Topological objects in gauge theories – Problem Set 6

July 9, 2010 (will be discussed on July 16, 2010)

Problem 11

Consider the following integrals:

$$\text{(a)} \quad I_1 = \int_{-\infty}^{+\infty} dx\, e^{-S_1} \quad , \quad S_1 = \lambda\Big(x^2 + a^2\Big)^2 \tag{1}$$

$$\text{(b)} \quad I_2 = \int_{-\infty}^{+\infty} dx\, e^{-S_2} \quad , \quad S_2 = \lambda\Big(x^2 - a^2\Big)^2 \tag{2}$$

$$\text{(c)} \quad I_3 = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dz\, e^{-S_3} \quad , \quad S_3 = \lambda\Big(x^2 + y^2 + z^2 - a^2\Big)^2, \tag{3}$$

$a > 0$, $\lambda > 0$.

Solve the integrals I_j in the "semiclassical approximation", i.e. consider quadratic fluctuations around the minimum/minima of S_j, but neglect cubic and higher order fluctuations.

Discuss, in which limit, expressed in terms of a and λ, the semiclassical approximations of I_j are rather accurate.

Evaluate the integrals I_j numerically for $a = 1.0$ and $\lambda \in \{0.01\,,\, 0.10\,,\, 1.00\,,\, 10.00\,,\, 100.00\}$ and compare with the corresponding semiclassical results.