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Exercise sheet 8

To be discussed on 06 and 09 July

Exercise 1 [χ^2-*fitting*]

The potential between an infinitely heavy quark and an infinitely heavy anti-quark, the so-called *static potential*, as function of the quark separation, can be computed using lattice QCD techniques and Monte Carlo methods. In this framework, both the quark separation $a\,\hat{r} \equiv r$ and the potential $\hat{V}_{Q\bar{Q}} \equiv a\,V_{Q\bar{Q}}$ are rescaled using the lattice spacing a, so that only dimensionless quantities appear[1].

At the web address

`http://th.physik.uni-frankfurt.de/~mwagner/teaching/numerik/V.dat`

you can find the result of a lattice QCD computation of the static potential, which has been done on a $24^3 \times 48$ discretized space-time, with $a = 0.0967\,\text{fm}$. The file contains three columns with, respectively, $\hat{r}$, $\hat{V}_{Q\bar{Q}}$ and the statistical error on the static potential $\Delta\hat{V}_{Q\bar{Q}}$.

(i) It is known that the parametrization

$$V(r) = V_0 + \frac{\alpha}{r} + \sigma\,r$$

is appropriate to describe the r dependence of the static potential.

(a) Rewrite this parametrization in terms of dimensionless quantities using the lattice spacing a.

(b) Perform a χ^2-minimising fit to the data and determine the values of $\hat{V}_0$, $\hat{\alpha}$ and $\hat{\sigma}$. Ignore the data for $\hat{r} \leqslant 2$, because it is known that they are affected by sizeable discretization errors.

(c) Judge the quality of your fit by calculating $\chi^2/\text{d.o.f.}$ (χ^2 per degree of freedom).

(ii) Estimate the force F in N between two quarks at large separation. Use the conversion factors

$$\hbar c = 1 = 197\,\text{MeV}\cdot\text{fm} \quad \text{and} \quad \text{N} = \frac{\text{eV}}{\hbar c}$$

to translate quantities between the natural and SI unit systems. Which would be the mass of an object on Earth with weight F?

(iii) Perform a polynomial interpolation of the 10 data points using Lagrange polynomials. Compare the obtained potential with the outcome of your previous fit and discuss which approach is more appropriate to parametrize the static potential.

[1] Note that, this time, the natural units are used, $\hbar = c = 1$.