# Topological objects in gauge theories – Problem Set 2

April 30, 2010 (will be discussed on May 14, 2010)

**Problem 3**

The Lagrangian of the sine-Gordon model is

$$\mathcal{L} \;=\; \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - V(\phi) \quad , \quad V(\phi) \;=\; A\Big(1-\cos(2\pi\phi/F)\Big),$$

where $\phi$ is a real scalar field.

Consider this model in $D = 1+1$ dimensions.

(a) Which are the classical ground states?

(b) Which boundary conditions are possible for field configurations with finite energy? Discuss topological indices and topological sectors. Give an appropriate definition of topological charge.

(c) Derive a Bogomol'nyi bound for the energy of field configurations with non-trivial boundary conditions.

(d) Find solutions of the equations of motion with finite energy and non-trivial boundary conditions.

**Problem 4**

Apply the gauge principle to the action of a complex scalar field $\phi$,

$$S \;=\; \int d^D x\, \mathcal{L} \quad , \quad \mathcal{L} \;=\; (\partial^\mu\phi)^*(\partial_\mu\phi) - V(|\phi|),$$

i.e. modify $S$ in a minimal way such that $S$ is invariant under local U(1) transformations

$$\phi(\mathbf{x},t) \;\to\; \phi'(\mathbf{x},t) \;=\; G(\mathbf{x},t)\phi(\mathbf{x},t) \quad , \quad G(\mathbf{x},t) \;=\; e^{i\Lambda(\mathbf{x},t)} \;\in\; \mathrm{U}(1).$$

Specify the covariant derivative and the transformation law of the corresponding U(1) gauge field.