Exercise 1: Dimensional regularization - tensor integrals *(4+4+4 points)*
In the lecture you calculated the following (Wick rotated) integral using dimensional regularisation

$$I = \int \frac{1}{[k^2+m^2]^n} \frac{d^dk}{(2\pi)^d} = \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{1}{(m^2)^{n-\frac{d}{2}}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)}. \tag{1}$$

i) Use this result to calculate

$$I(p) = \int \frac{1}{[k^2+2p\cdot k+M^2]^n} \frac{d^dk}{(2\pi)^d}. \tag{2}$$

Hint: Introduce a variable transformation.

ii) Calculate the integral (in euclidean space)

$$I_\mu(p) = \int \frac{k_\mu}{[k^2+2p\cdot k+M^2]^n} \frac{d^dk}{(2\pi)^d}. \tag{3}$$

Hint: Take a derivative with respect to p_μ.

iii) Calculate the integral (in euclidean space)

$$I_{\mu\nu}(p) = \int \frac{k_\mu k_\nu}{[k^2+2p\cdot k+M^2]^n} \frac{d^dk}{(2\pi)^d}. \tag{4}$$

Exercise 2: Dimensional regularization - vanishing integral *(3+3+2 points)*
We want to show that we can set

$$I = \int \frac{1}{k^2} \frac{d^dk}{(4\pi)^d} = 0, \tag{5}$$

in the sense of dimensional regularization.

i) Replace the integral by

$$I_\alpha = \int \frac{1}{(-k^2)^\alpha} \frac{d^dk}{(2\pi)^d}, \tag{6}$$

perform a Wick rotation and show that it is given as

$$I_\alpha = \mathrm{i}\frac{1}{(4\pi)^{\frac{d}{2}}} \frac{1}{\Gamma\left(\frac{d}{2}\right)} \int\limits_0^\infty y^{\frac{d}{2}-\alpha-1} dy, \tag{7}$$

with $y = k^2$. Classify the occuring divercence for $d < 2\alpha$, $d > 2\alpha$ and $d = 2\alpha$.

ii) Split the integral into parts $y > \Lambda^2$, $y < \Lambda^2$ and integrate each part seperately, choosing appropriate $d_{1/2}$.

iii) Finally it is possible to perform an analytic continuation setting $d_1 = d_2$. Do this continuation and show that the two parts you obtained from splitting the integral add exactly to 0.