
Topological objects in gauge theories – Problem Set 4

May 28, 2010 (will be discussed on June 11, 2010)

Problem 7

When using the spherically symmetric ansatz

$$\phi = \psi(r)e^{in\varphi} \quad , \quad \psi(r) \geq 0 \quad , \quad \mathbf{A} = \mathbf{e}_\varphi \frac{n\alpha(r)}{r} ,$$

the equations of motion of the Abelian Higgs model reduce to

$$0 = \psi''(r) + \frac{\psi'(r)}{r} - \frac{e^2 n^2}{r^2} \left(\alpha(r) - \frac{1}{e} \right)^2 \psi(r) - \frac{\lambda}{2} (\psi^2(r) - a^2) \psi(r)$$

$$0 = \alpha''(r) - \frac{\alpha'(r)}{r} - 2e^2 \psi^2(r) \left(\alpha(r) - \frac{1}{e} \right) ,$$

where smooth field configurations require $\psi(0) = 0$ and $\alpha(0) = 0$ (if $n \neq 0$) and finite energy $\psi(\infty) = a$ and $\alpha(\infty) = 1/e$.

- (a) Solve these equations approximately for large r by linearizing in $X = a - \psi$ and $Y = 1/e - \alpha$.
- (b) Discuss the validity of the obtained asymptotic solutions.
- (c) At which value of the coupling constant λ is the exponential fall-off of the Higgs field and of the gauge field identical?

Problem 8

Since vortices are exponentially localized, one can construct approximate multi vortex solutions of the equations of motion of the Abelian Higgs model by suitable superposition of well separated individual vortices.

Design a method to construct such approximate multi vortex solutions assuming that solutions for individual vortices are at your disposal.