Exercise sheet III

May 10 [correction: May 15]

Problem 1 [*Green's functions in a free real scalar field*] Consider the action for a free real scalar field in Euclidean space

$$S_E[\phi] = \int \mathrm{d}^4x \left(\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{1}{2}m^2\phi^2\right)$$

Using the generating functional discussed in the lectures,

(i) compute the 4-point function $G_4(x_1, x_2, x_3, x_4)$;

(ii) compute the 6-point function $G_6(x_1, x_2, x_3, x_4, x_5, x_6)$;

(iii) show that $G_n(x_1, \ldots, x_n) = 0$ for odd n.

Problem 2 [*Complex scalar field*] Consider a free complex scalar field described by the action (in Euclidean space)

$$S_E[\phi] = \int \mathrm{d}^4x \left(\partial_\mu\phi^*\partial_\mu\phi + m^2|\phi|^2\right) .$$

Generalize the path integral formalism from the lecture to this case (complex field, instead of a real one). In particular,

(i) write down espressions for the transition amplitude and for VEVs.

(ii) Define a suitable generating functional and compute it explicitly.

(iii) Show how one can obtain VEVs from your generating functional.

(iv) Calculate the propagators

$$\langle\Omega\,|\phi(x_2)\phi(x_1)|\,\Omega\rangle$$

and

$$\langle\Omega\,|\phi^*(x_2)\phi(x_1)|\,\Omega\rangle$$

for $t_2 > t_1$.