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# ################################
# Einfuehrung in die Quantenphysik
# 4. Uebung
# Aufgabe 1
# ################################
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```
restart:
with(linalg):
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#  In the follwing: E > V_0.
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# To determine B1, A2, B2 and A3, one has to solve a 4x4 linear
system.

M:=matrix(4,4):
v:=vector(4):

M[1,1] := 1:
M[1,2] := -1:
M[1,3] := -1:
M[1,4] := 0:
v[1] := -1:

M[2,1] := -k:
M[2,2] := -K:
M[2,3] := K:
M[2,4] := 0:
v[2] := -k:

M[3,1] := 0:
M[3,2] := exp(I*K*a):
M[3,3] := exp(-I*K*a):
M[3,4] := -exp(I*k*a):
v[3] := 0:

M[4,1] := 0:
M[4,2] := K*exp(I*K*a):
M[4,3] := -K*exp(-I*K*a):
M[4,4] := -k*exp(I*k*a):
v[4] := 0:

evalm(M);
evalm(v);
```

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -k & -K & K & 0 \\ 0 & e^{IKa} & e^{-IKa} & -e^{Ika} \\ 0 & Ke^{IKa} & -Ke^{-IKa} & -ke^{Ika} \end{bmatrix}$$

$$\begin{bmatrix} -1 & -k & 0 & 0 \end{bmatrix} \tag{1}$$

```
result := simplify(multiply(inverse(M), v)):

B1 := result[1];
A2 := result[2];
B2 := result[3];
A3 := result[4];
```

$$B1 := -\frac{2\,I\sin(Ka)\,(-K^2+k^2)}{2\,kKe^{-IKa}+K^2e^{-IKa}+2\,kKe^{IKa}-K^2e^{IKa}+e^{-IKa}k^2-e^{IKa}k^2}$$

$$A2 := \frac{2\,ke^{-IKa}\,(k+K)}{2\,kKe^{-IKa}+K^2e^{-IKa}+2\,kKe^{IKa}-K^2e^{IKa}+e^{-IKa}k^2-e^{IKa}k^2}$$

$$B2 := -\frac{2\,ke^{IKa}\,(k-K)}{2\,kKe^{-IKa}+K^2e^{-IKa}+2\,kKe^{IKa}-K^2e^{IKa}+e^{-IKa}k^2-e^{IKa}k^2}$$

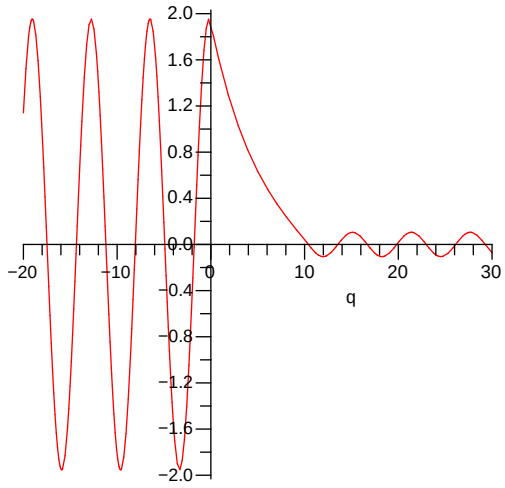
$$A3 := \frac{4\,ke^{-Ika}K}{2\,kKe^{-IKa}+K^2e^{-IKa}+2\,kKe^{IKa}-K^2e^{IKa}+e^{-IKa}k^2-e^{IKa}k^2} \tag{2}$$

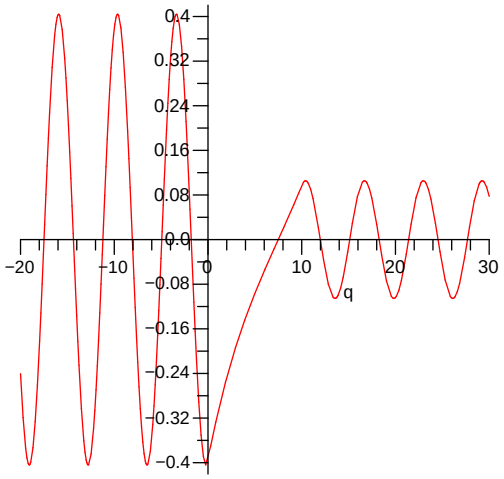
```
# Check the results.

# No barrier.
subs(k=K, B1);
subs(k=K, A2);
subs(k=K, B2);
subs(k=K, A3);

# Infinite barrier.
simplify(subs(k=0, B1));
subs(k=0, A2);
subs(k=0, B2);
subs(k=0, A3);
```

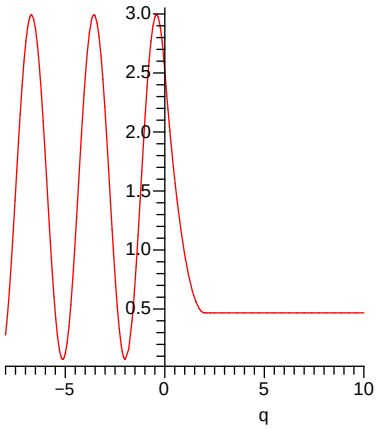
$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

```
# Two examples.

psi1 := exp(I*k*q) + B1*exp(-I*k*q):
psi2 := A2*exp(I*K*q) + B2*exp(-I*K*q):
psi3 := A3*exp(I*k*q):

phi := piecewise(q<0, psi1, 0<q and q<a, psi2, a<=q, psi3):

# First example (wave function).
plot(Re(subs(a=10.0, k=1.0, K=0.2*I, phi)), q =-20.0..30.0);
plot(Im(subs(a=10.0, k=1.0, K=0.2*I, phi)), q =-20.0..30.0);

# Second example (probability distribution).
plot(abs(subs(a=2.0, k=1.0, K=0.2*I, phi))^2, q =-8.0..10.0);
```







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```
# Compute the reflection and the transmission.

assume(a, real):
assume(k, real):
assume(K, real):

R := simplify(Re(B1)^2+Im(B1)^2);
T := simplify(Re(A3)^2+Im(A3)^2);

simplify(R+T);
```

$$R := \left((-1+\cos(K\sim a\sim)^2)\ (-K\sim^2+k\sim^2)^2\right) \Big/ \left(-2\,k\sim^2 K\sim^2 \cos(K\sim a\sim)^2 - K\sim^4 + K\sim^4 \cos(K\sim a\sim)^2 - 2\,k\sim^2 K\sim^2 - k\sim^4 + k\sim^4 \cos(K\sim a\sim)^2\right)$$

$$T := -(4\,k\sim^2 K\sim^2) \Big/ \left(-2\,k\sim^2 K\sim^2 \cos(K\sim a\sim)^2 - K\sim^4 + K\sim^4 \cos(K\sim a\sim)^2 - 2\,k\sim^2 K\sim^2 - k\sim^4 + k\sim^4 \cos(K\sim a\sim)^2\right)$$

1 (4)

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