Lecture II

- QCD and its basic symmetries
- Renormalisation and the running coupling constant
- Experimental evidence for QCD based on comparison with perturbative calculations

The road to QCD: SU(3) quark model

Experimental evidence in 1960's:

Many kind of baryons and mesons [e.g.: p,n,Δ,Λ,π,ρ,ω,K,ϕ,D, ...]
Attempt to order them leads to the concept of quarks†.

Quarks and gluons
carry color: R, G, B
[Δ^{++}-wavefunction must be antisymmetric]

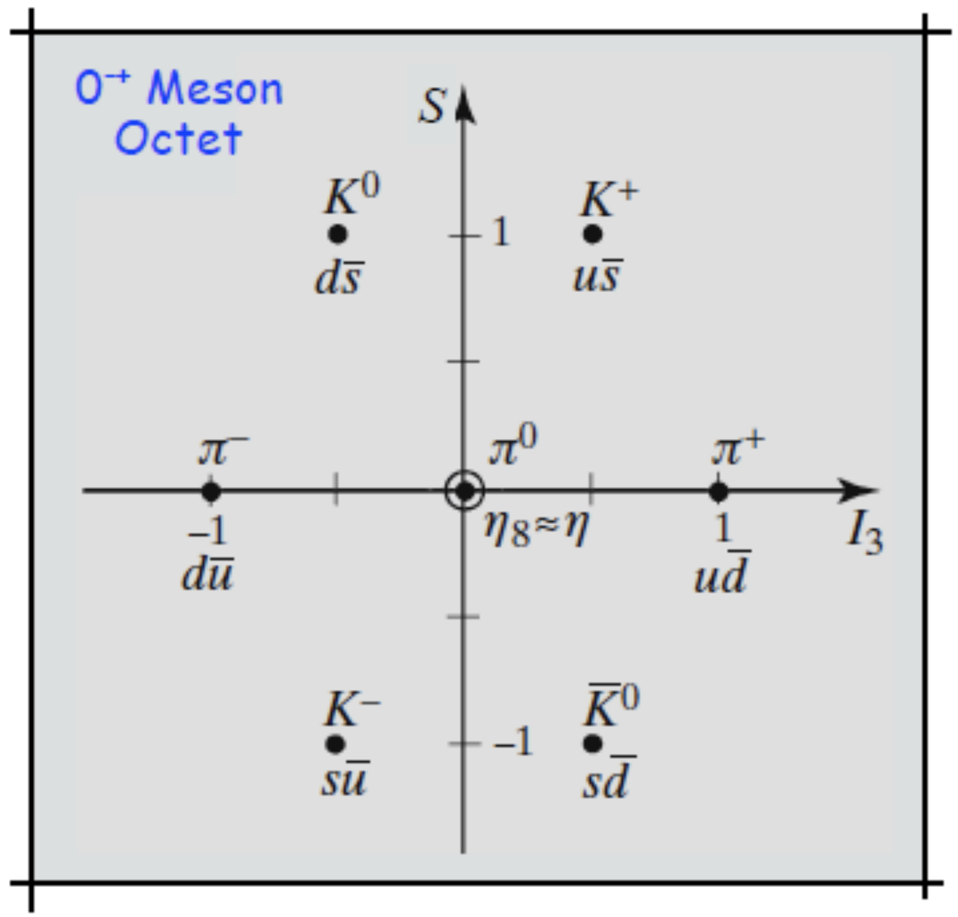
There exist:

u,d,s,c,b,t

3 Colours

- 36 Quarks [6x2x3]
- 8 Gluons

Anti-Quarks



† Three quarks for Muster Mark!
Sure he hasn't got much of a bark
And sure any he has it's all beside the mark.
[James Joyce, Finnegans Wake]

Symmetry considerations: group theory

Hadron multipletts are product representations of the fundamental triplett representation

Analogous to spin, SU(2): $$\frac{1}{2} \otimes \frac{1}{2} = 1, \frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2}$$

Analogous to spin, SU(3): quarks in fundamental rep., triplet and anti-triplet

$$3 \otimes 3 \otimes 3 = 1 + 8 + 8 + 10$$

$$3 \otimes \bar{3} = 1 + 8$$

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}$$

The need for colour

$\Delta^{++}(1232)$ Baryon: $|\Delta^{++}\rangle = |u^{\uparrow}u^{\uparrow}u^{\uparrow}\rangle$ $J^P = 3/2^+$

$$\psi(\Delta^{++}) = \underbrace{\psi\ (r)}_{\substack{l,\ l' = 0 \\ \Rightarrow \text{sym.}}} \cdot \underbrace{\psi_{\text{Spin}}(J)}_{\substack{\uparrow\uparrow\uparrow \\ \Rightarrow \text{sym.}}} \cdot \underbrace{\psi_{\text{Flavor}}}_{\substack{\text{3 u-Quarks} \\ \Rightarrow \text{sym.}}}$$

Contradiction with Pauli principle:
Total wave function must be anti-symmetric!

$$\psi(\Delta^{++}) = \psi\ (r) \cdot \psi_{\text{Spin}}(J) \cdot \psi_{\text{Flavor}} \cdot \psi_{\text{Colour}}$$

Cure: introduce new, unobservable, quantum number: colour

Observable states must be colourless!

$$\psi_{\text{Colour}} = 1/\sqrt{6} \cdot \sum_{i=r,g,b} \sum_{j=r,g,b} \sum_{k=r,g,b} \varepsilon_{ijk}\, q_i\, q_j\, q_k$$

Quark spinors now in colour triplet: $\psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$

Also explains absence of qq or qqqq states:

$3 \otimes 3 \otimes 3 = 1 + 8 + 8 + 10$ contains singlet

$3 \otimes \bar{3} = 1 + 8$ contains singlet

$3 \otimes 3 = \bar{3} + 6$ does not contain singlet!

etc.

QCD, theory of strong interactions

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\,\mu\nu}(x) + \sum_{c=1}^{N_c} \sum_{f=1}^{N_f} \bar{\psi}_{c,f}(x) \left(i\gamma^\mu D_\mu - m_f\right) \psi_{c,f}(x)$$

Quark fields: $\psi_{\alpha,c,f}(x)$

spinor colour flavour

$$\begin{aligned} \alpha &= 1,\dots 4 \quad \left(\text{spin up/down, particle/anti-particle}\right) \\ c &= 1,2,3 \quad \left(\text{red,blue,green}\right) \\ f &= u,d,s,c,b,t \end{aligned}$$

Covariant derivative:

$$D_\mu = \partial_\mu - igT^a A^a_\mu(x)$$

Field strength tensor:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$$

Parameters:

$$\begin{aligned} \alpha_s(q^2) &= \frac{g^2(q^2)}{4\pi} \\ m_u &\approx 3\text{MeV}, m_d \approx 6\text{MeV} \\ m_s &\approx 120\text{MeV}, m_c \approx 1.5\text{GeV} \\ m_b &\approx 4.5\text{GeV}, m_t \approx 175\text{GeV} \end{aligned}$$

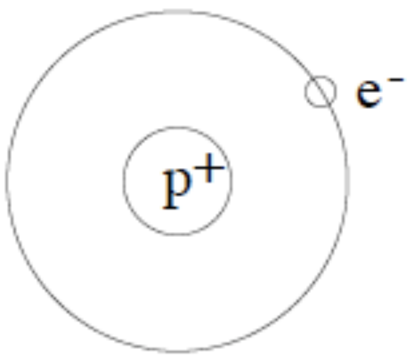
QCD, theory of strong interactions

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2}\,\mathrm{Tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{3}\bar{\psi}_i[\gamma_\mu D_\mu + m_i]\psi_i$$

$$m_u \sim 3\mathrm{MeV}, \quad m_d \sim 6\mathrm{MeV}, \quad m_s \sim 120\mathrm{MeV} \quad \Rightarrow \quad N_f \approx 2+1$$

weak vs. strong coupling:

QED



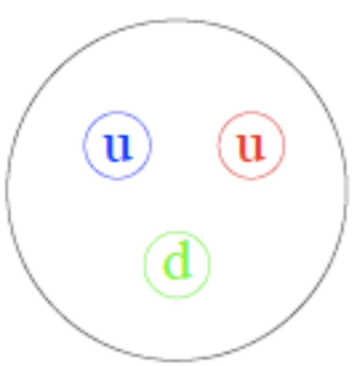
$M_e = 0.5$ MeV
$M_p = 938$ MeV
$E_{bind} = 13.6$ eV

hydrogen (e.m. force)

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

gluon self-interaction!

QCD



$M_u \sim 3$ MeV
$M_d \sim 6$ MeV
$M_p = 938$ MeV

proton (strong force)

$$\alpha_s = \frac{g^2}{4\pi} \approx 1$$

$\Rightarrow$ **Confinement, non-perturbative**

Interaction dictated by gauge symmetry

$$\bar{\psi} i \gamma_\mu D^\mu \psi = \bar{\psi} i \gamma_\mu (\partial^\mu - iqA^\mu)\psi$$

$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi - \mathbf{q}\left(\bar{\psi}\gamma^\mu\psi\right)\mathbf{A}_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\mathcal{L}_{QCD} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi - \sum_{\mathbf{a}=1}^{8}\mathbf{g_s}\left(\bar{\psi}\gamma_\mu\mathbf{T_a}\psi\right)\mathbf{G}_\mathbf{a}^\mu - \frac{1}{4}\sum_{a=1}^{8}G_{a\mu\nu}G_a^{\mu\nu}$$

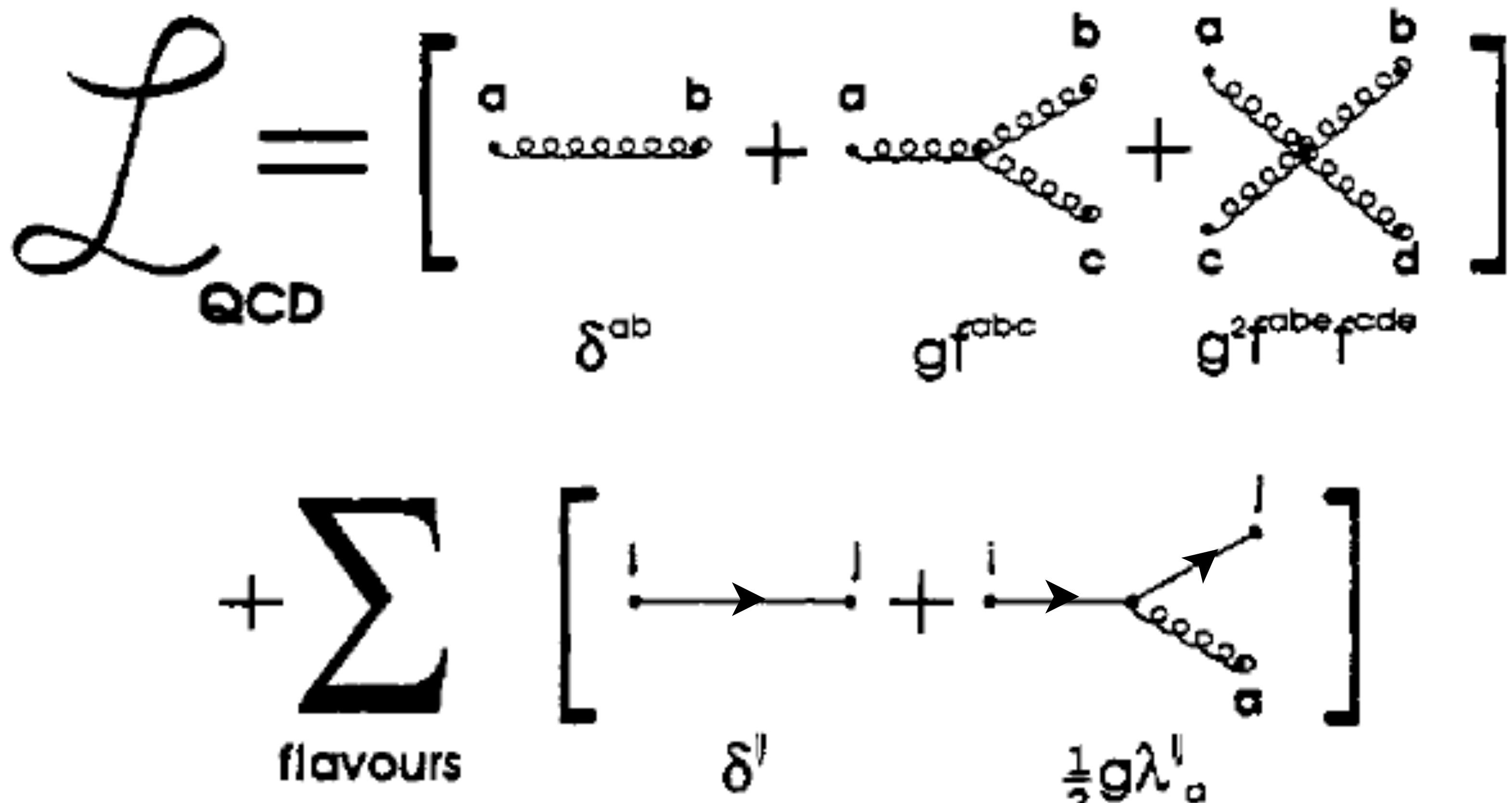
$$\bar{\psi} i \gamma_\mu D^\mu \psi = \bar{\psi} i \gamma_\mu (\partial^\mu - ig_s T^a G^{a\mu})\psi$$

Parameters: couplings and masses need to be determined by experiment

Feynman rules for perturbative QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\,\mu\nu}(x) + \sum_{c=1}^{N_c} \sum_{f=1}^{N_f} \bar{\psi}_{c,f}(x) \left(i\gamma^\mu D_\mu - m_f\right) \psi_{c,f}(x)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu \qquad\qquad D_\mu = \partial_\mu - igT^a A^a_\mu(x)$$



Global symmetries: example isospin

$$\psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

$$\psi' = U\psi, \quad U \in SU(2), U\,U^\dagger = 1$$

$$\bar{\psi}' = \psi'^\dagger\gamma^0 = \psi^\dagger U^\dagger \gamma^0 = \psi^\dagger\gamma^0 U^\dagger = \bar{\psi}U^\dagger$$

If $m_u = m_d$ we can write

$$\bar{\psi}'(i\gamma^\mu D_\mu - m)\psi' = \bar{\psi}U^\dagger(i\gamma^\mu D_\mu - m)U\psi = \bar{\psi}(i\gamma^\mu D_\mu - m)U^\dagger U\psi$$

For non-degenerate masses this does not work!

$$\bar{\psi}(x)i\gamma^\mu D_\mu\psi(x) - m_u\bar{\psi}_u(x)\psi_u(x) - m_d\bar{\psi}_d(x)\psi_d(x)$$

Symmetries of the QCD Lagrangian

Local $SU(3)_c$ transformations

$$\psi'_c(x) = \left(e^{i\theta^a(x)T^a}\right)_{cc'} \psi_{c'}(x)$$
$$a = 1, \ldots N_c^2 - 1$$

For degenerate quarks, $m_{f_1} = \ldots = m_{f_{n_f}}$
global $SU(n_f)$ transformations

$$\psi'_f(x) = \left(e^{i\theta^a T^a}\right)_{ff'} \psi_{f'}(x)$$
$$a = 1, \ldots n_f^2 - 1$$

Global $U(1)$ transformations:

$$\psi'(x) = e^{i\theta}\,\psi(x)$$

For massless quarks, $m_{f_1} = \ldots = m_{f_{n_f}}$ =0:

Global axial $SU(n_f)$ transformations

$$\psi'_f(x) = \left(e^{i\theta^a T^a \gamma_5}\right)_{ff'} \psi_{f'}(x)$$
$$a = 1, \ldots n_f^2 - 1$$

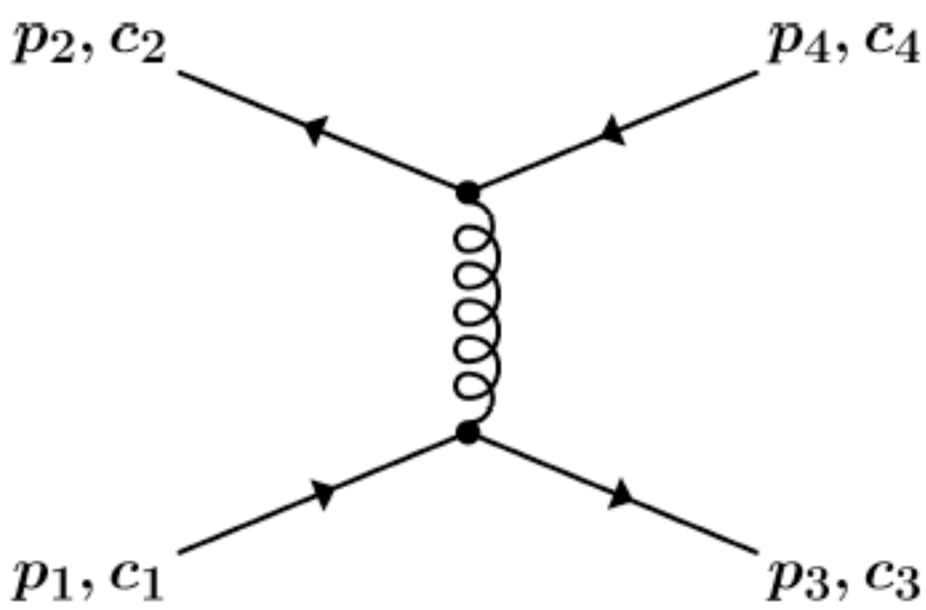
Global axial $U(1)$ transformations,
anomalous, broken by quantum effects

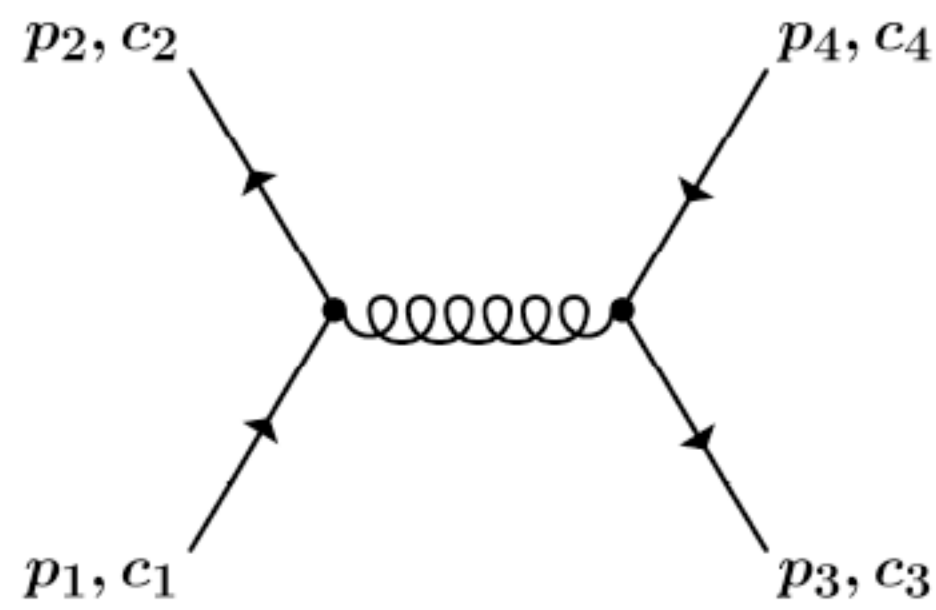
$$\psi'(x) = e^{i\theta\gamma_5}\,\psi(x)$$

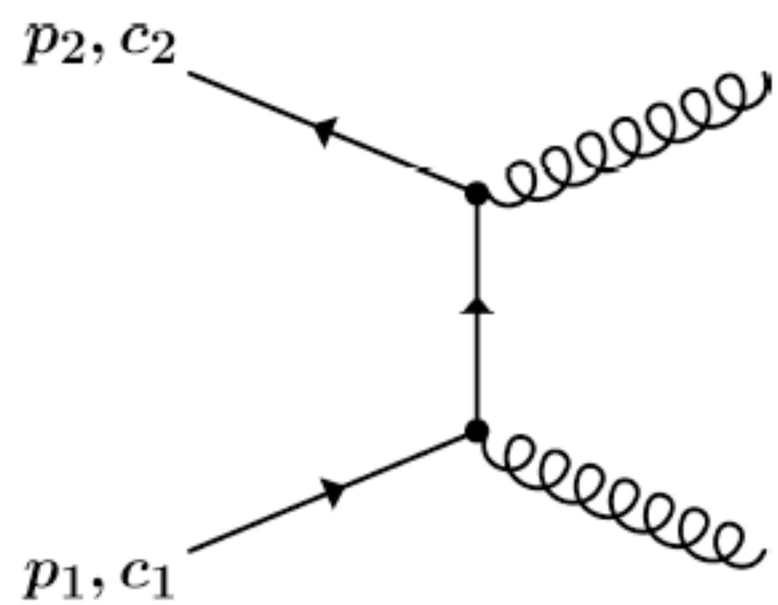
Symmetries for parameter values realised by nature

$SU(3)_c$	gauge symmetry, exact, only colour singlets observable		
$U(1)_B$	baryon number, exact		
$SU(2)_{\text{isospin}}$	approximate, O(few %),	$m_u \approx m_d$	
$SU(3)_{\text{flavour}}$	approximate, O(few 10 %),	$m_u \approx m_d \sim m_s$	(quark model!)
$SU(2)_{\text{axial}}$	approximate	$m_u \approx m_d \approx 0$	
$SU(2)_L \times SU(2)_R$	approximate chiral symmetry = isospin+axial flavour symmetry combined	$m_u \approx m_d \approx 0$	

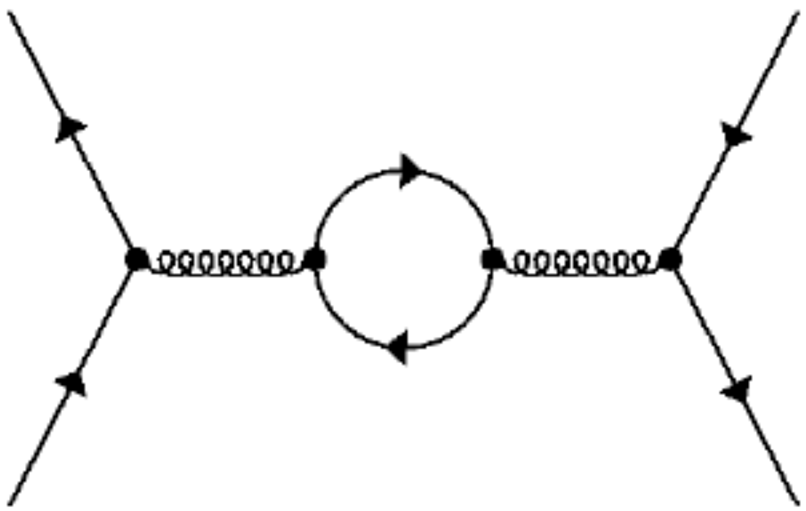
Elementary perturbative processes in QCD



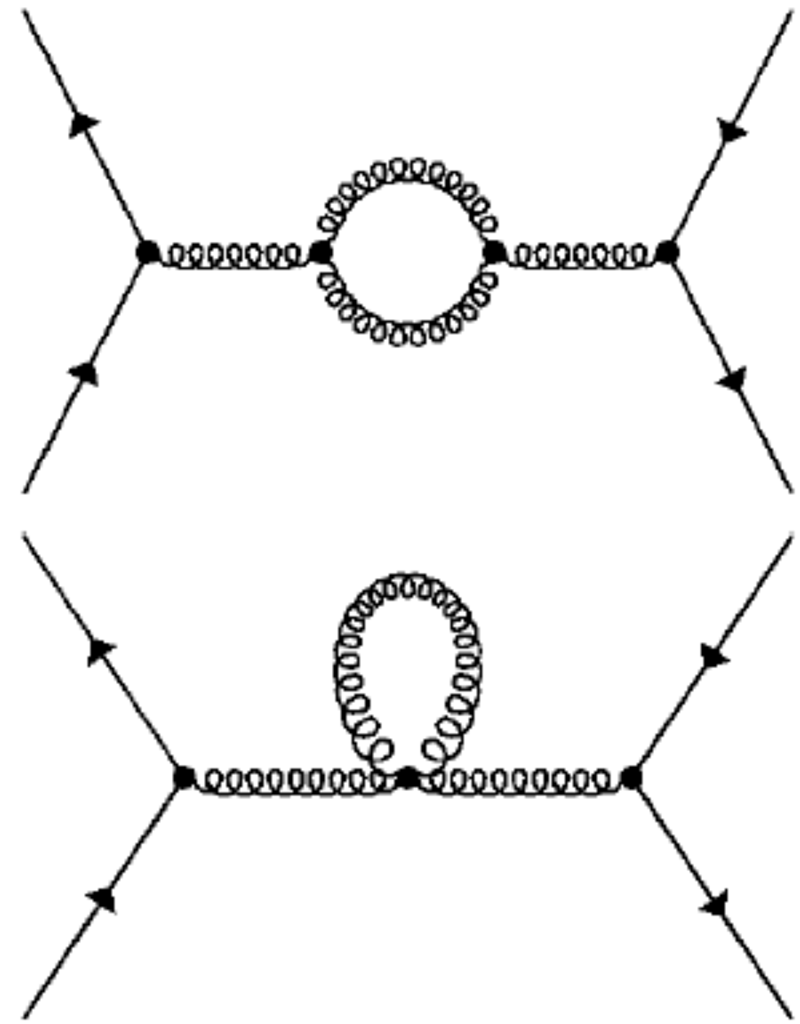




Higher order processes



as in QED

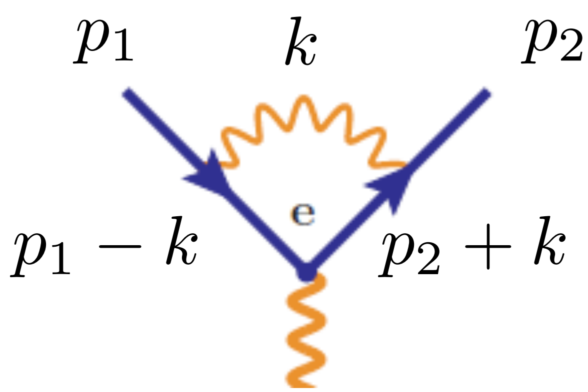


do not exist in QED!

Gluon self coupling: besides quark loops also gluon loops can occur!

Higher order corrections and renormalisation

Vertex correction:



$$\sim \int \frac{d^4k}{(2\pi)^4} \, \frac{(p_{2\mu} + k_\mu)\gamma^\mu + m}{p_2^2 + 2p_2 k + k^2 - m^2} \, \frac{(p_{1\mu} - k_\mu)\gamma^\mu + m}{p_1^2 - 2p_1 k + k^2 - m^2} \, \frac{1}{k^2}$$

$$\xrightarrow{k \to \infty} \int d^4k \, \frac{k^2}{k^6} \sim \ln k \to \infty$$

Regularisation by momentum cut-off: $$\int^\Lambda d^4k \, \frac{k^2}{k^6} \sim \ln \Lambda$$ finite

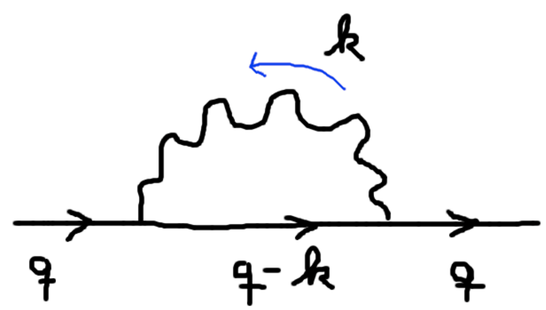
Altogether there are quadratic, linear and logarithmic divergencies from vacuum corrections to different n-point functions

Similar: correction to the mass

Remember Feynman propagator, free field $\quad i\int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip*x}}{p^2-m_0^2+i\varepsilon}$

Pole mass: parameter from Lagrangian, mass of the non-interacting particle

(One) correction in the interacting theory:



Full propagator: $\quad \langle 0|T\{\phi(x)\phi(y)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{e^{-ipx}}{p^2-m_0^2+\Sigma(p^2)}$

Pole of the propagator gets shifted! Correction to the mass:

$$p^2 - m_0^2 + \Sigma(p^2) = 0, \quad m^2 = m_0^2 + \delta m^2$$

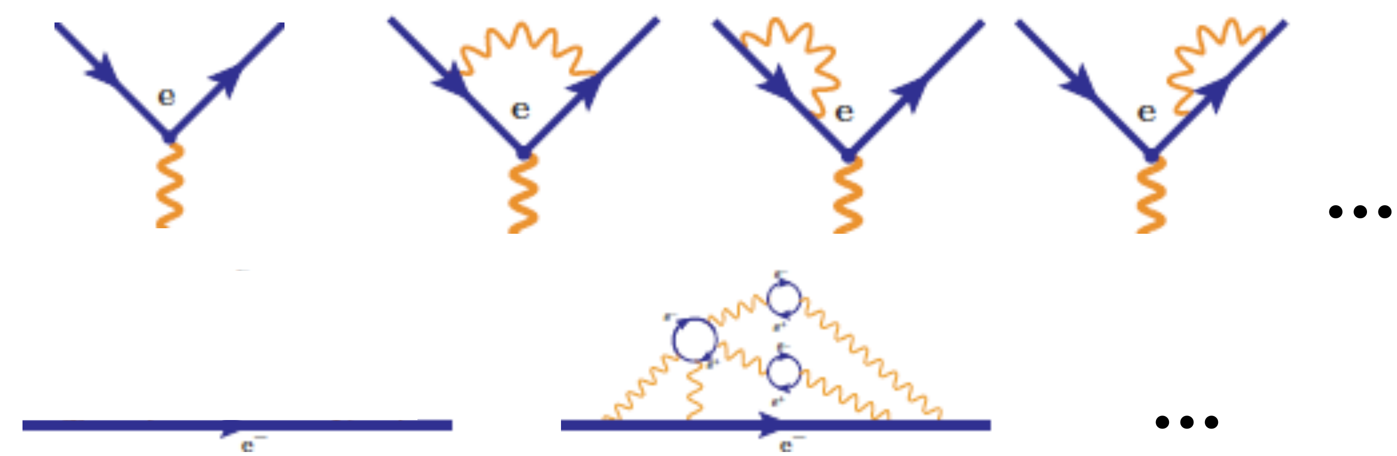
Renormalisation

Observation: "bare" parameters from L do not correspond to physical couplings, masses

not measurable!

Physical parameters: calculated from sum of all Feynman diagrams

$$
\begin{aligned}
e &= e_0\left(1 + a_1 e_0^2 \ln\frac{\Lambda}{\mu} + \ldots\right) \\
m &= m_0\left(1 + b_1 e_0^2 \ln\frac{\Lambda}{\mu} + \ldots\right)
\end{aligned}
$$



$$e_0 = e_0(e, m, \Lambda), m_0 = m_0(e, m, \Lambda)$$

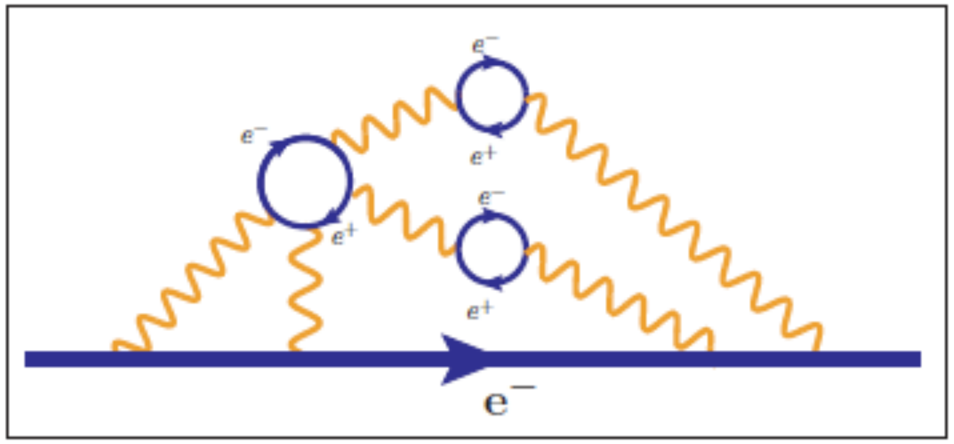
Observable $O(e_0, m_0) = O(e, m, \Lambda)$

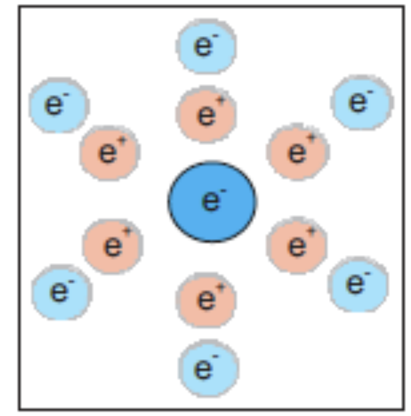
Renormalisation: absorb cut-off dependence in Z-factors, order by order

$$
\begin{aligned}
m_0 &= Z_m^{1/2}\, m \\
e_0 &= Z_e^{1/2}\, e
\end{aligned}
$$

$$O^R(e, m) = \lim_{\Lambda\to\infty} Z(e, m, \Lambda) O(e, m, \Lambda)$$

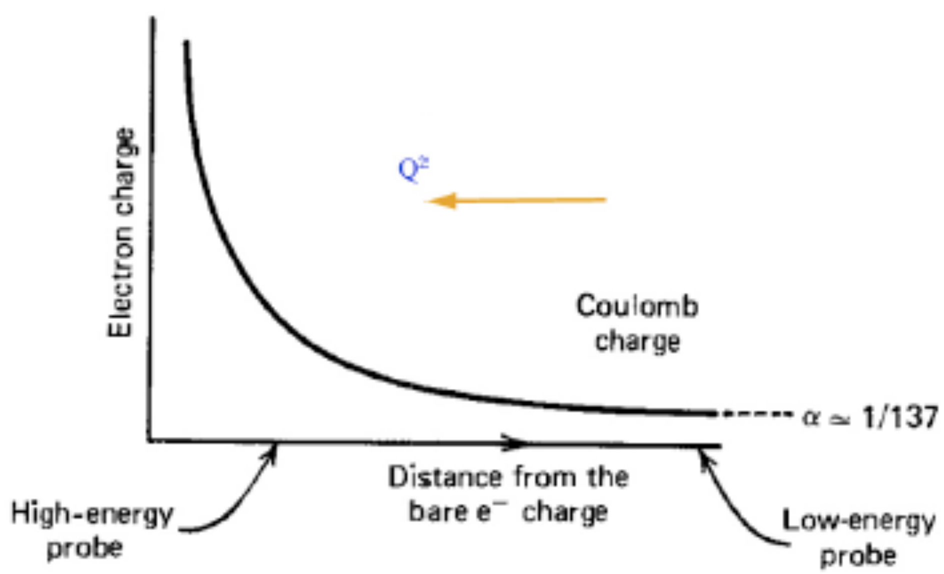
Running coupling constants: the vacuum as a medium

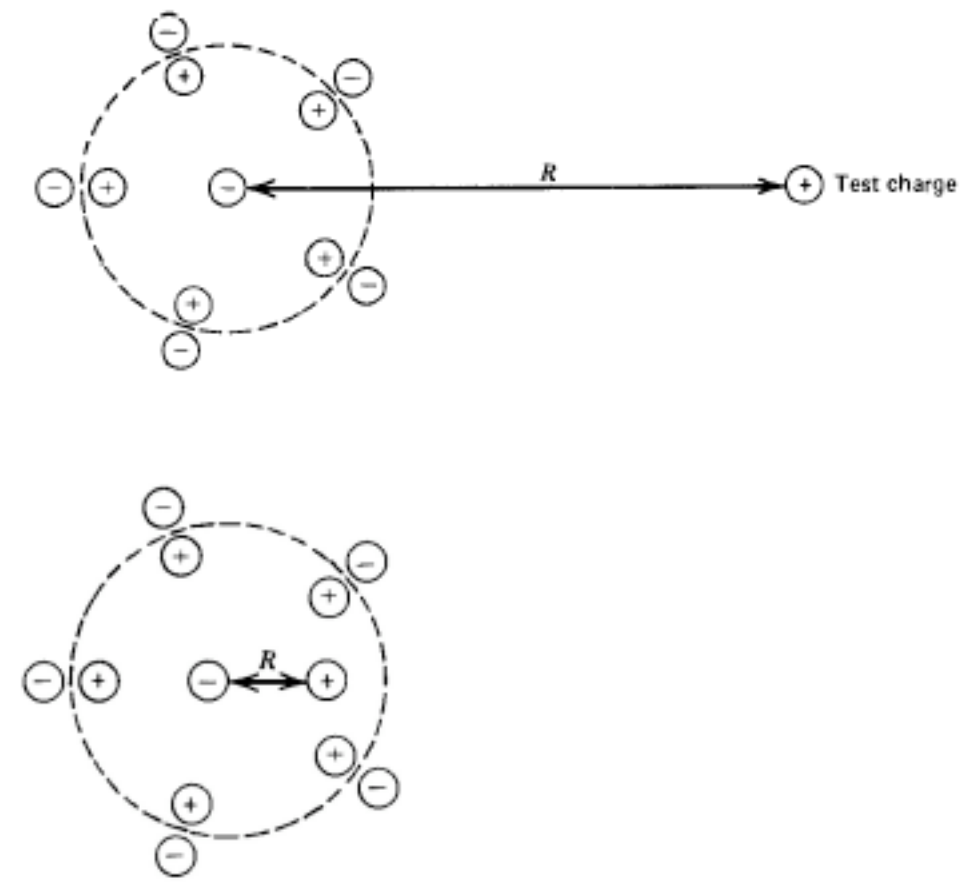




- "Free electron": idealisation, exists only in perturbation theory
- Vacuum permits quantum fluctuations, $\Delta t \cdot \Delta E \geq \frac{\hbar}{2}$
- Pair production of virtual electron-positron pairs
- Physical electron in continuous interaction with vacuum, surrounded by cloud of electrons, positrons, photons
- Polarisation of virtual dipoles, screening!

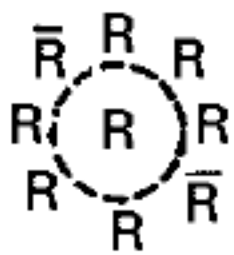
Effect of vacuum polarisation on the QED charge

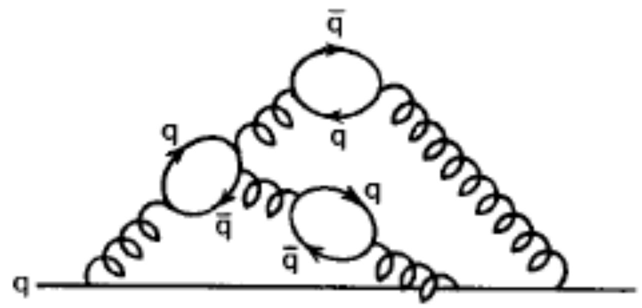


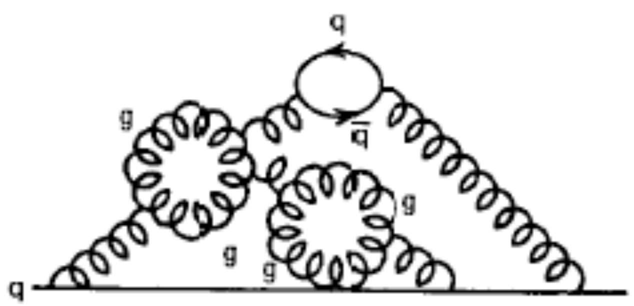


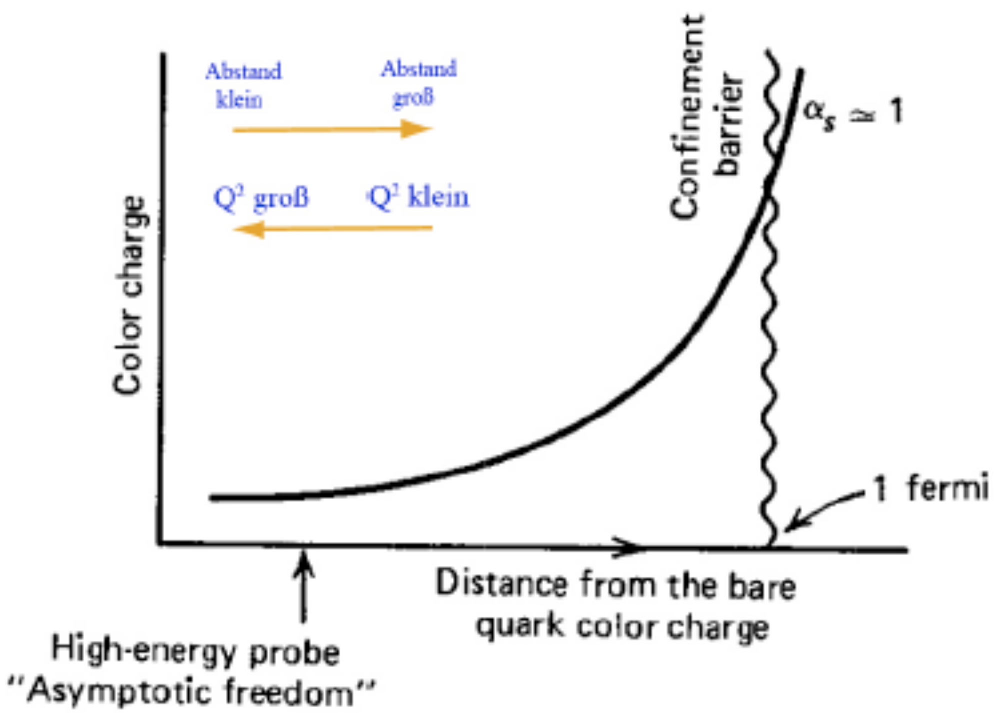
- Measure charge by scattering with 4-momentum transfer Q^2
- Small Q^2 = large distance ➔ small coupling $\alpha(Q^2)$
- Large Q^2 = small distance ➔ large coupling $\alpha(Q^2)$

Effect of vacuum polarisation on the QCD charge



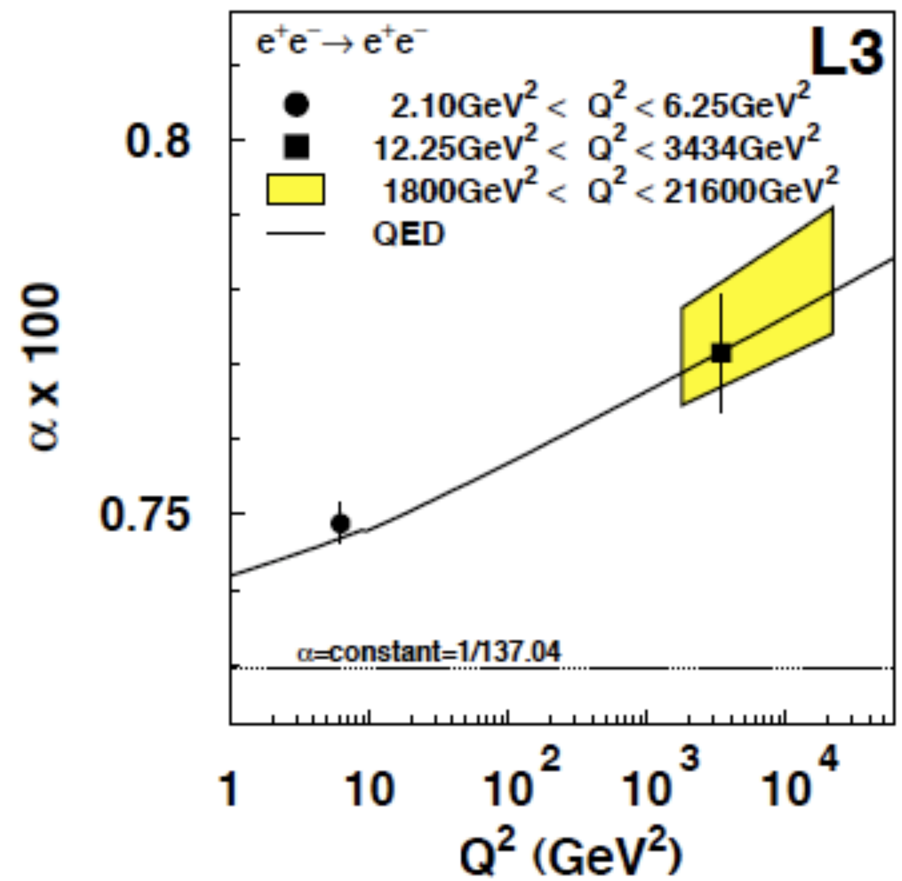


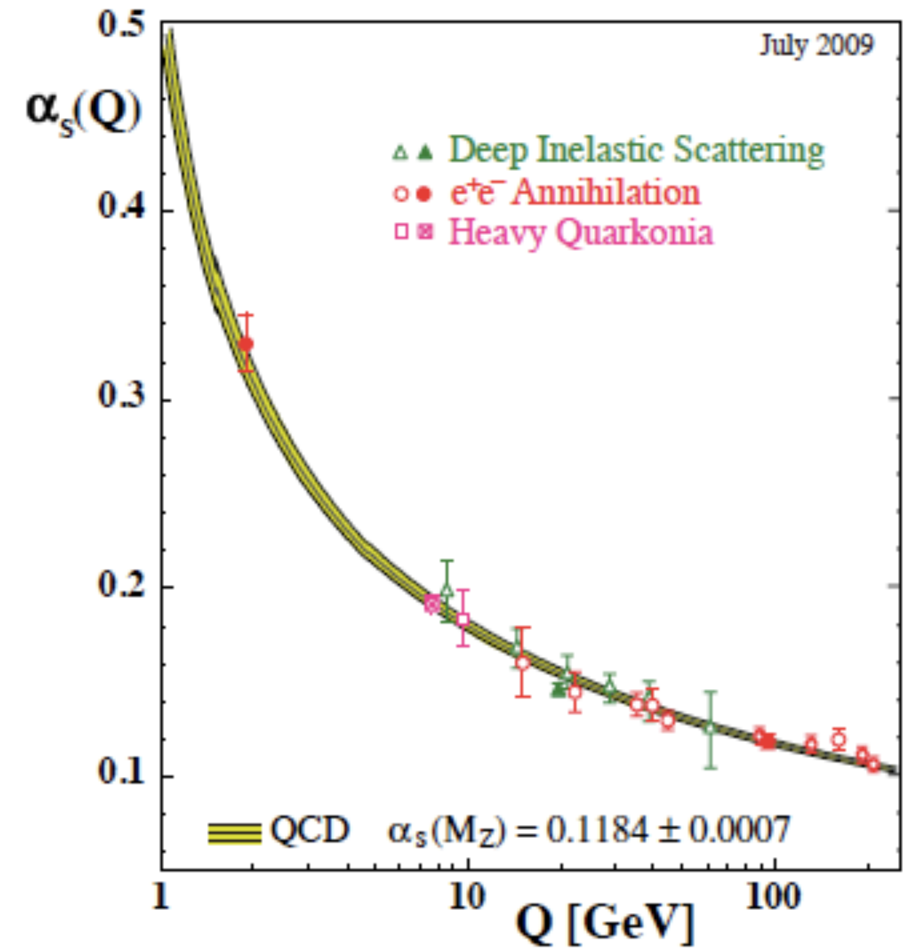




- Vacuum polarisation: quark anti-quark pairs
- Screening as in QED? Yes, but....
- Gluon loops in addition!
- In total: colour charge mostly surrounded by same charge gluonic cloud, anti-screening
- Small Q^2 = large distance ➡ large coupling
- Large Q^2 = small distance ➡ small coupling

Comparing calculation and experiment





QED

$$\alpha\left(q^2\right) = \frac{\alpha\left(\mu^2\right)}{1-\frac{\alpha\left(\mu^2\right)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

QCD

$$\alpha_S\left(Q^2\right) \equiv \frac{1}{\frac{33-2N_f}{12\pi}\ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

Running much faster than in QED!

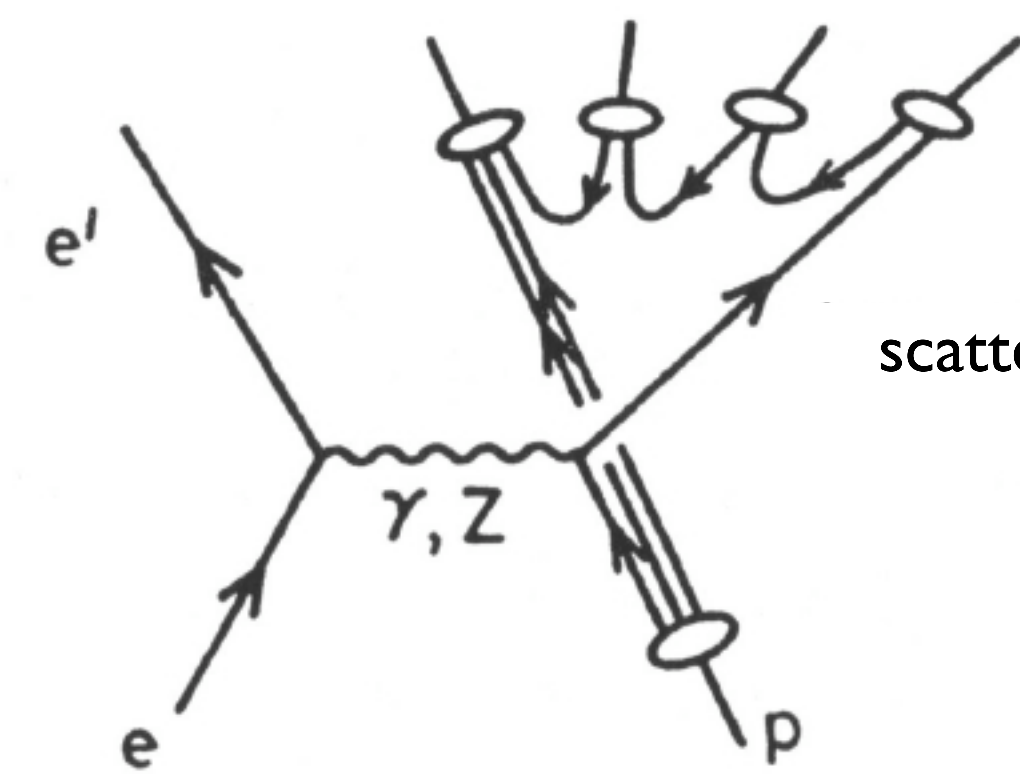
Confronting theory with experiment:

1. Perturbative high energy regime

Fragmentation (non-perturbative) and jets

Example e-p scattering

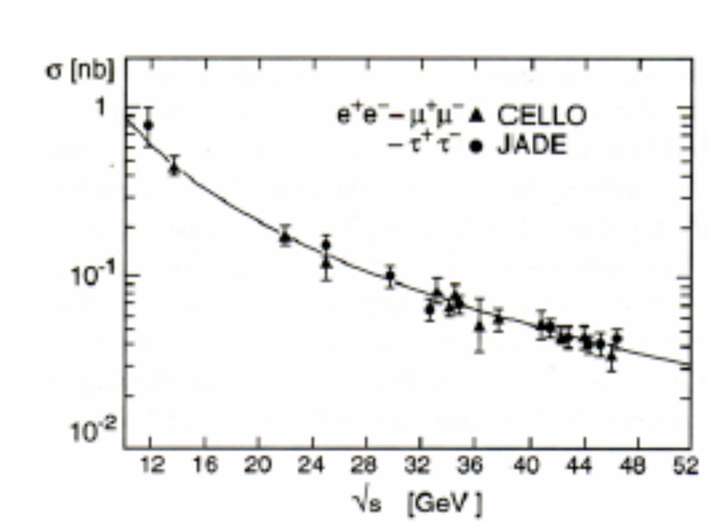
remainder of proton



scattered quark

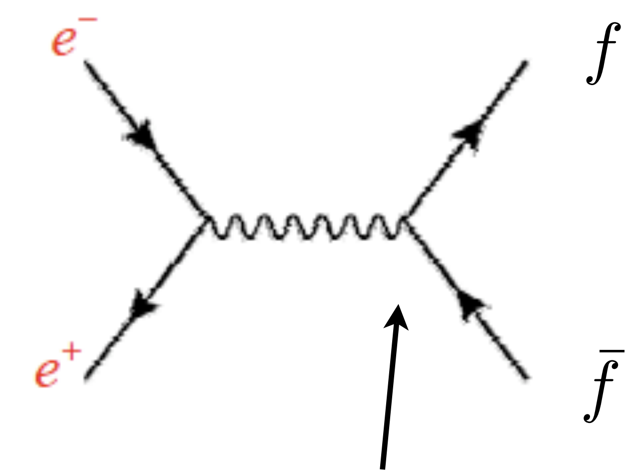
Test for Flavours and Colours via a QED process

$e^+ + e^- \rightarrow \mu^+ + \mu^-$



$$\sigma = \frac{4\pi\alpha^2}{3s}(\hbar c)^2$$

Fermion anti-fermion production



$Q_f = e$

$$R = \frac{\sigma(e^+e^- \rightarrow \text{Hadronen})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{\sigma(e^+e^- \rightarrow \text{alle } q,\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \cdot \sum_q Q_q^2$$

Evolution of R_{had}
with rising CMS-energy

$\sqrt{s}$	Quarks	$R_{had} = 3 \cdot \sum_i Q_i^2$
< ~3 GeV	uds	3·6/9=2.00
< ~10 GeV	udsc	3·10/9=3.33
< ~350 GeV	udscb	3·11/9=3.67
> ~350 GeV	udscbt	3·15/9=5.00

d,s u

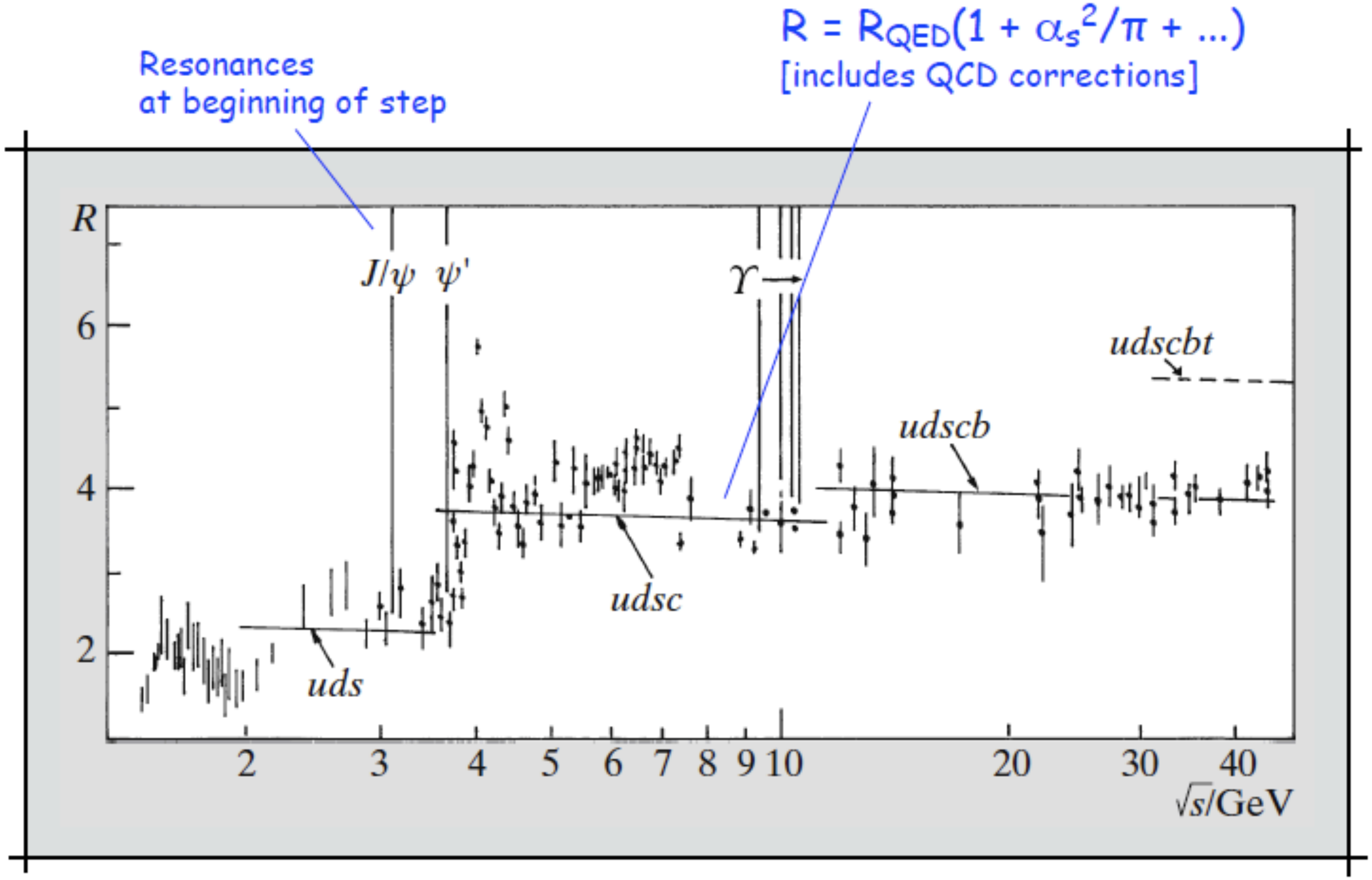
$3 \cdot [2 \cdot (1/3)^2 + (2/3)^2]$

$3 \cdot [2 \cdot (1/3)^2 + 2\,(2/3)^2]$

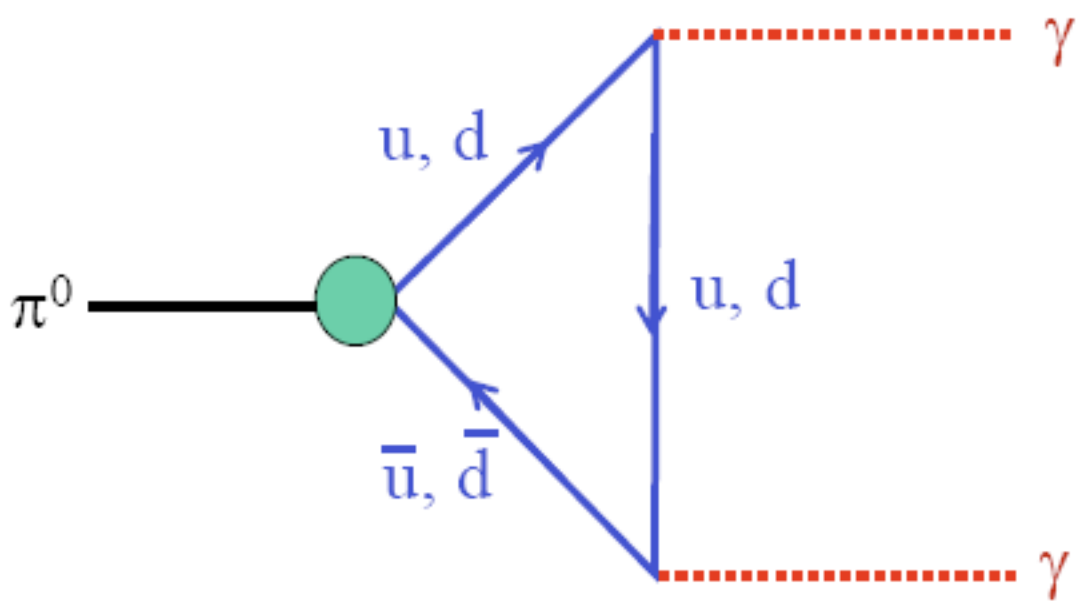
$3 \cdot [3 \cdot (1/3)^2 + 2\,(2/3)^2]$

d,s,b u,b

Evidence for 3 colours per quark flavour



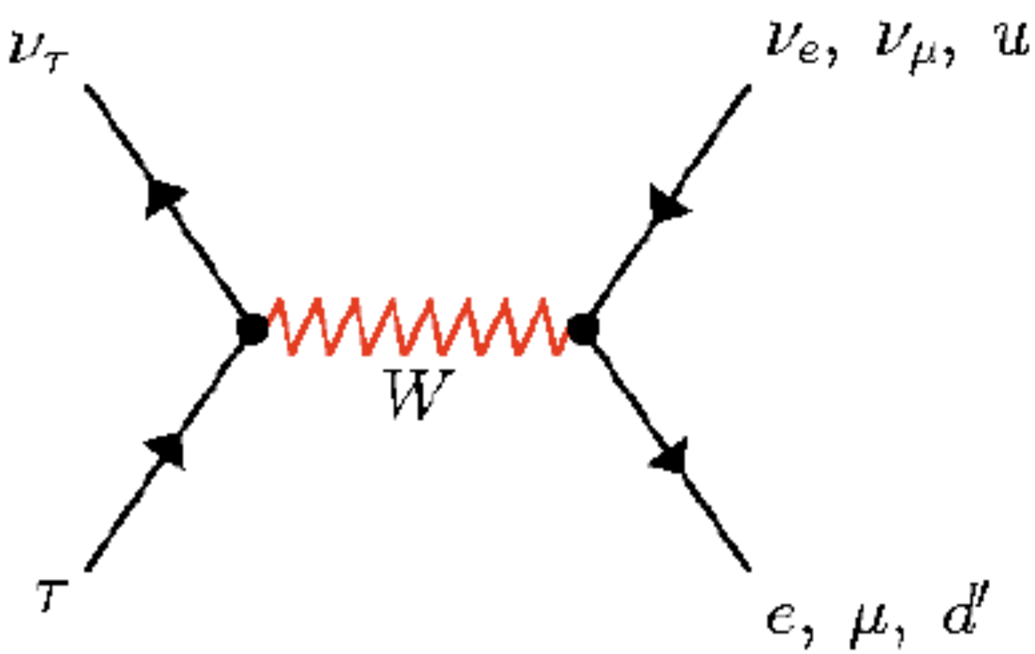
Evidence for 3 colours in pion and tau decay



$$\mathcal{M} = \frac{e^2}{4\pi^2}(Q_u^2 - Q_d^2)\cdot N_c$$

$$\Gamma \sim M^2 \sim N_c^2$$

Same principle: both processes sensitive to number of contributing quark states!



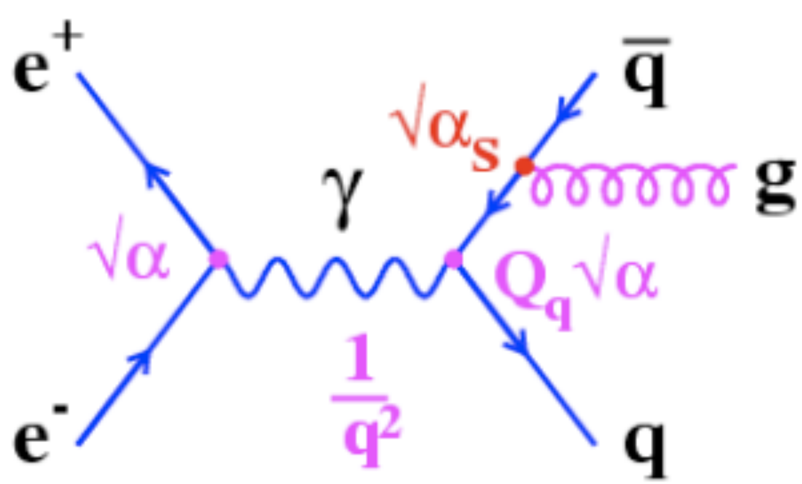
$$\begin{aligned}&\Gamma(\tau^- \to \nu_\tau + X)\\ &= 2\,\Gamma(\tau^- \to \nu_\tau + \ell^-\bar{\nu}_\ell)\\ &\quad + N_{\text{Farbe}}\,\Gamma(\tau^- \to \nu_\tau + d'\bar{u})\end{aligned}$$

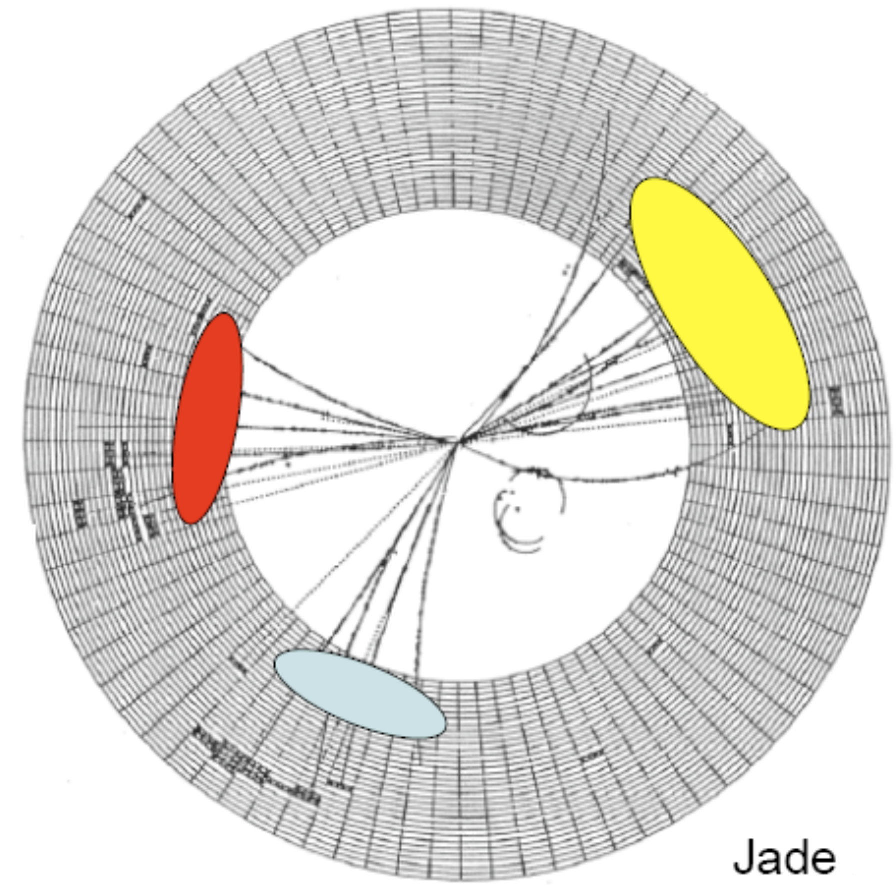
$$B_e = \frac{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}{\Gamma(\tau^- \to \nu_\tau X)} = \frac{1}{2 + N_{\text{Farbe}}} = 0.2$$

$B_e = 0.1784 \pm 0.0006$

Evidence for gluons: 3-jet events

Discovery: PETRA storage ring (DESY Hamburg), 1979



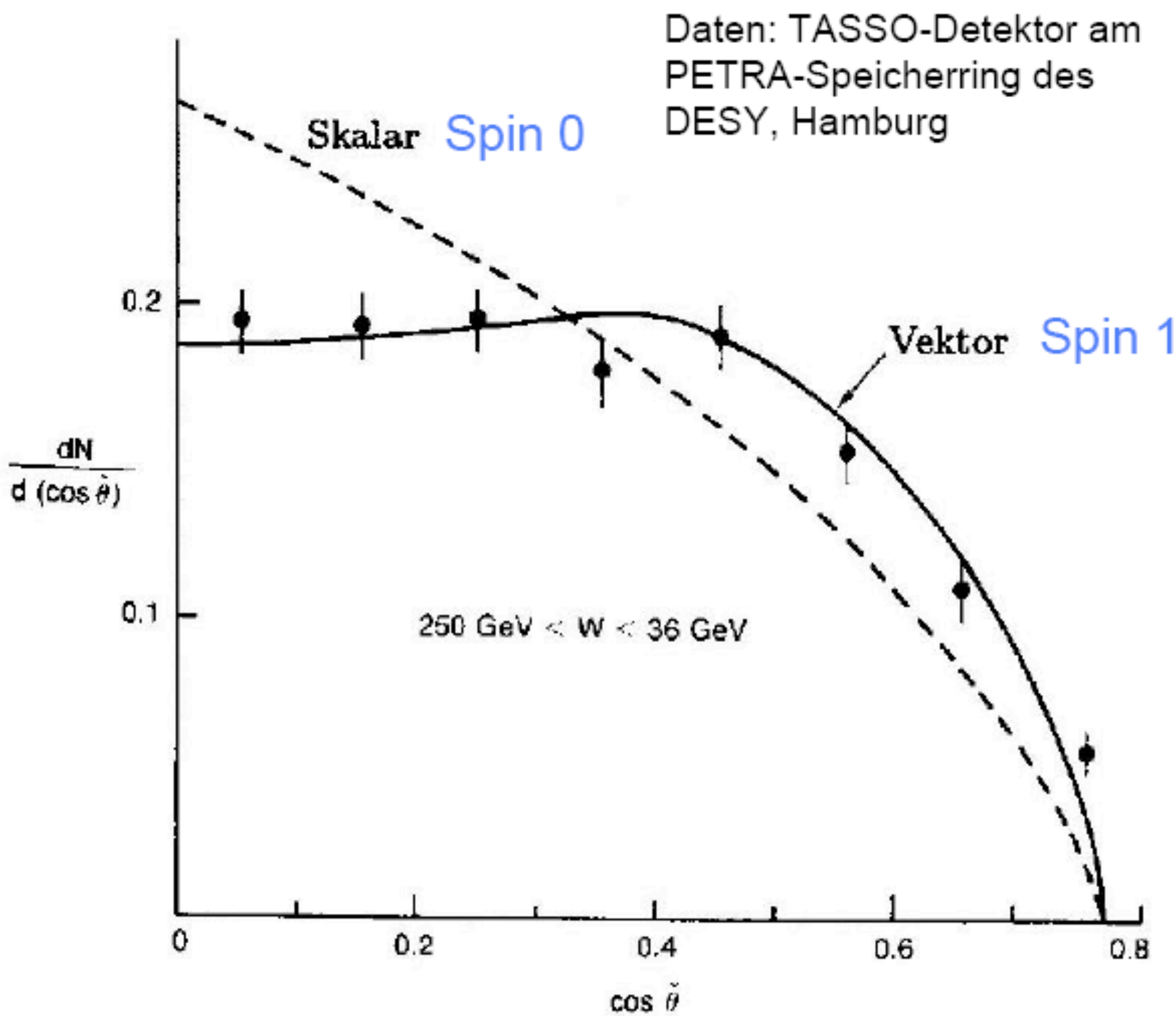


Three-quark final state not possible with leading order QCD Feynman rules

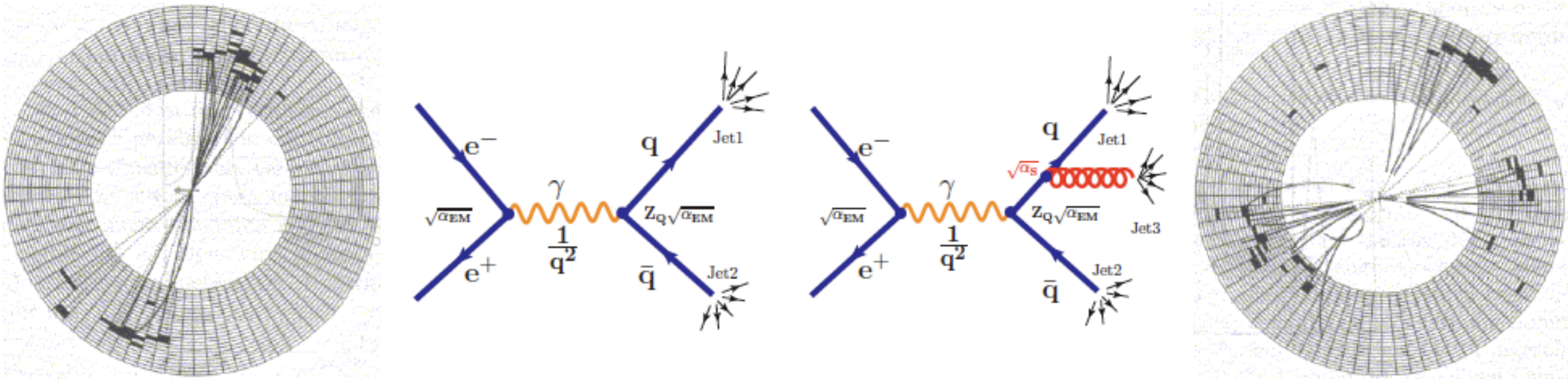
One jet comes from a gluon!

The gluon spin from 3-jet events

Theta: angle between axis of highest energy jet and the direction of the other two jets in their CMS

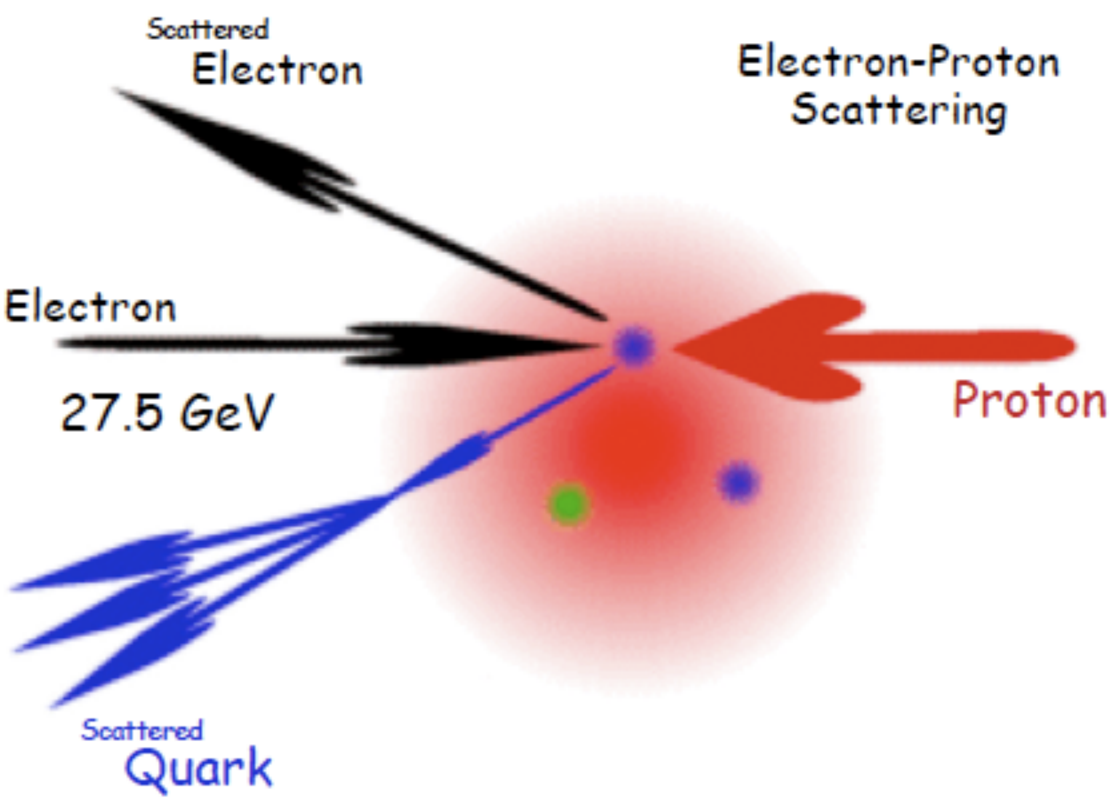


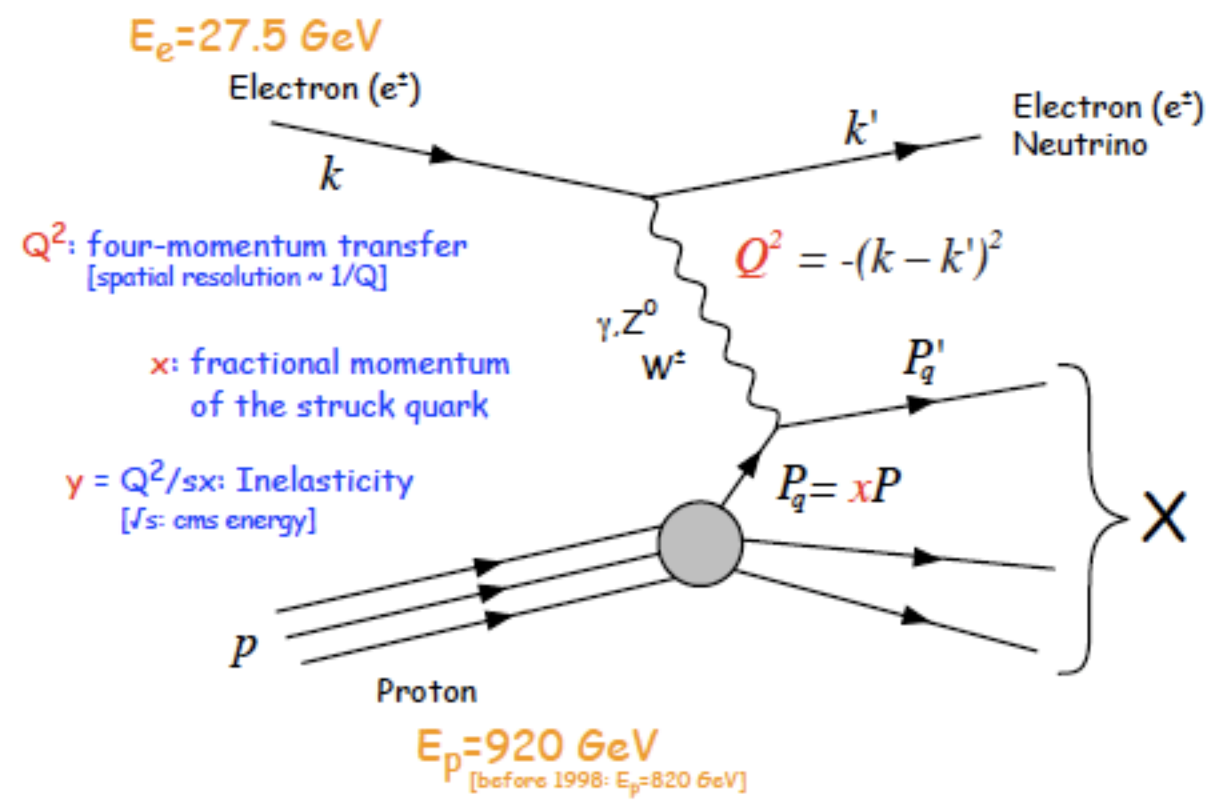
Strong coupling from 2-jet and 3-jet events



$$\frac{\sigma(e^+e^- \to 3\ \text{Jets})}{\sigma(e^+e^- \to 2\ \text{Jets})} \propto \alpha_s$$

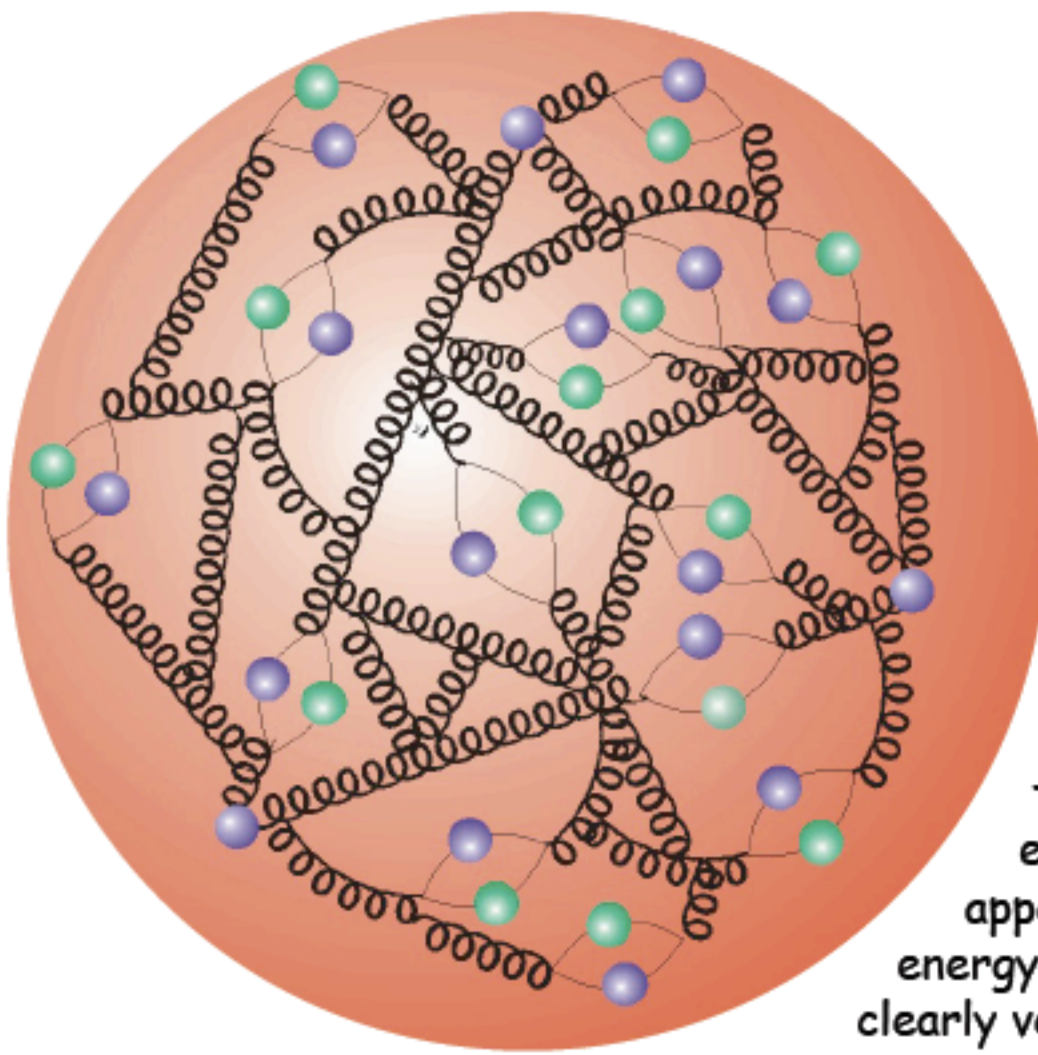
Evidence for quarks: Deep Inelastic Scattering





The difference to the quark model:

Todays Picture of the Proton



The most dramtic of these
[experimental consequences],
that the protons viewed at
ever higher resolution would
appear more and more as field
energy (soft glue), was only
clearly verified at HERA ...

F. Wilczek
[Nobel Prize 2004]

Confronting theory with experiment:

II. Non-perturbative low energy regime

Non-relativistic QCD (NRQCD)

Consider Dirac eqn for free particle + plane wave solution:

$$(i\gamma_\mu \partial^\mu - m)\psi(x) = 0$$

$$\begin{aligned}(E-m)\varphi - \sigma \cdot \mathbf{p}\chi &= 0 \\ (E+m)\chi - \sigma \cdot \mathbf{p}\varphi &= 0\end{aligned}$$

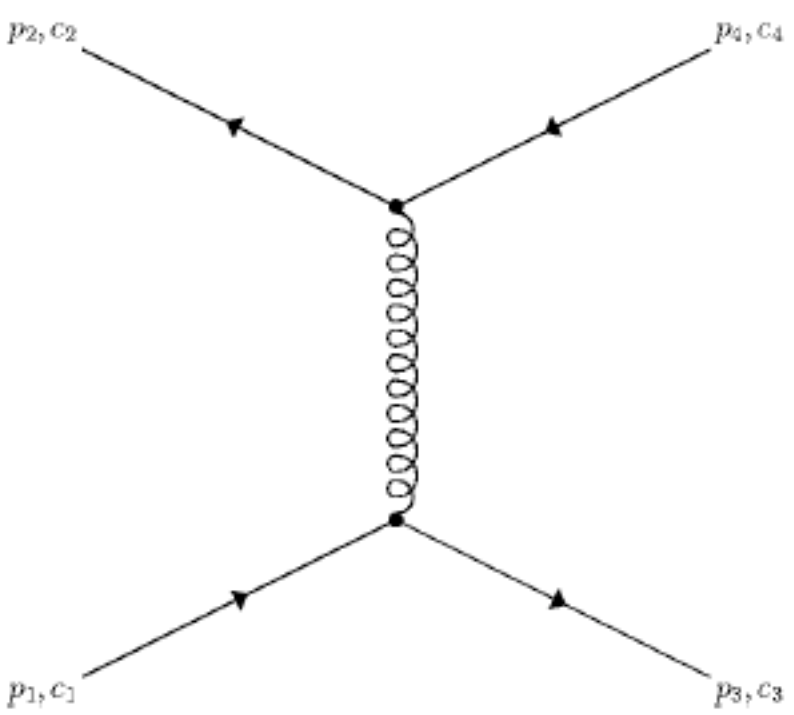
$$E = (\vec{p}^2 + m^2)^{1/2} = m\left(\frac{\vec{p}^2}{m^2} + 1\right)^{1/2} = m\left(1 + \frac{\vec{p}^2}{2m^2} + \dots\right)$$ expansion in small v/c

$\Rightarrow$ $$\chi \approx \frac{\sigma \cdot \mathbf{p}}{2m}\varphi$$ negligible, only φ left, two components (spin)

Wave equation: $$i\frac{\partial}{\partial t}\psi = -\frac{\vec{\nabla}^2}{2m}\psi$$ Schrödinger! Generalises to interactions

Bound states of heavy quarks

Short distance part perturbative

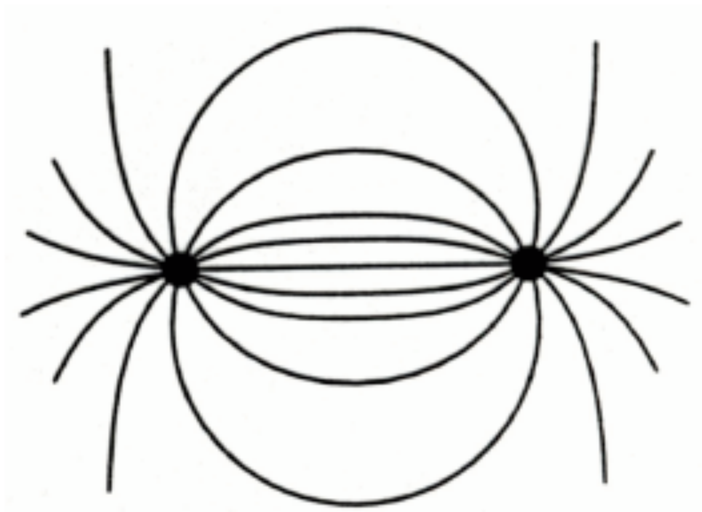


$$V(r) = -C\frac{\alpha_s}{r} \qquad \text{Singlet:} \quad C = \frac{4}{3}, \qquad \text{Octet:} \quad C = -\frac{1}{3}$$

Confinement, qualitative

Fields between charge and anti-charge:

QED



dipole field

QCD

colour electric fluc tube

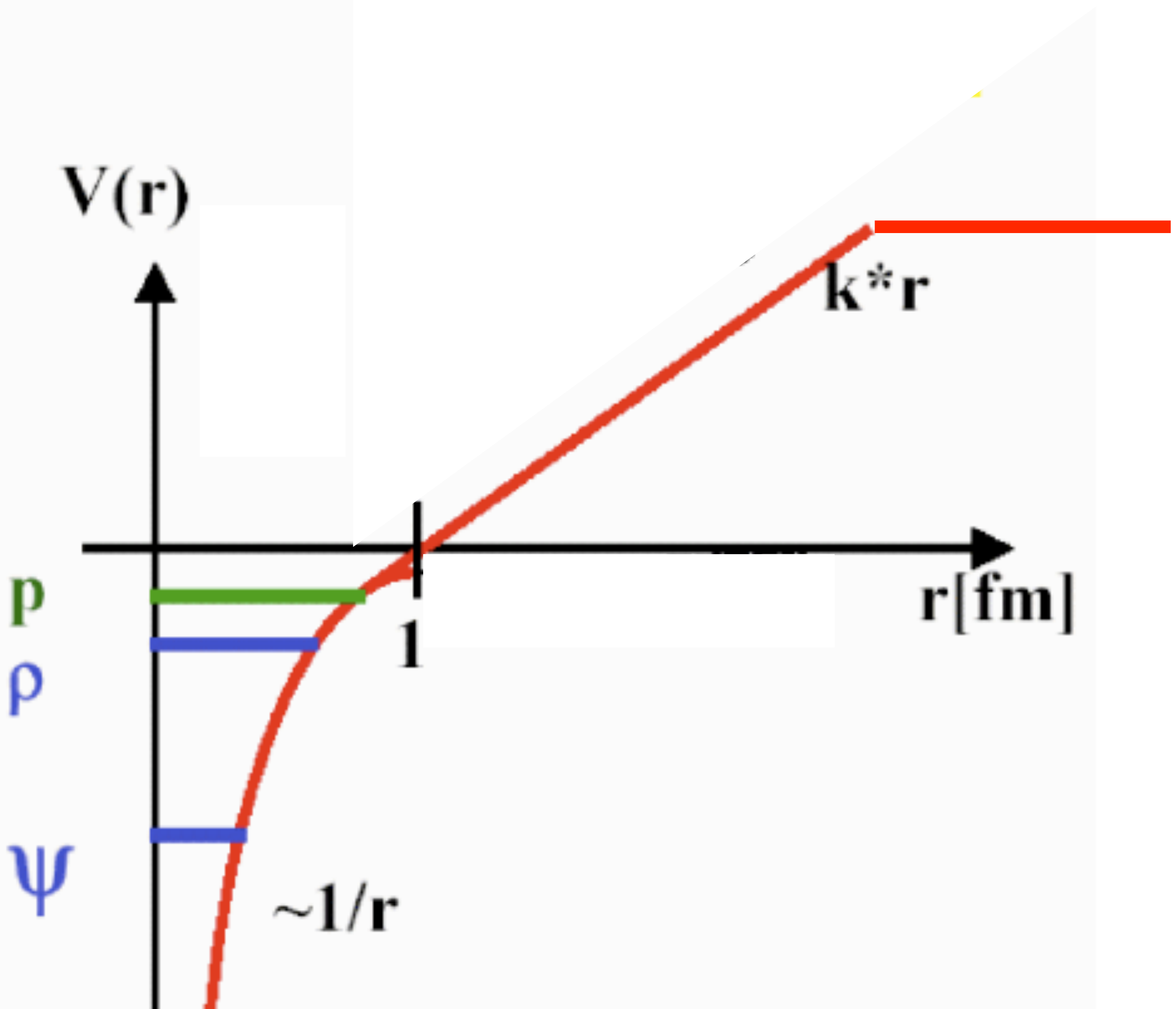
Field energy in the QCD flux tube grows linearly with separation

$V(x) = k \cdot x$ $\qquad$ $k \approx 1$ GeV / fm

At some point pair creation of light quarks is possible:

String breaking

Formation of heavy light mesons, saturation of the potential



Discovery of J/psi (1974)

Brookhaven National Laboratory:

$$p + Be \rightarrow \underbrace{J/\Psi}_{\hookrightarrow\, e^+e^-} + X$$

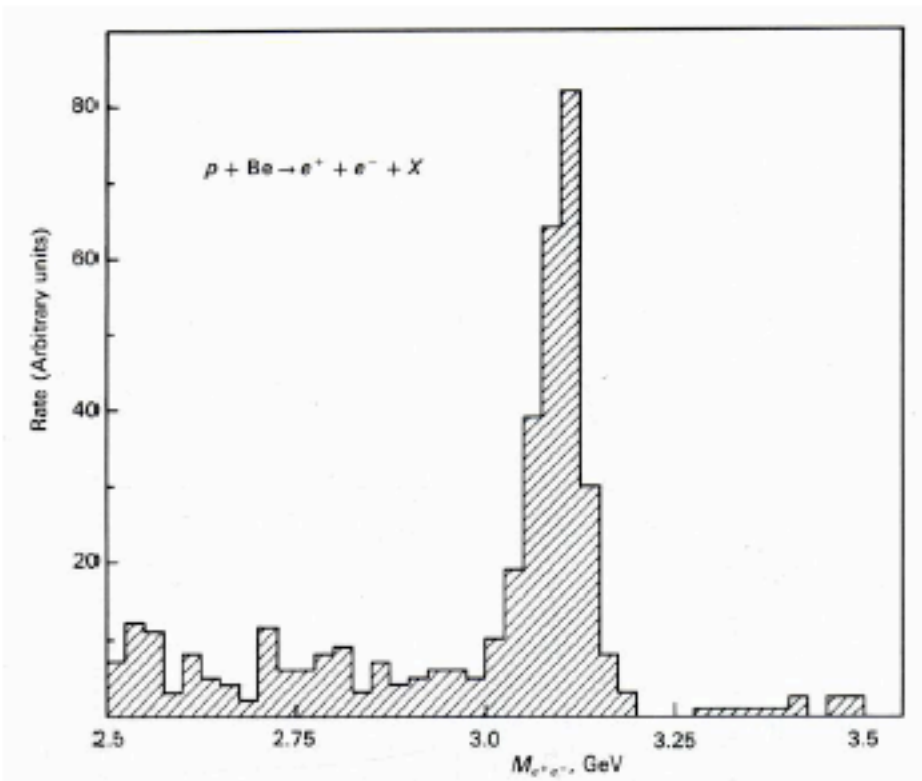


Figure 5.10 Results of Aubert *et al.* (1974) indicating the narrow resonance ψ/J in the invariant-mass distribution of e^+e^- pairs produced in inclusive reactions of protons with a beryllium target. The experiment was carried out with the 28-GeV AGS at Brookhaven National Laboratory.

e^+e^- Speicherring Spear in Stanford:

$$e^+e^- \rightarrow J/\Psi \rightarrow \text{Hadronen}$$
$$\rightarrow e^+e^-$$
$$\rightarrow \mu^+\mu^-$$

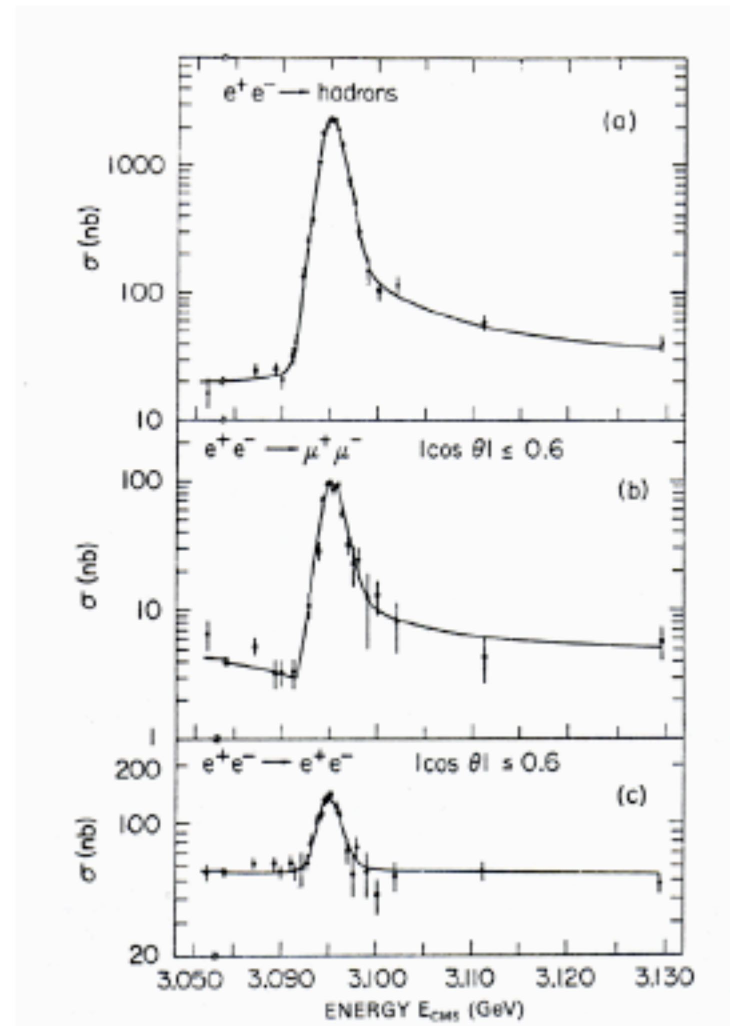


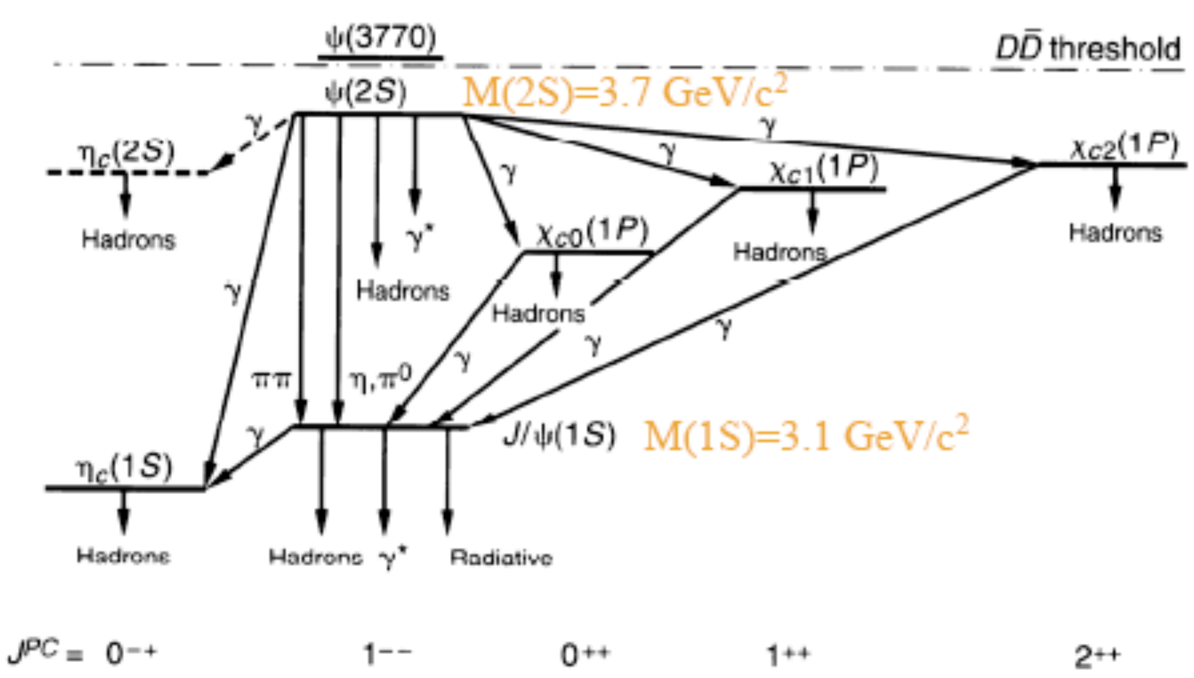
Figure 5.9 Results of Augustin *et al.* (1974) showing the observation of the ψ/J resonance of mass 3.1 GeV, produced in e^+e^- annihilation at the SPEAR storage ring, SLAC.

Predictions for spectra of quarkonia

c,b,t quarks are non-relativistic → solve Schrödinger eqn with potential

$$V(r) = \frac{4}{3}\frac{\alpha_s}{r} + kr$$

$\bar{c}c$ charmonia



Singlett Triplett Triplett

$\bar{b}b$ bottomonia