Quantum Field Theory and the Standard Model of Particle Physics

Exercise 1: Compton-scattering (*4+6+4 points*)

Consider the theory of Quantum Electrodynamics for electrons, positrons and photons,

$$\mathcal{L}_{QED} = \bar{\psi}\left[\mathrm{i}\not{D} - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \bar{\psi}\left[\mathrm{i}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}eA_{\mu}\right) - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{1}$$

Compton-scattering is the process of electron-photon scattering

$$e^{-}(p_1, r_1) + \gamma(p_2, \lambda) \to e^{-}(p_1', r_1') + \gamma(p_2', \lambda'). \tag{2}$$

i) Identify all tree level Feynman diagrams of the scattering process and calculate the matrix elements in momentum space using the Feynman rules.

ii) Compute the squared matrix element in the massless limit and average over spins and polarization states by calculating the corresponding sums, leading to

$$\overline{|\mathcal{M}_{f}|^2} = 2e^4\left(-\frac{u}{s} - \frac{s}{u}\right), \tag{3}$$

with Mandelstam variables s and u.

iii) Use your result to calculate the differential cross section in the center-of-mass frame. Why is it not possible to integrate the cross section and obtain the total cross section? How can the problem be fixed?

Hint: For each process involving two external photons the matrix element can be written as

$$\mathcal{M}_{f_i} = \mathcal{M}_{\alpha\beta}(k, k', \ldots)\epsilon^{\alpha}_{\lambda}(k)\epsilon^{\beta\star}_{\lambda'}(k'). \tag{4}$$

As a consequence of gauge invariance one has

$$k^{\alpha}\mathcal{M}_{\alpha\beta} = k^{\beta}\mathcal{M}_{\alpha\beta} = 0. \tag{5}$$

Therefore the second term in the completeness relation of the polarization vectors

$$\sum_{\lambda=1}^{2}\epsilon^{\alpha\star}_{\lambda}(k)\epsilon^{\beta}_{\lambda}(k) = -\left(g^{\alpha\beta} - \frac{k^{\alpha}k^{\beta}}{k^2}\right) \tag{6}$$

will not contribute to the matrix element $\mathcal{M}$ and can be discarded.

Exercise 2: Electron-electron scattering (*4 points*)

Draw and label the Feynman diagrams contributing to this process to leading order. Derive the spin-summed and averaged squared matrix element $\overline{|M_{fi}|^2}$ from that for electron-positron scattering by comparing the respective diagrams and making appropriate momentum substitutions.