Exercise 1: Muon pair production(*4+4+4 points*)

Consider the theory of Quantum Electrodynamics,

$$\mathcal{L}_{QED} = \sum_{f=1}^{2} \bar{\psi}_f \left[\mathrm{i}\not{D} - m_f \right] \psi_f - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \bar{\psi} \left[\mathrm{i}\gamma^\mu \left(\partial_\mu + \mathrm{i}eA_\mu \right) - m \right] \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \tag{1}$$

with $f = 1$ denoting the electron and positron fields with electron mass $m_1 = m_e$ and $f = 2$ the muon and anti-muon fields with muon mass $m_2 = m_\mu$.

Consider the creation of a muon (μ^-) and an anti-muon (μ^+) from electron (e^-) positron (e^+) annihilation

$$e^+(p_1, r_1) + e^-(p_2, r_2) \to \mu^+(p_1', s_1) + \mu^-(p_2', s_2). \tag{2}$$

i) Identify all the Feynman diagrams contributing at tree level ($\mathcal{O}(e^2)$) and write down the corresponding matrix elements in momentum space using the Feynman rules familiar from the lecture.

ii) Compute the spin averaged, squared absolute value of the amplitudes in the high energy limit and in the center of mass frame using Feynman gauge ($\xi = 1$).

iii) Use your result from sheet 8 for the differential cross section of a $2 \to 2$ particle process and show that the differential cross section in the case of muon-pair-production is given as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left(1 + \cos^2 \theta \right), \tag{3}$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine-structure constant, θ the scattering angle. Also determine the total cross-section σ.

Exercise 2: Bhabha-scattering(*4+4 points + 5 bonuspoints*)

Consider the theory of Quantum Electrodynamics for electrons, positrons and photons,

$$\mathcal{L}_{QED} = \bar{\psi}\left[\mathrm{i}\not{D} - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \bar{\psi}\left[\mathrm{i}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}eA_{\mu}\right) - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{4}$$

Electron (e^-) - Positron (e^+) scattering is referred to as Bhabha-scattering

$$e^+(p_1, r_1) + e^-(p_2, r_2) \to e^+(p_1', r_1') + e^-(p_2', r_2'). \tag{5}$$

i) Identify all the Feynman diagrams contributing at tree level ($\mathcal{O}(e^2)$) and write down the corresponding matrix elements in momentum space using the Feynman rules familiar from the lecture.

ii) Compute the spin averaged squared absolute value of the amplitudes (keep in mind that there could be interference terms!) in the high energy limit and in the center of mass frame using Feynman gauge ($\xi = 1$).

*******************************Bonus Part********************************

iii) Recall from sheet 8 that the differential cross section of a $2 \to 2$ particle process was given as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{f_i}|^2, \tag{6}$$

in the case of equal masses for incoming and outgoing particles. Use your previous result to show that the differential cross section in the case of Bhabha-scattering is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left(\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} + \frac{1+\cos^2(\theta)}{2} - 2\frac{\cos^4(\theta/2)}{\sin^2(\theta/2)}\right), \tag{7}$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine-structure constant, θ the scattering angle and E is the respective energy of electron and positron in the center-of-mass system.

Hint: Use Mandelstam variables.