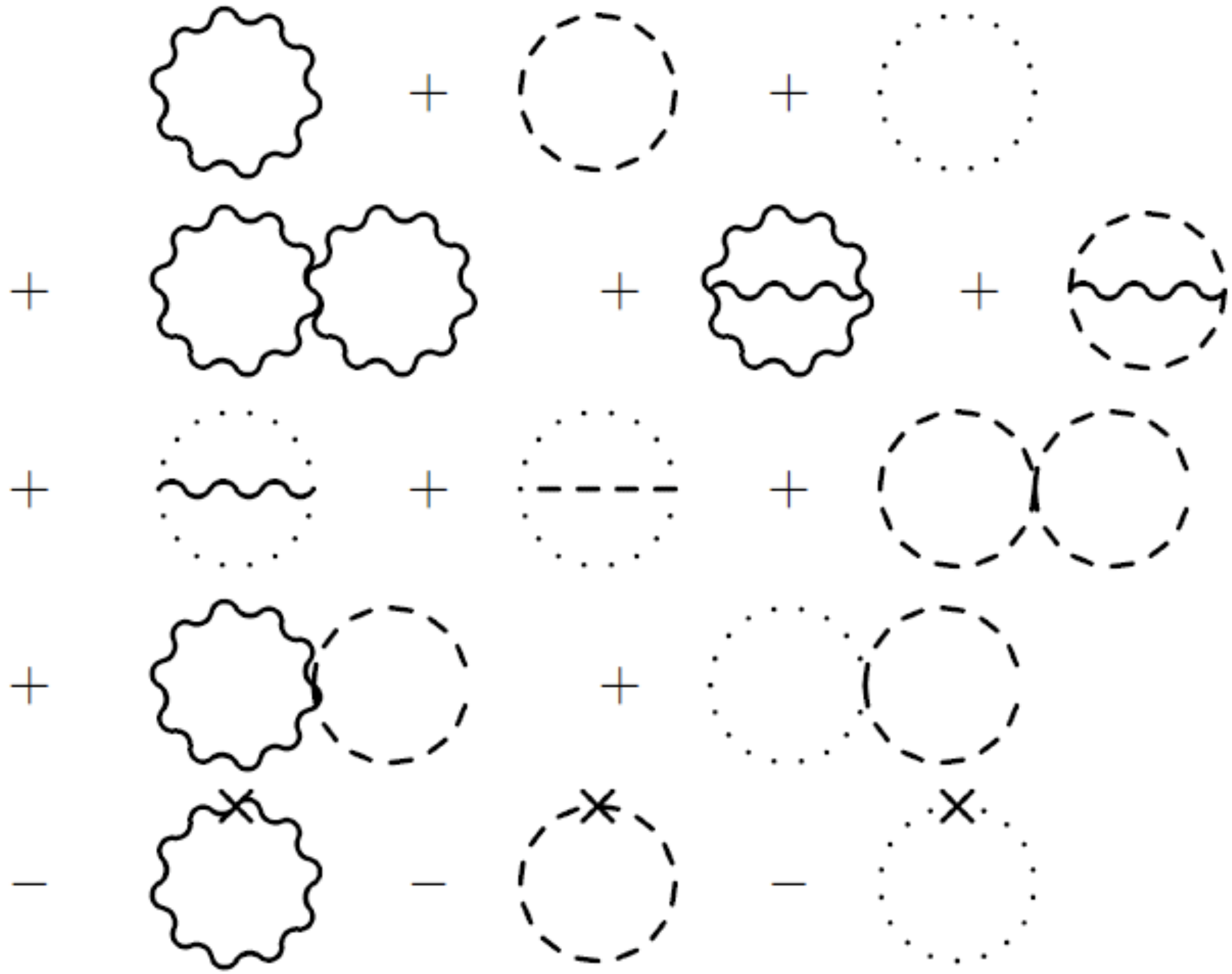


$\Longrightarrow \quad \Gamma^{ab}_{m}(\pi^2) = \xi m^2 l \delta^{ab},$



$\Longrightarrow \quad \Gamma^{ab}_{m}(c^2) = \xi m^2 l \delta^{ab}.$

Feynman diagrams through 2 loops



Result for 3d YM contribution

$$p_{\rm G}(T) = \frac{T}{V}\ln\int \mathcal{D}A_k^a \exp\left(-\int d^dx\,\frac{1}{2}{\rm Tr}\,F_{kl}^2\right)$$

$$\frac{p_{\rm G}(T)}{T\mu^{-2\epsilon}} = N^3(N^2-1)\frac{g_{\rm M}^6}{(4\pi)^4}\left[\alpha_{\rm G}\left(\frac{1}{\epsilon}+8\ln\frac{\bar{\mu}}{2m_{\rm G}}\right)+\beta_{\rm G}+O(\epsilon)\right]$$

$$p_{{\rm G},l^0}(T) = C^3(N^2-1)\frac{g^6}{6\pi}T^4,$$

1 loop

$$p_{\rm G,CT}(T) = -C^3l(N^2-1)\frac{g^6}{4\pi}T^4,$$

1 loop CT

$$\frac{p'_{{\rm G},l^1}}{\mu^{-2\epsilon}T^4} = C^2lN(N^2-1)\frac{g^6}{(4\pi)^2}\left[-\frac{21}{64}\left(\frac{1}{\epsilon}+4\ln\frac{\bar{\mu}}{2Cg^2T}\right)\right.$$
$$\left.+\frac{3}{16}-\frac{21}{32}-\frac{21}{16}\ln\frac{2}{3}+O(\epsilon)\right]$$

2 loop

Convergence?

The coefficients, numbers for

$N = 2,\ C = 0,28$

$$\alpha'_{\mathrm{G}} = -\frac{21}{4}\frac{C^2}{N^2}\pi^2 l \approx -1,015582 l$$

$$\beta_{\mathrm{G},l^0} = \frac{128}{3}\frac{C^3}{N^3}\pi^3 = 3,630132$$

$$\beta_{\mathrm{G},CT} = -64 l \frac{C^3}{N^3}\pi^3 = -5,445198 l$$

$$\beta'_{\mathrm{G},l^1} = -\left(\frac{15}{2} + 21 \ln\frac{2}{3}\right) l \frac{C^2}{N^2}\pi^2 = 0,196301 l$$

- LO + NLO CT naturally same order of magnitude
- NLO contribution ~10% of LO → looks promising(?)
- C~N (LO gap equations), coefficients N-independent!
- 3-loop under way..... 49 diagrams, 13 master integrals...

Conclusions

- Gauge invariant resummation methods for screened non-abelian p.t. at high T
- More flexibility than lattice, but convergence not guaranteed, to be observed in higher order calculations
- For 'magnetic mass,' pressure, NLO-correction ~15% of LO contribution
- In case of apparent convergence: apply to dynamical problems!