Exercise sheet VI

June 12 [correction: June 19]

Problem 1 [*"Higher order" perturbation theory, combinatorial (Feynman) rules*]

Consider Euclidean ϕ^4-theory,

$$S[\phi] \quad = \quad \int d^4x \left(\frac{1}{2}(\partial_\mu\phi(x))(\partial_\mu\phi(x)) + \frac{m}{2}\phi^2(x) + \frac{\lambda}{4!}\phi^4(x)\right).$$

(a) Draw all diagrams of order λ^3 contributing to the connected 2-point function $G_2^c(x_1, x_2)$.

(b) Draw all diagrams of order λ^3 contributing to the connected 4-point function $G_4^c(x_1, x_2, x_3, x_4)$.

(c) Truncate the diagrams drawn in (b). Which of them are 1-particle-irreducible?

Problem 2 [*Feynman rules in axial gauge*]

Write down the effective action (gauge-fixing term, ghost term) for QCD (i.e. the $SU(3)$ gauge theory) in the *axial gauge*:

$$n_\mu A_\mu^a = 0 \;\;,\;\; \text{for some fixed } n_\mu \;:\; n_\mu n_\mu = 1 \;\;;$$

What happens to the ghost? Derive the gluon propagator (in momentum space) and sketch the corresponding Feynman rules for the quark-gluon interactions.

Hint: when inverting the gluon operator, the symmetry-motivated Ansatz

$$\Big(-k^2 g_{\mu\nu} + k_\mu k_\nu - \frac{1}{\alpha}n_\mu n_\nu\Big)^{-1} = A g_{\mu\nu} + B k_\mu k_\nu + C(k_\nu n_\mu + k_\mu n_\nu)$$

will suffice.