Numerische Methoden der Physik

WiSe 2023-2024 – Prof. Marc Wagner

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Exercise sheet 3

To be handed in on 01.11.23 and discussed on 03.11.23 and 06.11.23.

Exercise 1 [*Pendulum motion*] (2+2+9+2+3+2=20 pts.)

Consider the so-called *simple* pendulum (mass m and length l), with gravitational force, $\vec{F} = -mg\vec{e}_z$. The angle ϕ describes the angular displacement of the pendulum from the neutral position $\phi = 0$. Consider in the following an initial displacement $\phi(t=0) = \phi_0$ and vanishing velocity $\dot{\phi}(t=0) = 0$.

(i) Derive the equation of motion of the pendulum.

(ii) Solve analytically the equation of motion assuming $\phi_0 \ll 1$.

(iii) Implement the 4^{th} order Runge-Kutta method discussed in the lecture, to solve the equation of motion obtained in task (i). Discuss in detail the usual preparatory steps to solve a second order ordinary differential equation with Runge-Kutta methods? What would you do if you had to solve a higher order ordinary differential equation?

(iv) Test your code for small-angles. Study several small values for ϕ_0 and compare the numerical solution to the previously calculated analytical solution.

(v) Use your program to solve the equation of motion for initial conditions $\phi_0 \in \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$. Calculate the motion $\phi(t)$ for at least a complete period of the pendulum. Compare again your analytical solution with the numerical one. Can you explain the discrepancy?

(vi) Set $\phi_0 = \frac{\pi}{2}$ and design and implement a way to calculate the period of the pendulum. Provide an error estimate for your result.