Exercise sheet V

May 18 [correction: May 25]

Problem 1 [*"Higher order" perturbation theory, combinatorial (Feynman) rules*] :

Consider Euclidean ϕ^4-theory,

$$S[\phi] \quad = \quad \int d^4x \left(\frac{1}{2}(\partial_\mu \phi(x))(\partial_\mu \phi(x)) + \frac{m}{2}\phi^2(x) + \frac{\lambda}{4!}\phi^4(x)\right).$$

(a) Draw all diagrams of order λ^3 contributing to the connected 2-point function $G_2^c(x_1, x_2)$.

(b) Draw all diagrams of order λ^3 contributing to the connected 4-point function $G_4^c(x_1, x_2, x_3, x_4)$.

(c) Truncate the diagrams drawn in (b). Which of them are 1-particle-irreducible?

(d) Determine for the perturbative expansion of $G_4^c(x_1, x_2, x_3, x_4)$ the prefactors of the diagrams drawn in (b).
(Do not write down lengthy mathematical expressions! Think about the underlying mathematics and figure out simple combinatorial rules ["Feynman rules"], to determine the correct prefactors. Check these rules by reproducing the order λ^2 prefactors determined, when solving exercise sheet 3.)