SS 2024
Prof. Dr. Owe Philipsen

Advanced
**Quantum Field Theory**

27.06.2024
Exercise sheet 10

Exercise 1: 1-loop correction to the gluon propagator *(6+6+4 points)*

In the lecture you discussed the 1-loop correction to the gluon propagator in QCD. You explicitly calculated the 1-loop correction including gluon 3-vertices and ghost field vertices. In this exercise we are going to extend this calculation to the gluon 4-vertex and the quark loop.

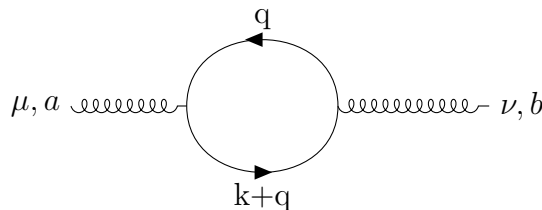   i) Show that the 1-loop correction to the gluon propagator caused by the 4-gluon interaction
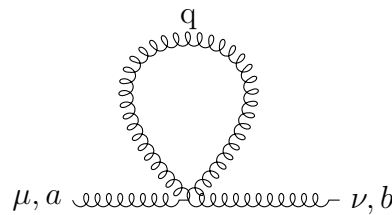
in d-dimensions is given as

$$\mathrm{i}\Pi_{\mu\nu}^{(c),ab} = -g_\epsilon^2 \delta^{ab} N_c(d-1)g_{\mu\nu} \int \frac{d^dq}{(2\pi)^d}\frac{1}{q^2+\mathrm{i}\epsilon} \tag{1}$$

and comment on the divergence.

  ii) Show that the fermion 1-loop correction to the gluon propagator

in d-dimensions is given as

$$\mathrm{i}\Pi_{\mu\nu}^{(d),ab} = -\frac{g_\epsilon^2}{2}\delta^{ab}4\sum_{f=1}^{N_f}\int\frac{d^dq}{(2\pi)^d}\frac{2q_\mu q_\nu - g_{\mu\nu}\left(g^2+k\cdot q-m_f^2\right)+k_\mu q_\nu+k_\nu q_\mu}{(q^2-m_f^2+\mathrm{i}\epsilon)((q+k)^2-m_f^2+\mathrm{i}\epsilon)} \tag{2}$$

and comment on the divergence.

 iii) Consider massless quarks. Use a Wick rotation to change to euclidean space for the fermion loop contribution and use the $J(k), J^\mu(k), J^{\mu\nu}(k)$ familiar from the lecture to regularize your result.