Exercise sheet IX

June 22 [correction: June 29]

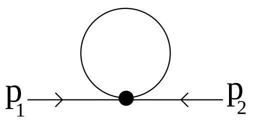
Problem 1 [*Divergences in the ϕ^3 theory*]

Consider the four-dimensional ϕ^3 theory. Draw all diagrams that have a non-negative "superficial degree of divergence".

For each such diagram, with explicit calculations, determine the actual degree of divergence.

Problem 2 [*Dimensional regularisation of the ϕ^4 theory*]

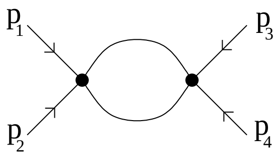
In the Lecture you have seen how, in Euclidean dimensional regularisation ($\epsilon = 4 - d$),



$$= \lambda \frac{1}{2} \delta(p_1 + p_2) \pi^2 m^2 \left(\frac{2}{\epsilon} + 1 - \gamma + \log\left(\frac{\mu^2}{\pi m^2} \right) + \mathcal{O}(\epsilon) \right)$$

(external lines are not explicitly written on the r.h.s.).

Prove that



$$= \lambda^2 \frac{1}{2} \mu^\epsilon \delta(p_1 + p_2 + p_3 + p_4) \pi^2 \quad \times$$

$$\left[\frac{2}{\epsilon} - \gamma - \int_0^1 \mathrm{d}z \log\left(\frac{\pi [(p_3 + p_4)^2 z(1 - z) + m^2]}{\mu} \right) + \mathcal{O}(\epsilon) \right] .$$

Hint: to proceed, you will need to use the *Feynman parametrisation* trick:

$$\underbrace{\frac{1}{(q_1^2 + m^2)}}_{a} \cdot \underbrace{\frac{1}{[(q_1 - q_2)^2 + m^2]}}_{b} = \frac{1}{ab} = \int_0^1 \frac{\mathrm{d}z}{[az + b(1 - z)]^2} .$$