Lecture III

- Putting the path integral on the lattice
- Lattice QCD
- Some numerical results on non-perturbative physics

Recall Euclidean path integral for scalar fields

$$\phi(\mathbf{x}, t)$$

$$\mathbf{x}$$

$$\prod_t \prod_{\mathbf{x}} \mathrm{d}\phi(\mathbf{x}, t) \equiv \mathcal{D}\phi$$

$$S = \int \mathrm{d}t \, \mathrm{d}^3 x \, \mathcal{L}$$

Example:

$$\mathcal{L}(x) = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{m_0^2}{2} \phi^2(x) - \frac{g_0}{4!} \phi^4(x)$$

$$t = -i\tau$$

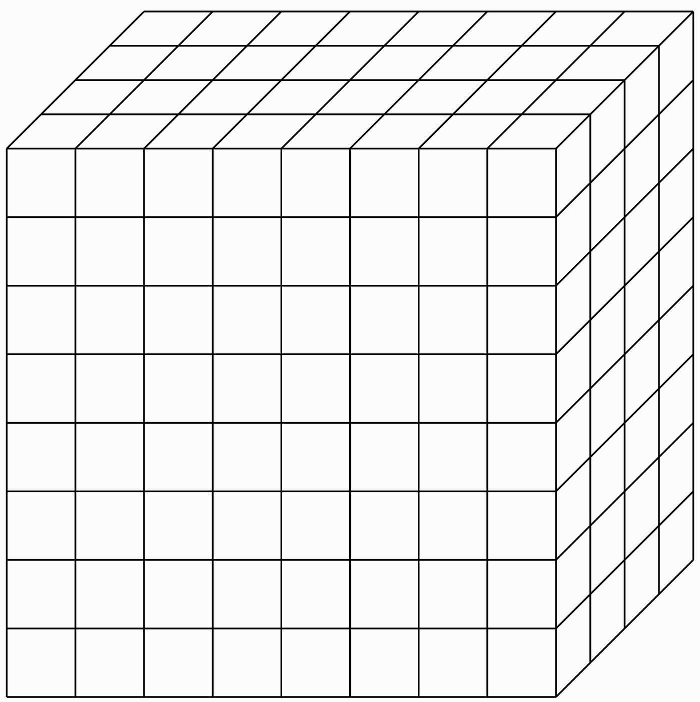
- no oscillatory integrand
- strongly fluctuating fields exp. suppressed

$$G_E(x_1, \ldots, x_n) = \frac{1}{Z} \int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) \, \mathrm{e}^{-S_E}$$

$$Z = \int \mathcal{D}\phi \, \mathrm{e}^{-S_E}$$

Lattice formulation of Euclidean QFT's

$\mathbb{R}^4 \quad \to \qquad x = \{x_\mu \,|\, \mu = 1, \ldots, 4\} \in a\mathbb{Z}^4 \qquad a \quad : \quad$ lattice spacing (or constant)



- $\phi(x)$: living on the lattice sites only
- partial derivatives $\to$ finite differences:

$$\partial_\mu \phi \quad \to \quad \triangle_\mu^{(*)} \phi(x) = \frac{\pm\phi(x \pm a\hat{\mu}) \mp \phi(x)}{a}$$

$\Rightarrow$ forward & backward lattice derivatives

Rotation symmetry:

$SO(4) \longrightarrow D_h^4$

$$\int \mathrm{d}^4 x \quad \to \quad \sum_x a^4 \qquad\qquad \mathcal{D}\phi \to \prod_x \mathrm{d}\phi(x) \equiv \mathcal{D}[\phi]$$

(infinite-dimensional) integration measure well defined on discrete system!

➡ finite numbers on finite lattice!

Lattice action:

$$S = \sum_x a^4 \left\{ \frac{1}{2} \sum_{\mu=1}^{4} [\triangle_\mu \phi(x)]^2 + \frac{m_0^2}{2} \phi^2(x) + \frac{g_0}{4!} \phi^4(x) \right\}$$

Fourier transform : $\tilde{\phi}(p) = \sum_x a^4 \mathrm{e}^{-ipx} \phi(x)$ periodic

$\Rightarrow$ restrict momenta to Brillouin zone $-\pi/a < p_\mu \leq \pi/a$

inverse Fourier transform:

$$\phi(x) = \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{e}^{ipx} \tilde{\phi}(p) \quad \Leftrightarrow \quad \text{ultraviolet cutoff } |p_\mu| \leq \frac{\pi}{a}$$

$\Rightarrow$ field theories on a lattice are naturally regularized

finite volume $V = L^3 L_\mathrm{t}$

coordinates $x_\mu = a n_\mu \,,\; n_\mu = 0, 1, 2, \ldots L_\mu - 1$

boundary cond. (periodic)

momenta $p_\mu = \dfrac{2\pi}{a L_\mu} \times l_\mu \,,\; l_\mu = 0, 1, 2, \ldots L_\mu - 1$

momentum integrations $\displaystyle \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \rightarrow \frac{1}{a^4 V} \sum_{l_\mu}$

Transfer matrix formalism

Euclidean time evolution operator over interval T:

$$e^{-\mathbb{H}T} = \lim_{\tau\to 0} \mathbb{T}^N \quad \Leftrightarrow \quad \mathcal{Z} = \lim_{\tau\to 0} \mathrm{Tr}\left\{\mathbb{T}^N\right\} \qquad \tau = T/N$$

governs evolution of states:

$$|\Psi_{\tau+1}\rangle = \mathbb{T}|\Psi_\tau\rangle \qquad \Psi_{\tau+1}(\Phi) = \int \prod_{\mathbf{x}} \mathrm{d}\phi'(\mathbf{x})\, \mathbb{T}[\Phi,\Phi']\Psi_\tau(\Phi')$$

Euclidean Green functions: $N_k = [x_0^{(k)} - x_0^{(k+1)}]/\tau \qquad x_0^{(1)} \geq x_0^{(2)} \geq \cdots \geq x_0^{(n)}$

$$\begin{aligned} G_{\mathrm{E}}(x_1,\ldots,x_n) &= \lim_{T\to\infty}\lim_{\tau\to 0} \frac{\mathbb{T}^{N_0}\phi(\mathbf{x}_1,0)\mathbb{T}^{N_1}\phi(\mathbf{x}_2,0)\cdots\phi(\mathbf{x}_n,0)\mathbb{T}^{N_n}}{\mathrm{Tr}\left\{\mathbb{T}^N\right\}} \\ &= \lim_{T\to\infty}\lim_{\tau\to 0} \langle\phi(x_1)\phi(x_2)\cdots\phi(x_n)\rangle \end{aligned}$$

$\Rightarrow$ the lattice Hamiltonian is defined by

$$\mathbb{H} = -\frac{1}{a}\ln\mathbb{T}$$

- for physical theories, the transfer matrix is self-adjoint and positive
- provides connection between Euclidean path integral and Hamiltonian
- useful for spectral analysis of correlation functions

$$\alpha = 0, 1, 2, \ldots \qquad \mathbb{T}|\alpha\rangle = \lambda_\alpha|\alpha\rangle$$

$$\lambda_0 \geq \lambda_1 \geq \lambda_2 \geq \cdots \qquad E_\alpha = -\frac{1}{a}\ln\lambda_\alpha$$

$$\mathbb{T} = \sum_{\alpha=0}^{\infty}\lambda_\alpha|\alpha\rangle\langle\alpha|$$

Use in two-point euclidean Green function:

$$\langle\phi(x)\phi(y)\rangle = \sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty} \mathrm{e}^{-(E_\alpha-E_0)(T-t)} M_{\alpha\beta}(\mathbf{x}) \mathrm{e}^{-(E_\beta-E_0)t} M_{\beta\alpha}(\mathbf{y})$$

$$\xrightarrow{T\to\infty} \sum_{\beta=0}^{\infty} \mathrm{e}^{-(E_\beta-E_0)t} M_{0\beta}(\mathbf{x}) M_{\beta 0}(\mathbf{y})$$

$$M_{\alpha\beta}(\mathbf{x}) = \langle\alpha|\phi(\mathbf{x},0)|\beta\rangle : \quad \text{matrix elements}$$

Extract low lying spectrum of the theory from exponential decay at large t

(in momentum space, these are the poles of the propagators)

Formalism carries over to gauge theories!

The continuum limit

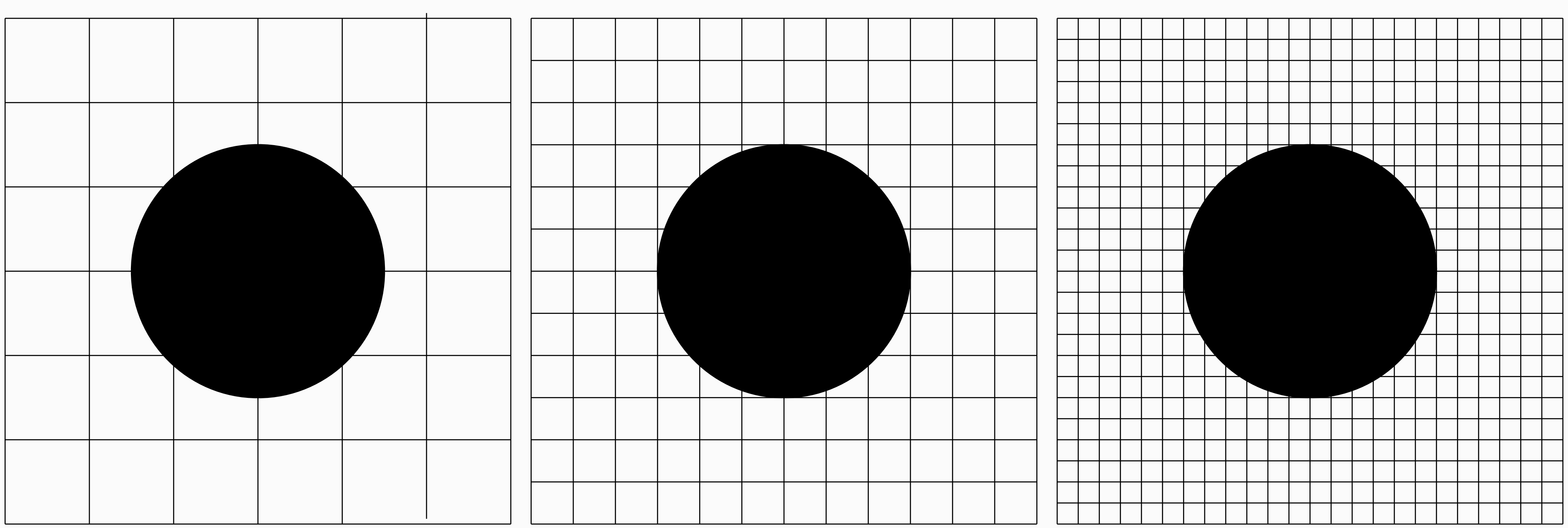
To be taken in a controlled way through series of simulations:

lattice spacing $a \to 0$ while physical (= renormalized) masses

should take *finite* limits   line of constant physics in bare parameter space

$$\begin{array}{lcl} a \text{ dimensionful} & \Rightarrow & \text{fix some mass scale } m \\ \mathrm{CL}: \quad am \to 0 & \Leftrightarrow & \dfrac{1}{am} \equiv \xi \to \infty \\ \xi & : & \text{correlation length} \end{array}$$

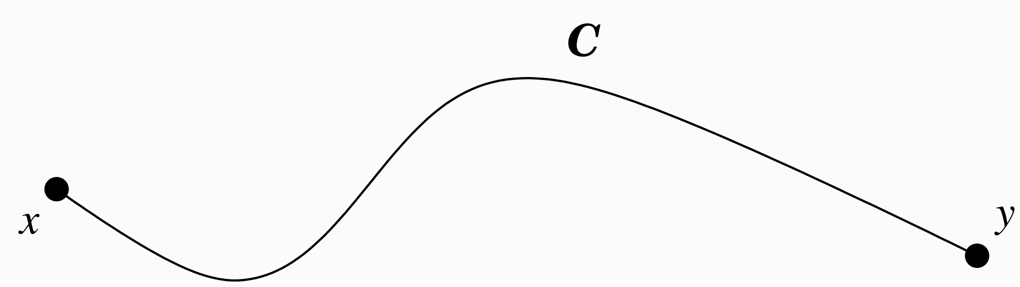
thus: the CL is associated with a *critical point* of the theory



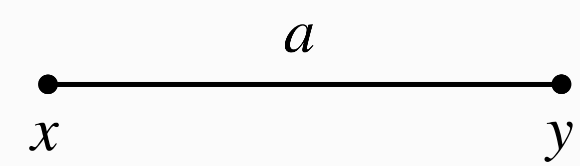
SU(N) gauge theory on a lattice

Gauge fields: cf. continuum parallel transport

$\psi(y) = \mathcal{P}\exp\left[ig\int_x^y dz_\mu A_\mu(z)\right]\psi(x)$



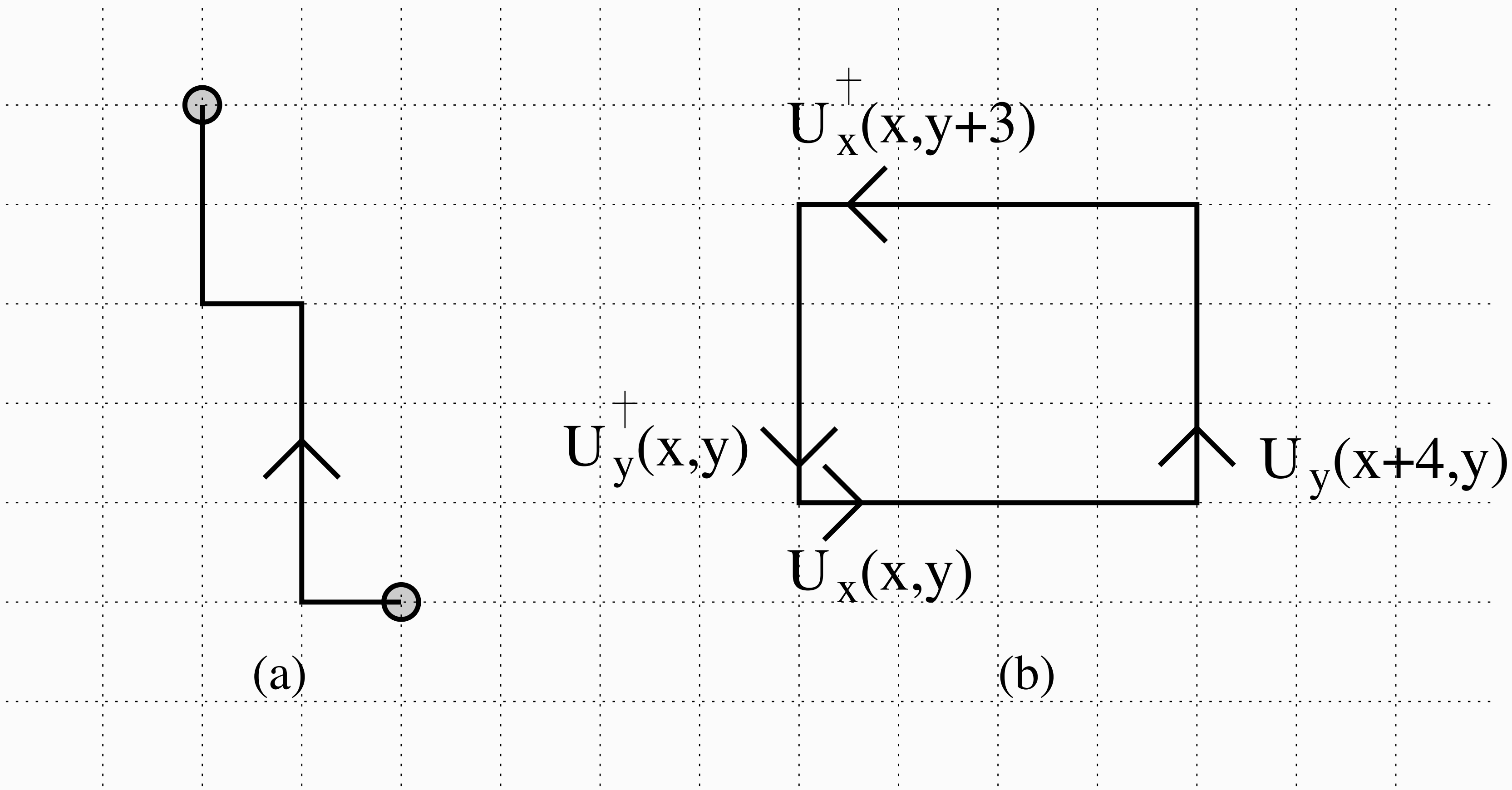
Links=parallel transp. by a: $U_\mu(x) = \mathrm{e}^{-\mathrm{i}agA_\mu(x)}$



Gauge trafo: $\psi^g(x) = g(x)\psi(x)$, $U^g_\mu(x) = g(x)U_\mu(x)g^\dagger(x+\hat{\mu})$

Covariant derivative: $D_\mu\psi(x) \to a^{-1}\left(U_\mu(x)\psi(x+\hat{\mu}) - \psi(x)\right) + O(a)$

Two kinds of gauge invariant objects $\longrightarrow$ observables



transforms adjoint:

$$U_C^g(x) = g(x)U_C(x)g^{-1}(x)$$

$$\Rightarrow \quad \mathrm{Tr}U_C^g = \mathrm{Tr}U_C(x)$$

Discretisation respects gauge invariance, independent of a!

smallest loop: "plaquette" $\quad U_p(x) \equiv U^\dagger(x,\nu)U^\dagger(x+a\hat{\nu},\mu)U(x+a\hat{\mu},\nu)U(x,\mu)$

$$\square \rightarrow \; 1 + ia^2 gF_{\mu\nu} \; - \; \frac{a^4 g^2}{2} F_{\mu\nu}F^{\mu\nu} \; + \; O(a^6) \; + \; \ldots$$

$U_\mu(x) = \mathrm{e}^{-\mathrm{i}agA_\mu(x)}$

$$\square \rightarrow 1 + ia^2 g F_{\mu\nu} - \frac{a^4 g^2}{2} F_{\mu\nu} F^{\mu\nu} + O(a^6) + \ldots$$

Wilson action:

$$S_\mathrm{g}[U] = \beta \sum_p \left\{ 1 - \frac{1}{N} \mathrm{Re}\, [\mathrm{Tr}\, U(p)] \right\} \qquad \sum_p = \sum_x \sum_{1 \le \mu < \nu \le 4}$$

$$\beta = \frac{2N}{g_0^2}$$ lattice gauge coupling

reproduces SU(N) Yang-Mills in continuum limit; for finite a not unique!

- action gauge-invariant for any lattice spacing
- real, positive

Quantum theory: $\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U]\, O\, \mathrm{e}^{-S_\mathrm{g}[U]} \qquad \mathcal{Z} = \int \mathcal{D}[U]\, \mathrm{e}^{-S_\mathrm{g}[U]}$

$$\mathcal{D}[U] = \prod_b \mathrm{d}U(b)$$

with Haar measure $\int_G f(U)\,\mathrm{d}U = \int_G f(VU)\,\mathrm{d}U = \int_G f(UV)\,\mathrm{d}U \qquad \int_G \mathrm{d}U = 1$

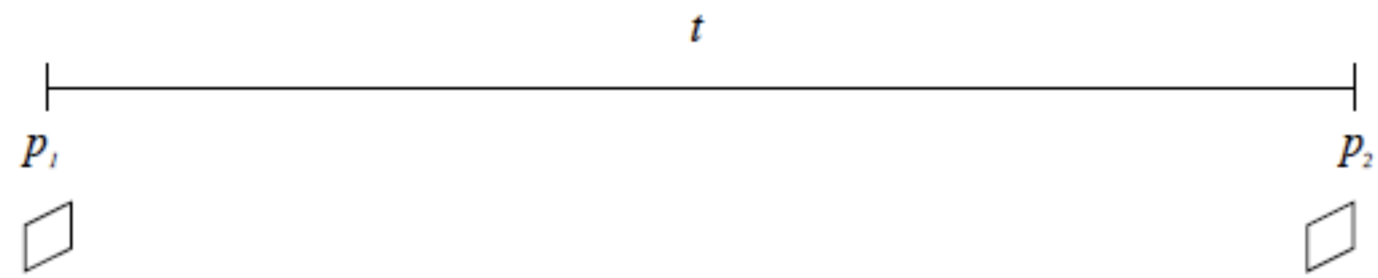
- Path integral exists: $S_g[U]$ real positive, compact integration range, finite no. of integrations

  ➡ result finite

- Integration requires no gauge fixing, except for doing perturbation theory

Observables in lattice pure gauge theory

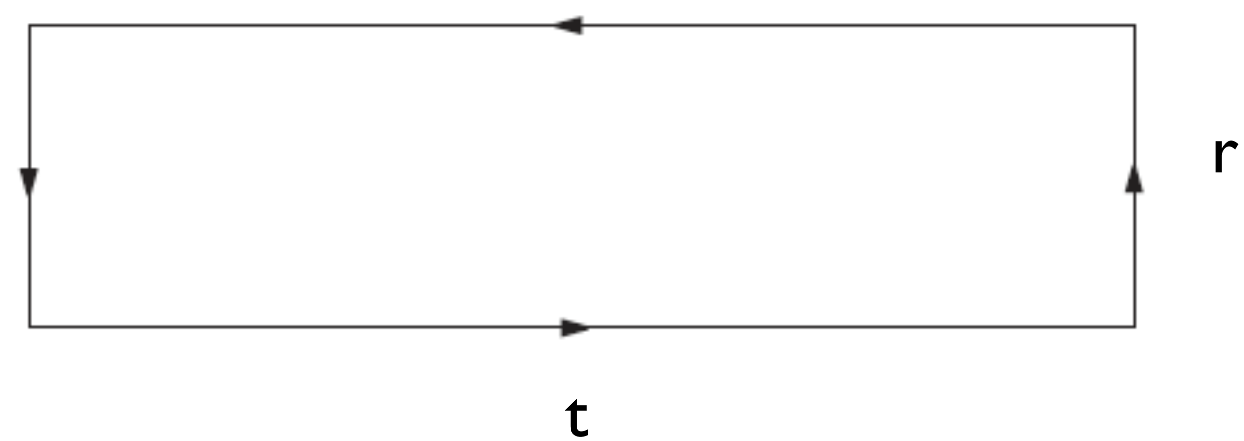
Plaquette correlation function:



Spectral analysis, large t: $\langle \mathrm{Tr}(U(p_1))\mathrm{Tr}(U(p_2))\rangle_c \sim \exp -mt$

Glueball: colour singlet, bound state of gluons

Wilson loop:



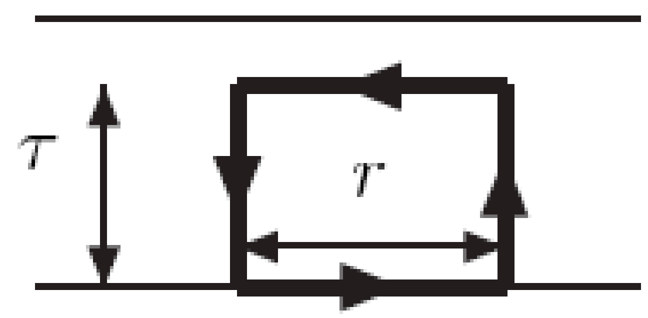
Spectral analysis, large t: $\langle \mathrm{Tr}(U(\mathcal{C}))\rangle \sim \exp -\alpha\, \mathrm{r}\, \mathrm{t}$

area law = confinement

The static potential from the Wilson loop

- Euclidean correlator of gauge invariant meson operator
- integrate out quarks in the limit $M \to \infty$

$$\langle \bar{\psi}(\mathbf{x},\tau)U(\mathbf{x},\mathbf{y};\tau)\psi(\mathbf{y},\tau)\bar{\psi}(\mathbf{y},0)U^\dagger(\mathbf{x},\mathbf{y};0)\psi(\mathbf{x},0)\rangle \longrightarrow \mathrm{e}^{-2M\tau}W_E(|\mathbf{x}-\mathbf{y}|,\tau)$$



$U(\mathbf{x},\mathbf{y};0) \equiv U_r(0)$

$$r = |\mathbf{x}-\mathbf{y}|$$

- spectral decomposition, $T \to 0$ $\qquad W_E(r,\tau\to\infty) \longrightarrow c_{01}^2[U(\mathbf{x},\mathbf{y};0)]\,\mathrm{e}^{-V(r)\tau}$

energy of static quark-antiquark pair

Area law: signal for confinement!

=static potential $\quad V(r) \overset{r\to\infty}{\longrightarrow} \sigma r$

Fermions on the lattice = headache

functional integral quantization: define fermionic path integrals

$$\langle 0|A|0\rangle = \frac{1}{Z}\int \mathcal{D}[\psi]\mathcal{D}[\overline{\psi}]\, A\, \mathrm{e}^{-S_{\mathrm{f}}[\psi,\overline{\psi}]}$$

with

$$\text{measure} \quad : \quad \mathcal{D}[\psi]\mathcal{D}[\overline{\psi}] = \prod_x \prod_\alpha \mathrm{d}\psi_\alpha(x)\,\mathrm{d}\overline{\psi}_\alpha(x)$$

$$\text{free field action} \quad : \quad S_{\mathrm{f}}[\psi,\overline{\psi}] = \int \mathrm{d}^4x\, \overline{\psi}(x)\,(\gamma_\mu\partial^\mu + m)\,\psi(x)$$

Grassmann variables:

$$\int \mathcal{D}[\psi]\mathcal{D}[\overline{\psi}]\, \exp\left\{-\int \mathrm{d}^4x\,\overline{\psi}(x)Q\psi(x)\right\} = \det Q$$

Fermion determinant

Naive lattice action: $$S_{\mathrm{f}}[\psi,\overline{\psi}] = \frac{1}{2}\sum_x \sum_\mu \overline{\psi}(x)\,(\gamma_\mu \triangle_\mu + m)\,\psi(x) + \mathrm{h.c.}$$

with momentum space propagator

$$\tilde{\triangle}(k) = \frac{-i\sum_\mu \gamma_\mu \sin k_\mu + m}{\sum_\mu \sin^2 k_\mu + m^2}$$

physical pole at zero momentum

additional poles at $k_\mu = \pm\pi$ because of periodicity!

→ describes 16 instead of 1 particle: `fermion doubling' problem

Nielsen-Ninomiya theorem:

There exists no local, chiral fermion actions without doublers

Fixes: pick your poison

- Wilson fermions
  add irrelevant ops. (going away in CL) to make doublers very massive
  breaks chiral symmetry for non-zero a

- staggered (Kogut-Susskind) fermions
  distribute spinor components on different sites, reduces to 4 flavours
  take 4th root of determinant to get to one flavour, keeps reduced chiral symm.
  non-local operation, have to take CL before chiral limit, mixing of spin, flavour

- domain wall fermions
  introduce 5th dimension, fermions massive in that dim. and chiral in the other
  expensive

- overlap fermions
  non-local formulation with modified chiral symmetry even for finite a
  two orders of magnitude more expensive than Wilson

Wilson fermions

- Add a term to D_{naive} which formally vanishes as $a \to 0$:

$$D_{\mathrm{w}} + m_f = \frac{1}{2}\gamma_\mu \left(\nabla_\mu + \nabla_\mu^*\right) + ar\nabla_\mu^*\nabla_\mu + m_f$$

$$\widetilde{D}_{\mathrm{w}}(p) = i\gamma_\mu \frac{1}{a}\sin(ap_\mu) + \frac{2r}{a}\sin^2\left(\frac{ap_\mu}{2}\right) \quad \text{(free theory)}$$

$\Rightarrow$ Mass of doubler states receives contribution $\propto r/a$: pushed to cutoff scale

:-) Complete lifting of degeneracy

:-(Explicit breaking of chiral symmetry:

even for $m_f = 0$ the action is no longer invariant under

$$\psi(x) \to \mathrm{e}^{i\alpha\gamma_5}\psi(x), \qquad \overline{\psi}(x) \to \overline{\psi}(x)\mathrm{e}^{i\alpha\gamma_5}$$

Mostly acceptable, but makes things more complicated

Lattice perturbation theory

Classical vacuum: $A_\mu = 0 \rightarrow U_\mu(x) = 1$

➡ expand $U(x,\mu) = \exp\left\{ig_0 a A_\mu^b(x) T_b\right\}$ in powers of the bare coupling g_0

➡ $$S_{\rm g}[U] = \sum_x a^4 \left\{L_2 + g_0 L_3 + g_0^2 L_4 + \cdots\right\}$$

➡ $$\prod_{x,\mu} {\rm d}U(x,\mu) \;=\; {\rm e}^{-S_{\rm m}[A]} \prod_{x,\mu,a} {\rm d}A_\mu^a(x)$$

$$S_{\rm m}[A] \;=\; \sum_{x,\mu} \frac{N}{24} a^2 g_0^2 A_\mu^b(x) A_\mu^b(x) + {\rm O}(A^4)$$

- propagators and vertices different from continuum, n-point self-interactions!
- complicated and clumsy, but sometimes necessary for
  -determination of RG functions
  -matching of continuum and lattice renormalisation schemes
  -determination of lattice cut-off effects
  ......

Strong coupling expansion for pure gauge theory

expand the Boltzmann factor in powers of $\beta \propto 1/g_0^2$ as

$$\exp\left\{\beta\,\frac{1}{N}\mathrm{Re}\,[\mathrm{Tr}\,U(p)]\right\} = 1 + \beta\,\frac{1}{N}\mathrm{Re}\,[\mathrm{Tr}\,U(p)] + \cdots$$

amounts to diagrammatic representation in terms of plaquettes

'famous' leading order predictions: Wilson 1974

$$\langle \mathrm{Tr}\,U\,(\mathcal{C}_{R,T})\rangle \overset{R,T\to\infty}{\sim} C\,\mathrm{e}^{-\sigma RT} \quad \text{with} \quad \sigma = -\ln\beta + \cdots$$

$$C_{\mathrm{p}}(t) \overset{t\ \text{large}}{\sim} \mathrm{e}^{-m_{\mathrm{G}}t} \quad \text{with} \quad m_{\mathrm{G}} = -4\ln\beta + \cdots$$

[SU(N) LGT is confining for all values of the gauge coupling]

- Rewrite partition function as sum over sets of plaquettes

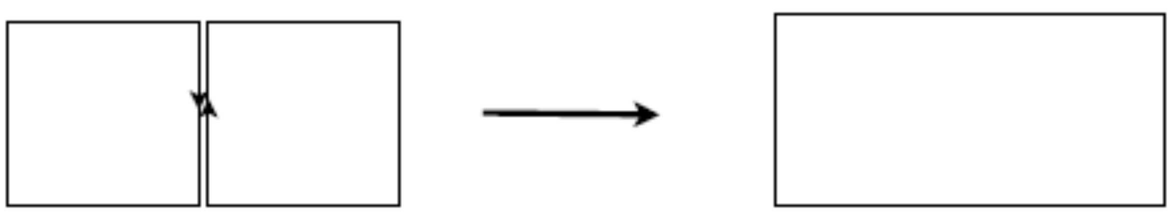
$$Z = \prod_p \int DU_b \exp(-S_p)$$

$$= \prod_p c_0(\beta) \int DU_b \left\{ 1 + \sum_{r\neq 0} c_r(\beta)\chi_r(U_p) \right\} = c_0^{6\Omega} \sum_G \Phi(G)$$

- Integration rule 1

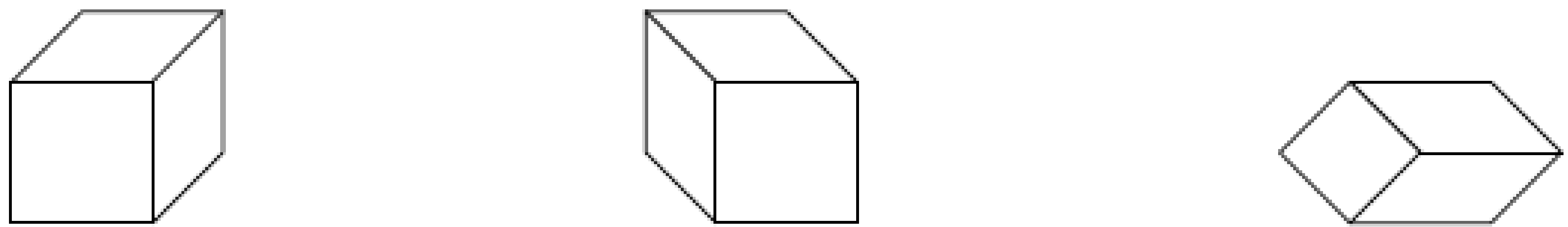
$$\int dU \chi_r(U) = \delta_{r,0}$$

- $\longrightarrow$ Graphs must have closed surfaces
- Integration rule 2

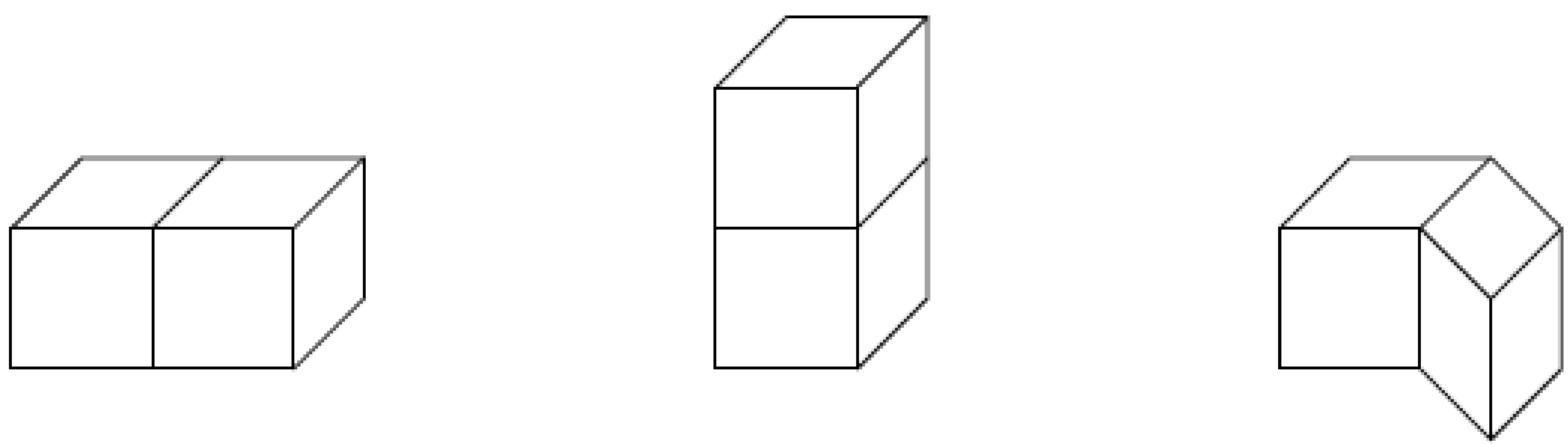


$$\int dU \chi_r(UV)\chi_s(U^{-1}W) = \delta_{rs}\frac{1}{d_r}\chi_r(VW)$$

- Leading order graphs



- Corrections



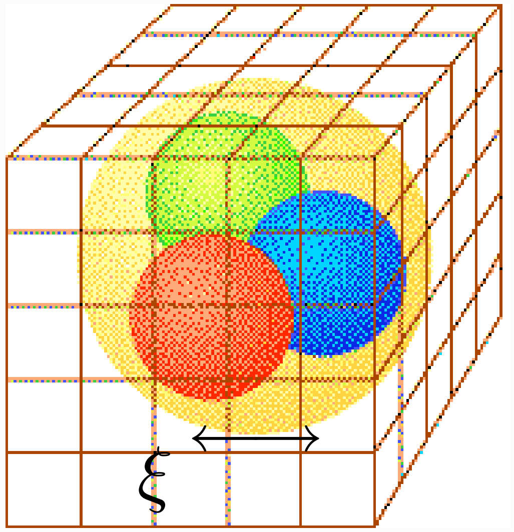
Monte Carlo evaluation

$$Z = \int D\bar{\psi} D\psi DU \, \mathrm{e}^{-S_g[U]-S_f[U,\bar{\psi},\psi]} = \int DU \prod_f (\det M) \, \mathrm{e}^{-S_g[U]}$$

Systematics: finite V,a effects

for hadron with m_H, $\xi \sim m_H^{-1}$

$a \ll \xi \ll aL$!

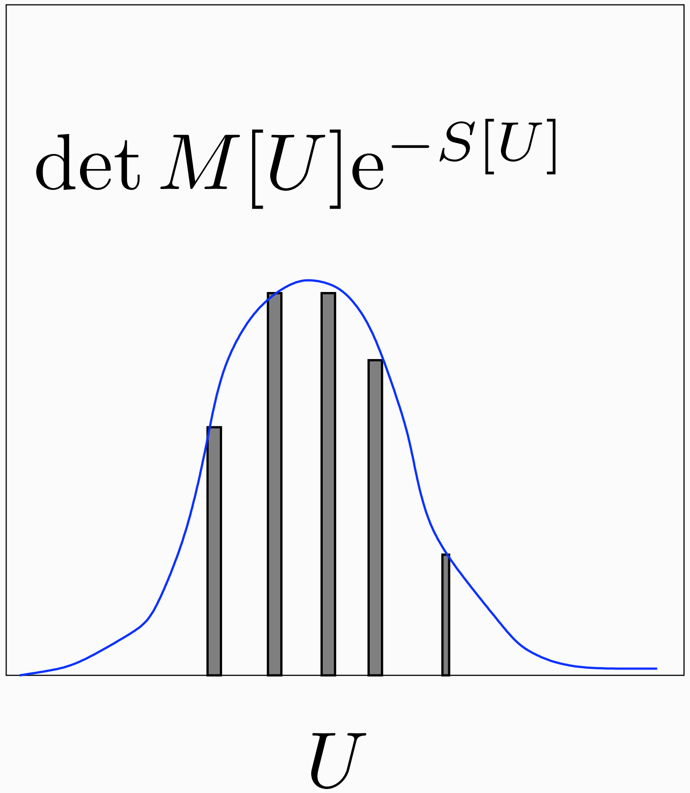


$\Rightarrow$e.g. $30^4 \sim 10^6$ lattice points

every point $\Rightarrow$4 U's, every $U \in$ SU(3) $\Rightarrow$8 independent components $\boxed{\Rightarrow 10^8\text{-dimensional integral!}}$

very peaked integrand!

$\Rightarrow$Monte Carlo integration, importance sampling



Markov process: ensemble

$$\{U_1\} \rightarrow \{U_2\} \rightarrow \{U_3\} \ldots \{U_N\}$$

"$\rightarrow$": updating algorithm with associated probability, ergodic

$$\langle \mathcal{O} \rangle \quad = \quad Z^{-1} \int DU \ \det M \mathcal{O} \, \mathrm{e}^{-S_g[U]} \quad \approx \quad \frac{1}{N} \sum_{n=1}^{N} (\det M \mathcal{O})[U]$$

$\Rightarrow N$ "measurements" of $\mathcal{O}$ $\Rightarrow$statistical error $\sim 1/\sqrt{N}$

Light fermions expensive:

$$\det M[U] = \lambda_1[U] \cdot \lambda_2[U] \cdot \lambda_3[U] \ldots , \qquad \text{cost}(\det M) \sim \frac{1}{m_q^n}, \quad n > 2$$

Non-local: every eigenvalue depends on every link

Computing hadron masses

$$C(t) = \langle 0| \sum_x \mathcal{O}_f(\vec{x},t)\mathcal{O}_i(0)|0\rangle = \sum_n \langle 0|\mathcal{O}_f|n\rangle\langle n|\mathcal{O}_i|0\rangle\ e^{-E_n t}$$

glueball: $\mathcal{O}(x) = \mathrm{Tr}F^2_{\mu\nu} = \mathrm{Tr}U_p(x)$ $\qquad \overrightarrow{\tau \to \infty}\ \langle 0|\mathcal{O}_f|\pi\rangle\langle\pi|\mathcal{O}_i|0\rangle\ e^{-M_\pi\tau}$

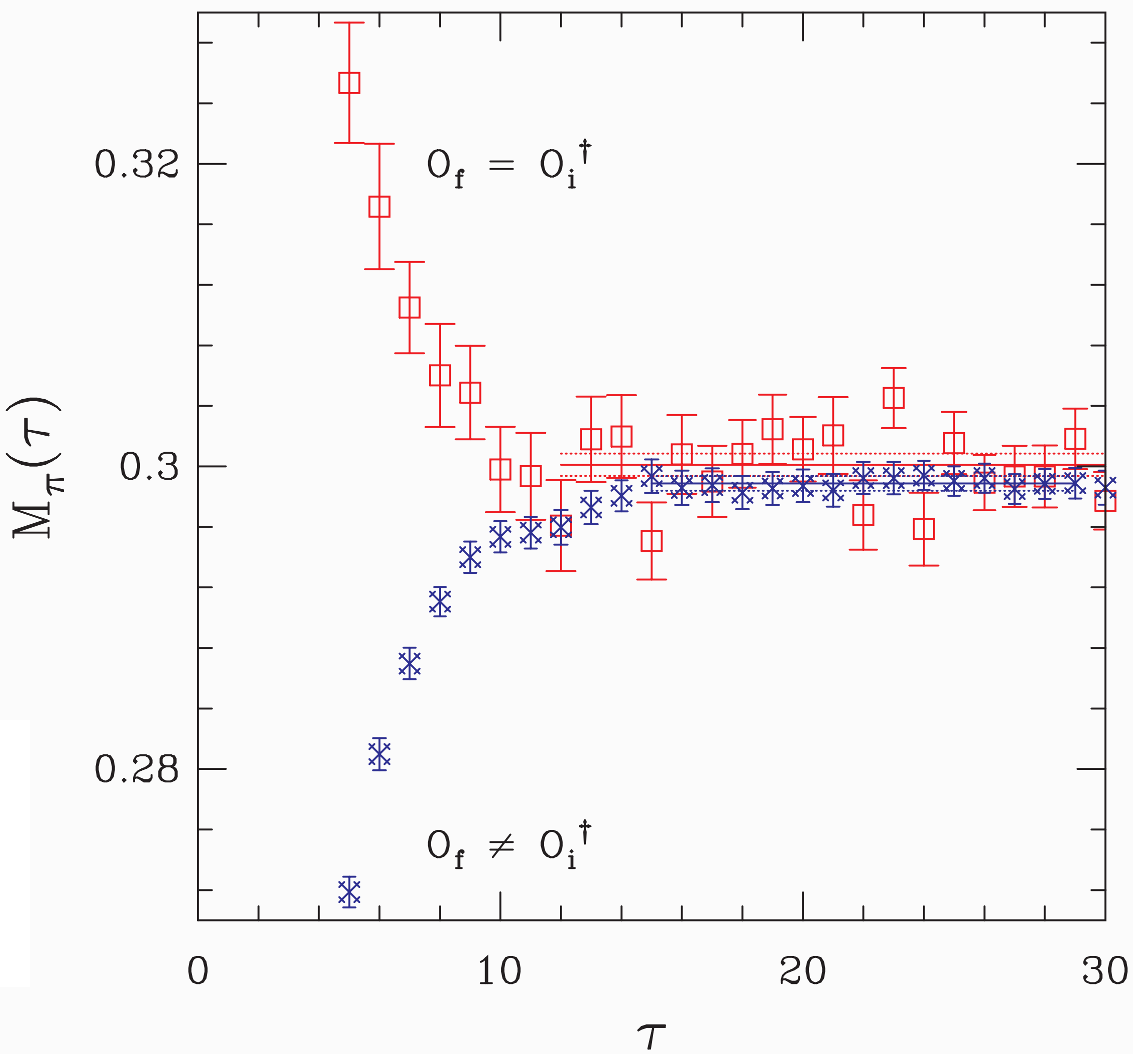
PS-meson: $\mathcal{O}(x) = \bar{\psi}(x)\gamma_5\psi(x)$ pion, ...

V-meson: $\mathcal{O}(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ rho, ...

$\sum_x$: zero momentum projection, $E_n \to M_n$

eff.mass $M_\pi(t) = -\ln[C(t+1)/C(t)]$

matrix elements related to f_π



Eliminating bare lattice parameters

- Bare parameters: $g_0,\ m_u,\ m_d,\ m_s,\ldots$
- Can freely choose bare quark mass m_q in simulations;
  Which value of m_q corresponds to $m_u,\ m_d,\ldots$?
- Obtain hadron masses as functions of m_q, e.g. $am_{\mathrm{PS}}(m_{q_1},m_{q_2})$
- Quark mass dependence of hadron masses:

$$m_{\mathrm{PS}}^2 \propto m_q, \quad m_{\mathrm{V}},\, m_N \propto m_q$$

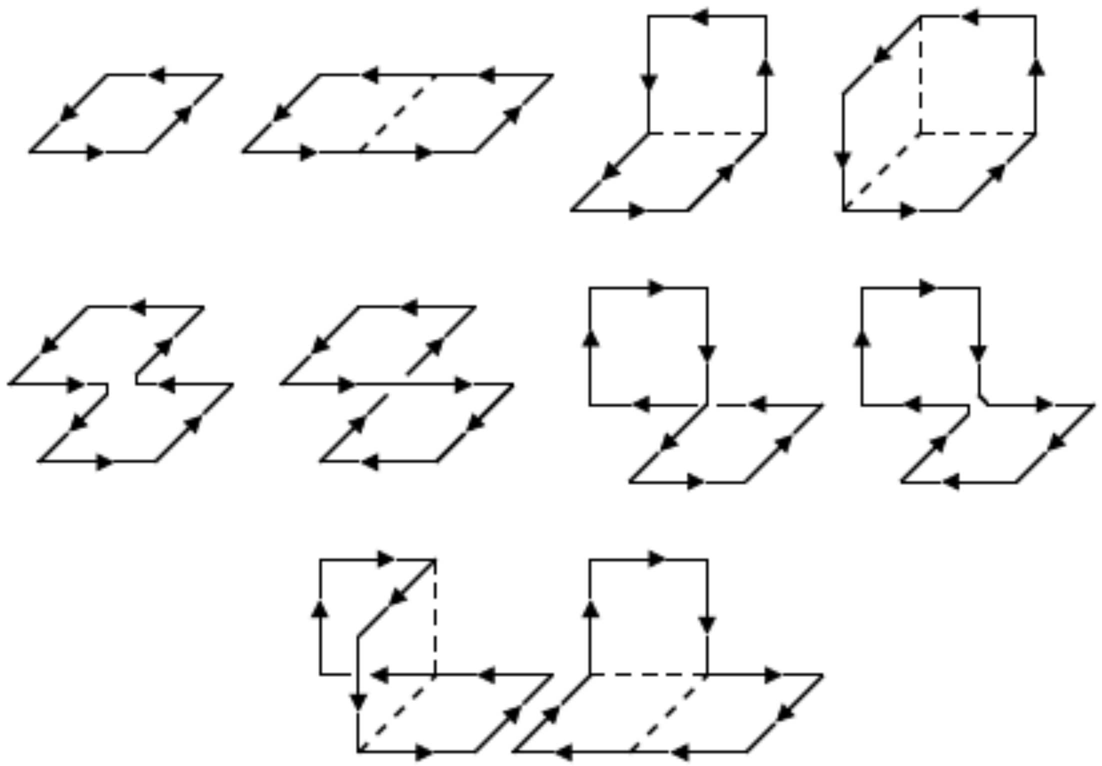
- Eliminate the bare parameters in favour of hadronic input quantities:

$$g_0 \sim 1/\ln a: \quad a^{-1}\,[\mathrm{GeV}] = \frac{Q\,[\mathrm{GeV}]}{(aQ)}, \quad Q = f_\pi, m_N, \Delta^{\Upsilon}_{\mathrm{1P-1S}},\ldots$$

$$\hat{m} = \tfrac{1}{2}(m_u+m_d): \quad \frac{m_{\mathrm{PS}}^2}{f_\pi^2} \to \frac{m_\pi^2}{f_\pi^2}, \quad m_s: \quad \frac{m_{\mathrm{PS}}^2}{f_\pi^2} \to \frac{m_{\mathrm{K}}^2}{f_\pi^2}$$

Pure gauge theory: glueball spectrum

Glueball interpolating operators



- contamination of higher spin states;

  e.g. 0^{++} can mix with 4^{++}

- Matrix correlators:

$$C_{ij}(x_0) = \sum_{\vec{x}} \langle O_i(x) O_j(0) \rangle$$

- $\{O_1, \ldots, O_n\}$:

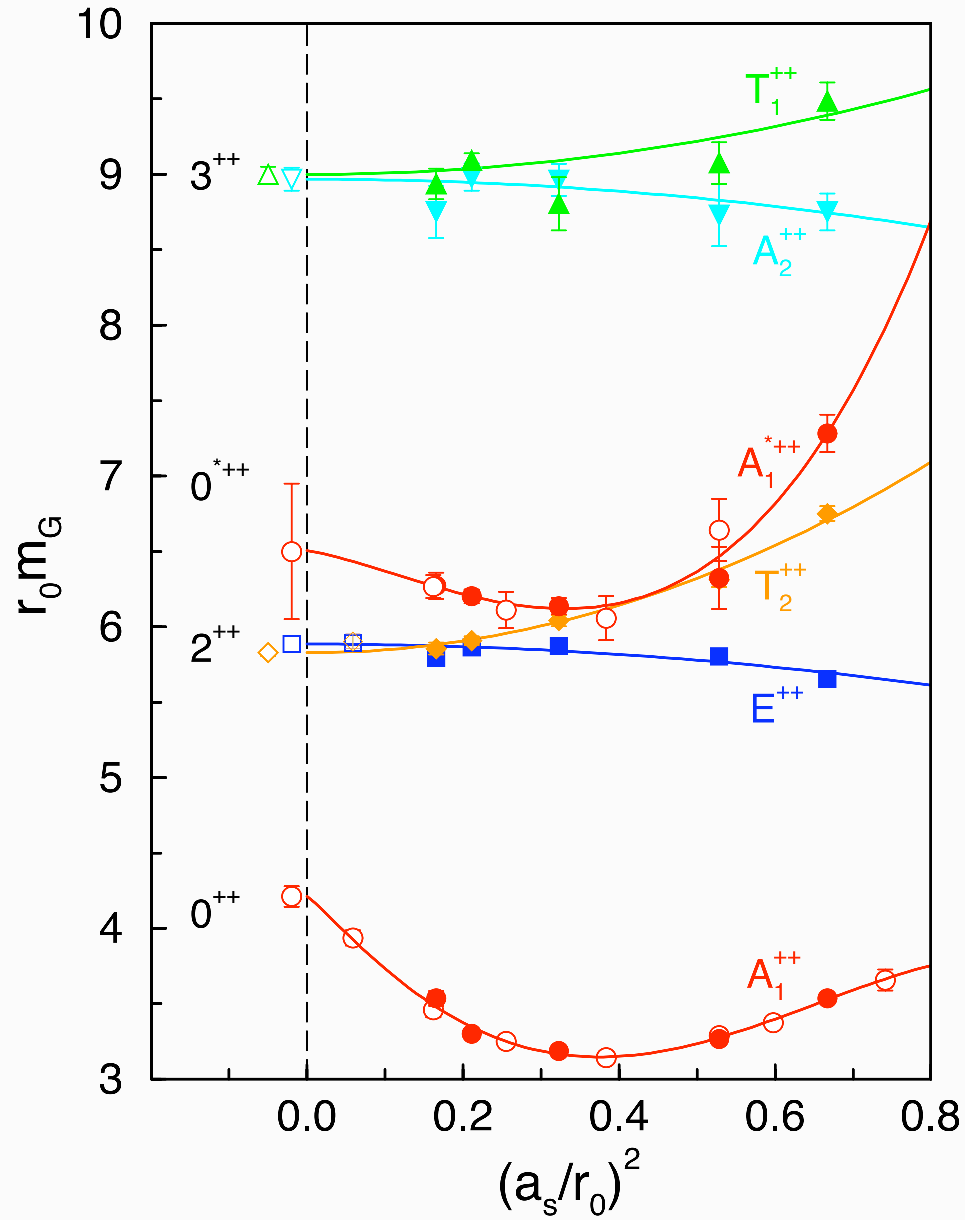
  basis of interpolating operators for given irrep. of the hypercubic group

$\rightarrow$ Recover a given spin-parity in the continuum limit

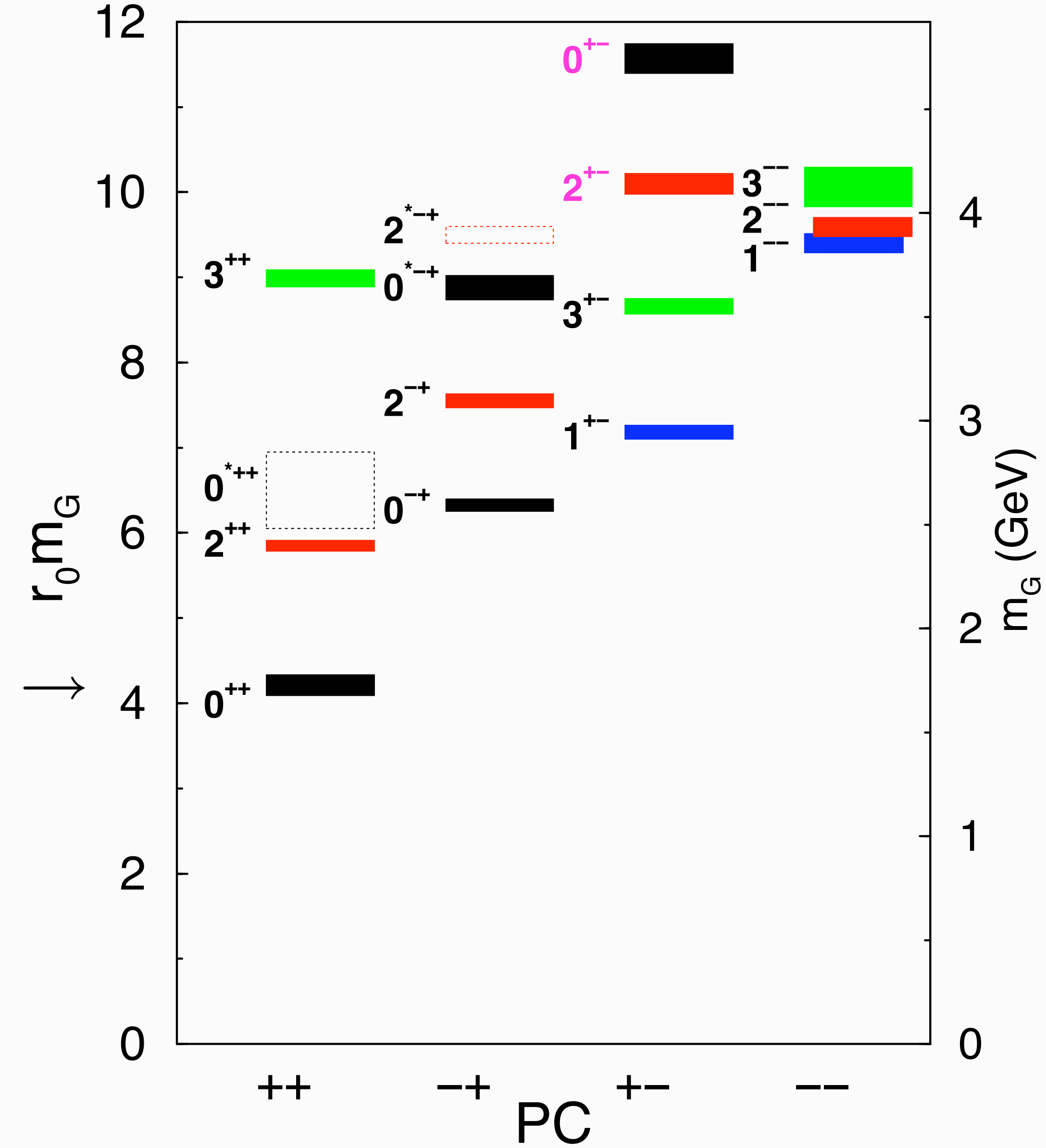
Pure gauge theory: glueball spectrum

continuum extrapolation

Candidates: $f_0(1370),\ f_0(1500),\ f_0(1710),\ f_0(2220)$



1600 MeV $\longrightarrow$

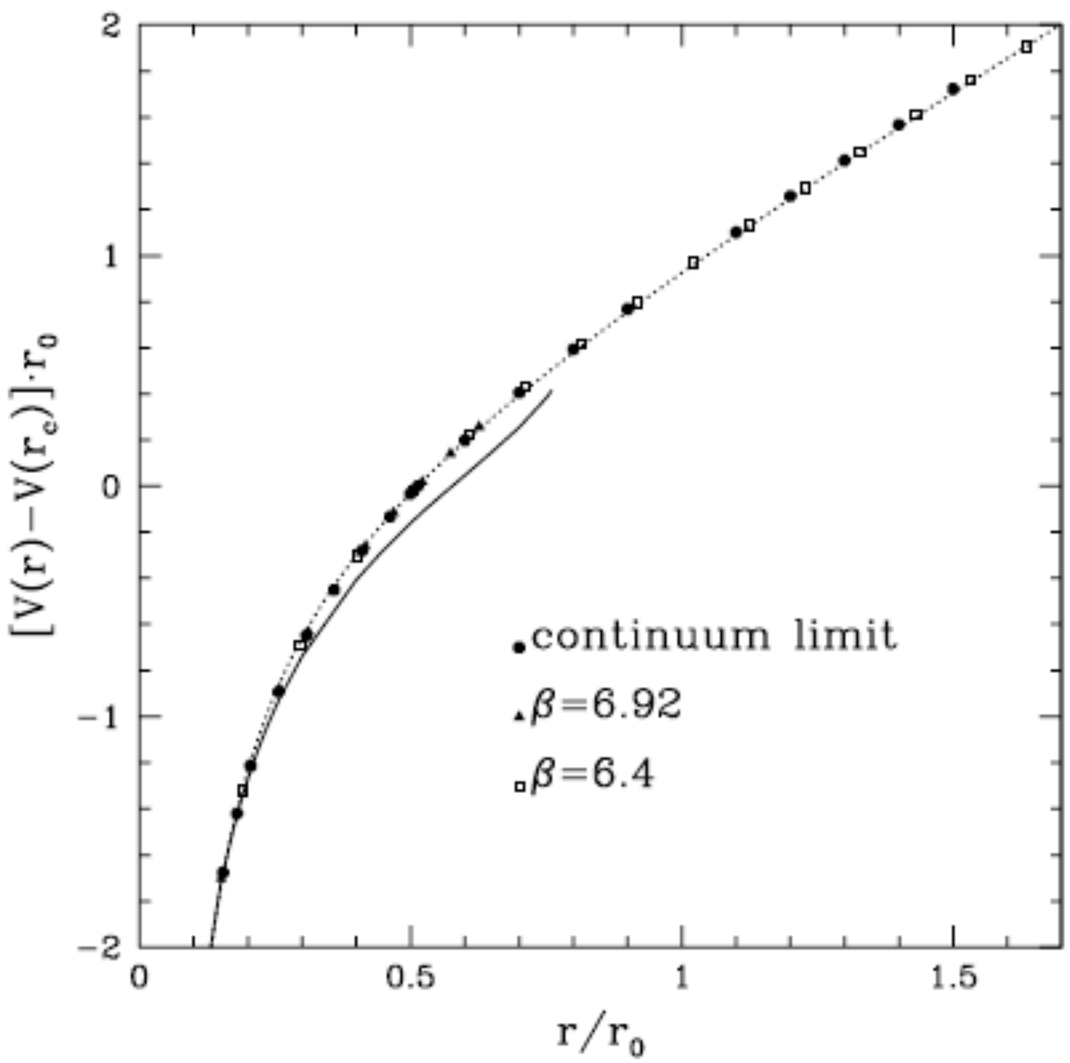


$r_0 \approx 0.5$ fm

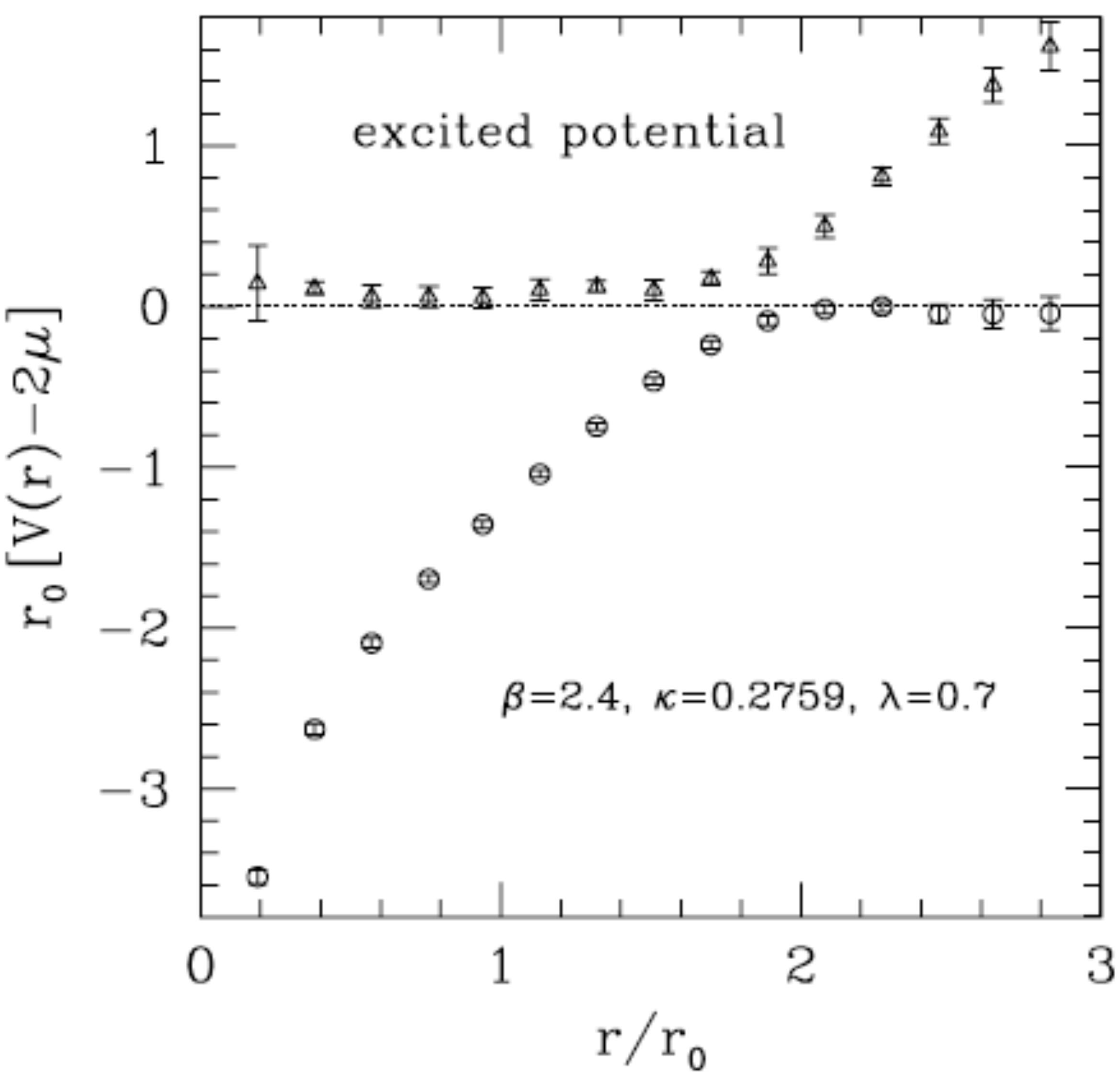
Recovery of SO(4)!

Morningstar, Peardon PRD 99

The static potential



pure gauge theory

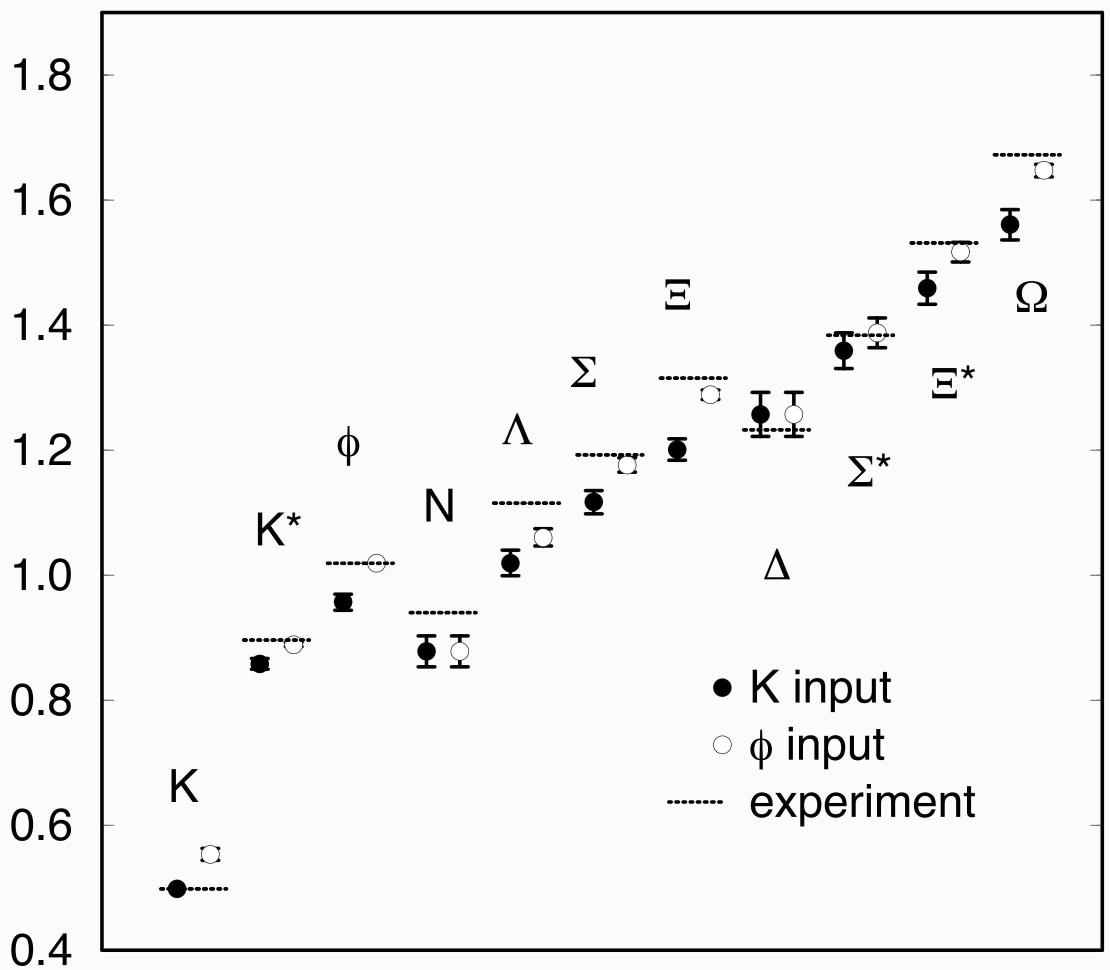


SU(2) gauge theory with matter fields

Alpha collaboration

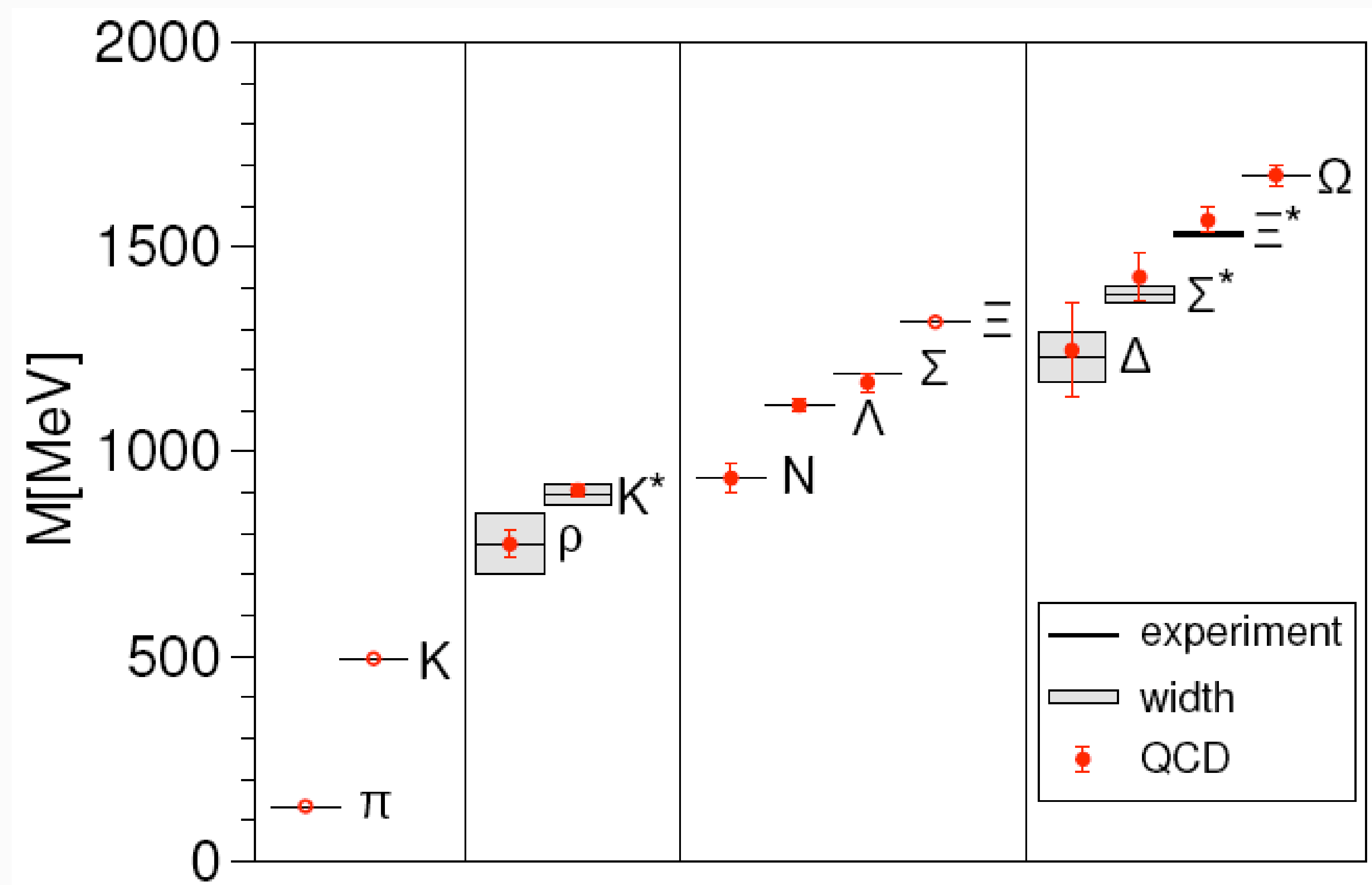
QCD: the light hadron spectrum

quenched



CP-PACS 99

unquenched



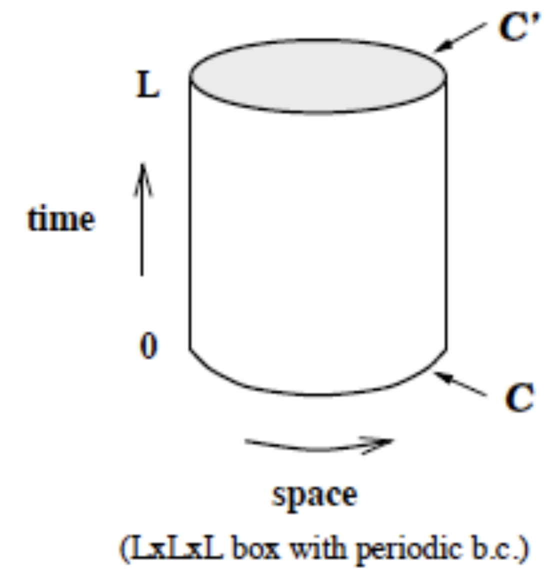
Budapest-Marseille-Wuppertal, LAT08

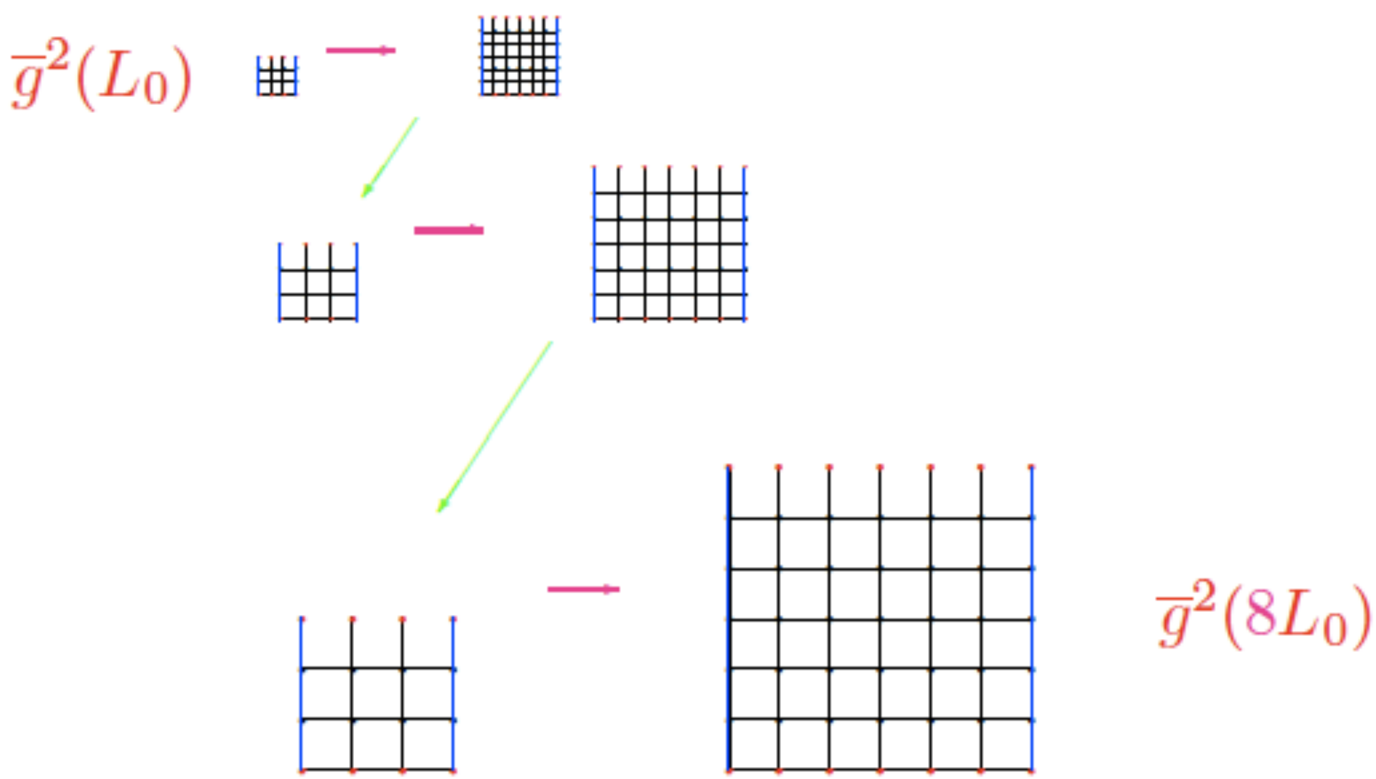
Nf=2+1, nearly physical, one lattice spacing

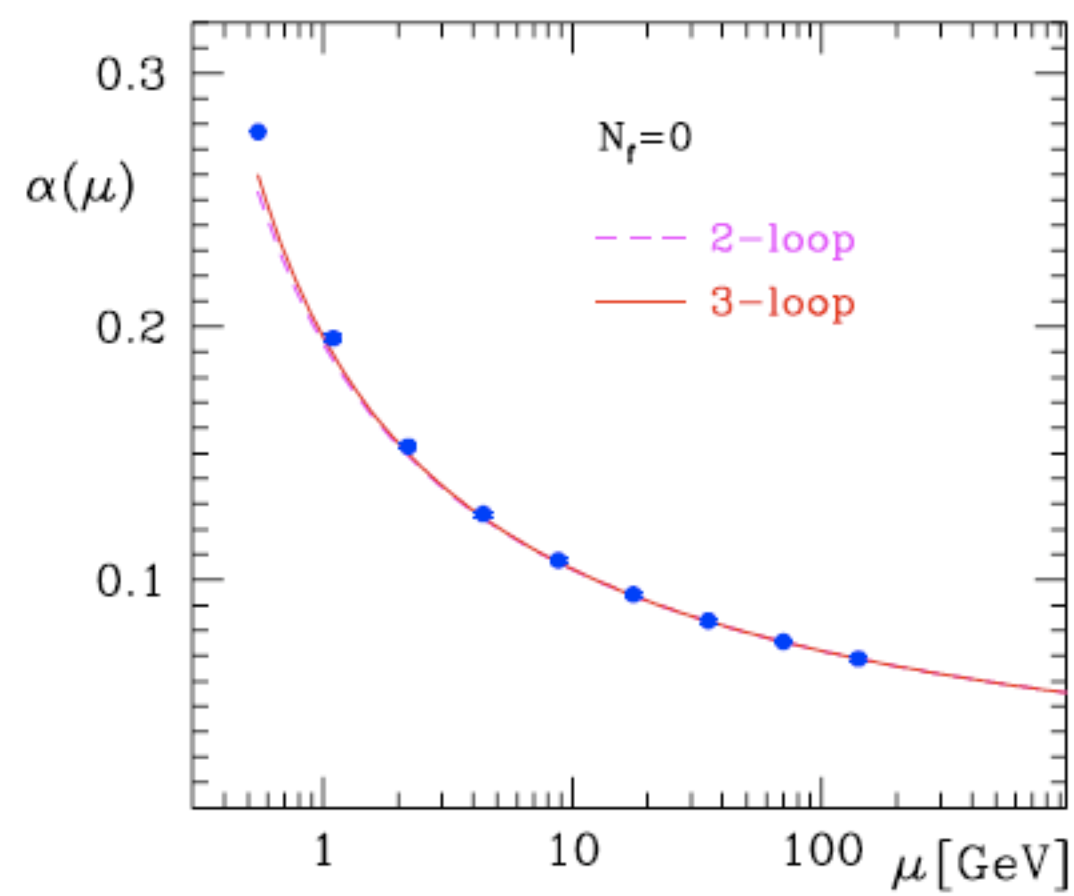
Running coupling from Schrödinger functional

Schrödinger functional:
finite-volume renormalisation scheme

$\bar{g}^2_{\mathrm{SF}}$ runs with the box size: $\bar{g}^2_{\mathrm{SF}}(L)$

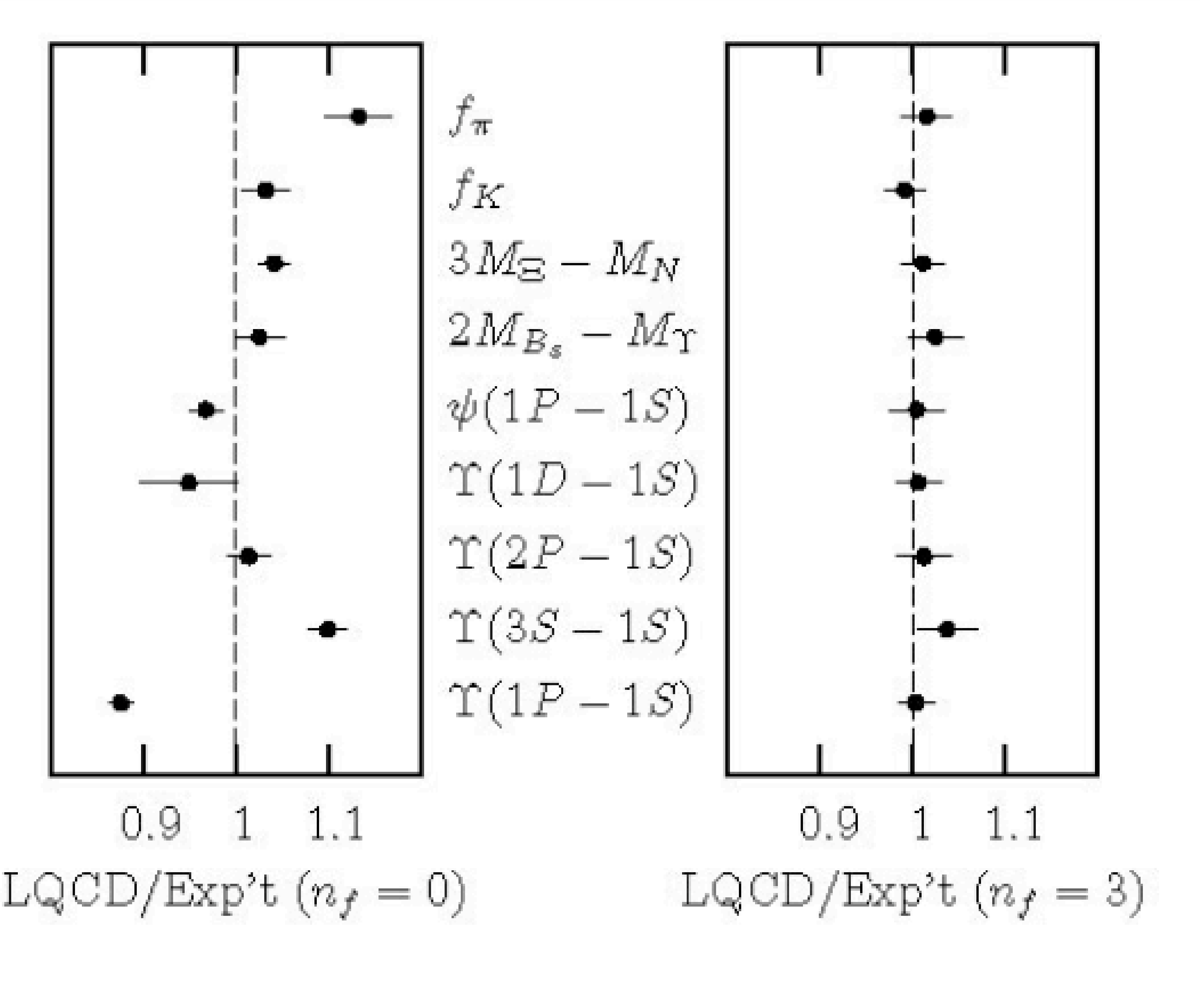






Capitani, Lüscher, Sommer, Wittig 99

Precision physics results!



HPQCD-MILC-FNAL

staggered u,d,s + NRQCD for c,b