Exercise sheet II

October 26 [correction: November 2]

Problem 1 [*Electrons in LEP*] In the LEP storage ring at CERN, head-on collisions between (equally accelerated) electrons and positrons were produced, such that the total energy in the center of mass frame was equal to the mass of the Z boson ($m_Z = 91$ GeV). What is the velocity of each particle before the collision? If an electron is accelerated toward a positron at rest, what velocity does it need in order to reach the same center-of-mass total energy?

Problem 2 [*Continuity equation revisited*] The Klein-Gordon equation for a free particle reads

$$-\frac{\partial^2}{\partial t^2}\phi \; = \; -\nabla^2\phi + m^2\phi \, .$$

Derive the corresponding continuity equation using a similar procedure as in Problem 3 of exercise sheet I.

Verify that the continuity equation can be written as $\partial_\mu j^\mu = 0$, where

$$j_\mu \; = \; i\left(\phi^*(\partial_\mu\phi) - (\partial_\mu\phi^*)\phi\right) \, .$$

What happens to $j_0 = \rho$ for a negative energy solution $\phi \sim \exp(ipx)$, and what does this imply for the interpretation of ρ?

Problem 3 [*Invariant measure*] Given the relativistic invariance of the measure d^4k, show that the integration measure

$$\frac{\mathrm{d}^3k}{(2\pi)^3 2E(\mathbf{k})}$$

is Lorentz invariant, provided that $E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$. Use this result to argue that $2E(\mathbf{k})\delta^3(\mathbf{k} - \mathbf{k}')$ is a Lorentz invariant distribution. [**Hint:** Start from the Lorentz invariant expression $\frac{\mathrm{d}^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k_0)$ and use $\delta(x^2 - x_0^2) = \frac{1}{2|x|}\left(\delta(x - x_0) + \delta(x + x_0)\right)$. What is the significance of the δ and the θ functions above?]

Problem 4 [*Lorentz transformations*] An infinitesimal Lorentz transformation and its inverse can be written as

$$x'^\alpha \; = \; (g^{\alpha\beta} + \epsilon^{\alpha\beta})x_\beta \; , \quad x^\alpha \; = \; (g^{\alpha\beta} + \epsilon'^{\alpha\beta})x'_\beta \, ,$$

where $\epsilon^{\alpha\beta}$ and $\epsilon'^{\alpha\beta}$ are infinitesimal. Show from the definition of the inverse that

$$\epsilon'^{\alpha\beta} = -\epsilon^{\alpha\beta} \, .$$

Show from the preservation of the norm that

$$\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha} \, .$$