Advanced quantum mechanics

SS 2019 – Prof. Dr. Marc Wagner

Organization: Room GSC 0|21

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Exercise sheet 6

To be handed in before 30.05.19, 11:00 by e-mail or in office 2.107.
To be discussed in the week of 03.06.19.
23.05.19

Exercise 1 [*Higher partial waves*] (2+3+3=8 pts.)

In the lecture we calculated the contribution of the partial wave with $l = 0$ for a scattering process in the potential of a hard sphere

$$V(r) = \begin{cases} +\infty & \text{if } r \leq R \\ 0 & \text{otherwise} \end{cases} . \tag{1}$$

(a) Consider the cases $kR = 2$ and $kR = 0.2$. Is the partial wave contribution σ_0 a good approximation of the total cross-section, i.e. $\sigma \approx \sigma_0$?

(b) Calculate the contributions of the partial waves with $l = 1$ and $l = 2$. Determine both the phase shift $\delta_l(E)$ and the contribution to the total cross-section σ_l. Do you find convergence with increasing l? *Note that the equation to determine the phase shift can only be solved analytically for $l = 0$. To find the solution, use a computer, or determine the solution graphically.*

(c) Plot the differential cross-section, including contributions of the partial waves with

- $l = 0$
- $l = 0$ and $l = 1$
- $l = 0$, $l = 1$ and $l = 2$.

For which of the cases is the scattering isotropic? Do you observe convergence in the calculations and plots for the three cases?

Exercise 2 [*Yukawa potential, Born approximation*] (4+4+4=12 pts.)

Consider scattering at a Yukawa-potential

$$V(r) = A\frac{e^{-\lambda r}}{r} \ , \ \lambda > 0, \tag{2}$$

which is frequently used in physics, e.g. to describe forces between neutrons and protons due to pion exchange.

(a) Using the Born approximation, calculate the scattering amplitude $f(\vartheta)$.

(b) Using your result from (a), show that the differential cross-section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\vartheta) = \frac{A^2}{(4E_k(\sin(\vartheta/2))^2 + \hbar^2\lambda^2/2m)^2}. \tag{3}$$

(c) Calculate the potential and differential cross-section in the limit $\lambda \to 0$. Are the Born approximation and the assumptions used to derive scattering theory in the lecture applicable in this limit? Compare your result for the differential cross-section with the correct result from the literature.