
Topological objects in gauge theories – Problem Set 5

June 25, 2010 (will be discussed on July 2, 2010)

Problem 9

Show that the n -sphere $S^n = \{\mathbf{r} \in \mathbb{R}^{n+1} \mid |\mathbf{r}| = 1\}$ and the punctured \mathbb{R}^{n+1} , $\mathbb{R}^{n+1} \setminus \{\vec{0}\}$, are homotopically equivalent.

Problem 10

Consider $SU(2)$ Yang-Mills-Higgs theory with the Higgs field in the adjoint representation in $d = 3$:

$$\mathcal{L} = \text{Tr}\left((D^\mu\phi)(D_\mu\phi)\right) - V(\text{Tr}(\phi\phi)) - \frac{1}{2}\text{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right)$$
$$V(\text{Tr}(\phi\phi)) = \frac{\lambda}{4}\left(2\text{Tr}(\phi\phi) - a^2\right)^2.$$

Give an example of the asymptotic form ($|\mathbf{r}| \rightarrow \infty$) of a finite energy field configuration ϕ , A_μ with

- (a) $\text{deg}(\phi) = -1$,
- (b) $\text{deg}(\phi) = +2$.

Perform an explicit calculation of the degree. Determine the corresponding asymptotic magnetic field.