Z states with lattice QCD

"Workshop on Z states", Gießen

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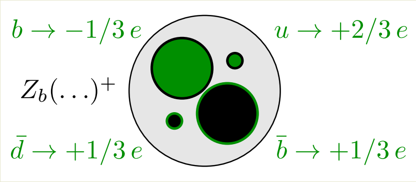
Introduction

- **Goal**:
  - Study tetraquark candidates (= 4-quark states) using lattice QCD.
  - In particular study tetraquarks with a heavy $\bar{c}c$ or $\bar{b}b$ pair.
- **Motivation**: Experimentally measured
  - charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states,
  - charged charmonium states, e.g. $Z_c(3900)^\pm$, $Z_c(4020)^\pm$, $Z_c(4050)^\pm$, $Z_c(4250)^\pm$, $Z_c(4430)^\pm$ ... must be four quark states,
  - ...
- A very challenging problem in lattice QCD $\rightarrow$ no solid results yet.



Outline

(1) Introduction to QCD and lattice QCD.

(2) Two lattice QCD approaches to study Z states:

- Creation operators with 4 quarks of finite mass
  $\rightarrow$ more suited for Z_c states.
- Creation operators with 2 quarks of finite mass and 2 static quarks
  $\rightarrow$ more suited for Z_b states.

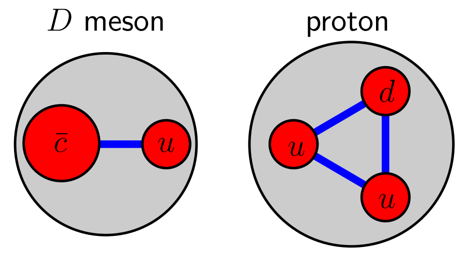
Part 1: Introduction to QCD and lattice QCD

QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.
- Part of the standard model explaining the formation of hadrons (mesons with integer spin, usally $q\bar{q}$; baryons with half-integer spin, usually $qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.
- Definition of QCD simple:

$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu\Big) + m^{(f)}\Big) \psi^{(f)} + \frac{1}{2g^2} \text{Tr}\Big(F_{\mu\nu} F_{\mu\nu}\Big)\right)$$

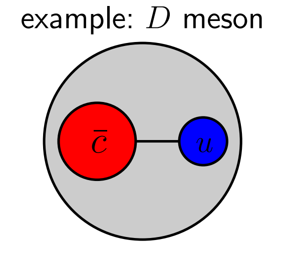
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$



- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of
  - non-linear field equations,
  - the absence of any small parameter/coupling constant (i.e. perturbation theory not applicable).

$\rightarrow$ Solve QCD numerically by means of lattice QCD.

Hadron spectroscopy



- Proceed as follows:

  (1) **Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator O.**

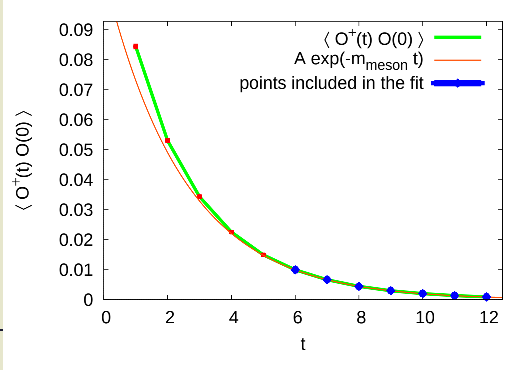
  - An operator O, which generates the quantum numbers (flavor, J^{PC}) of the hadron of interest, when applied to the vacuum $|\Omega\rangle$.
  - An operator O, which crudely generates the hadron of interest (in particular same number of quarks), when applied to the vacuum $|\Omega\rangle$.

  (2) **Determine the corresponding hadron mass from the asymptotic exponential decay of $C(t)$ in time.**

- Example: D meson mass m_D (valence quarks $\bar{c}$ and u, $J^P = 0^-$),

$$O \equiv \int d^3r \, \bar{c}(\mathbf{r})\gamma_5 u(\mathbf{r})$$

$$C(t) \equiv \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle \overset{t\to\infty}{\propto} e^{-m_D t}.$$



Lattice QCD (1)

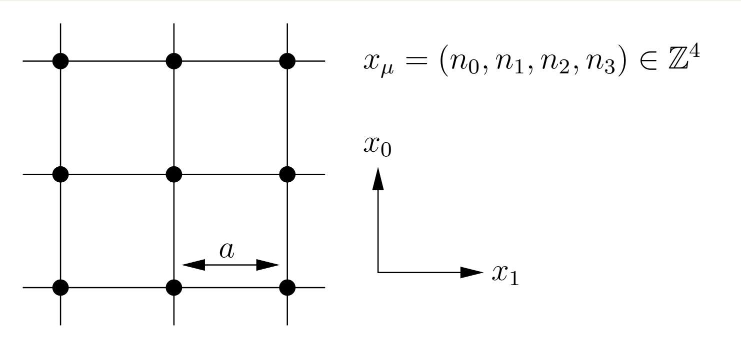
- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle = \\ = \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,O^\dagger(t)O(0)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slide).
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{r},t)$ and $A_\mu(\mathbf{r},t)$.
  - $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite volume effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \bigg) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    - x_μ: $32^4 \approx 10^6$ lattice sites.
    - $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    - $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    - In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

  $\rightarrow$ Standard approaches for numerical integration not applicable.
  $\rightarrow$ Sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

Part 2a: Creation operators with 4 quarks of finite mass (Z_c states)

[S. Prelovsek, C. B. Lang, L. Leskovec and D. Mohler, Phys. Rev. D **91**, 014504 (2015) [arXiv:1405.7623 [hep-lat]]]

[J. O. Daldrop *et al.* [ETM Collaboration], PoS **LATTICE2012**, 161 (2012) [arXiv:1211.5002 [hep-lat]]]
[C. Alexandrou *et al.* [ETM Collaboration], JHEP **1304**, 137 (2013) [arXiv:1212.1418 [hep-lat]]]
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[M.W. *et al.*, PoS **LATTICE2013**, 258 (2014) [arXiv:1309.0850 [hep-lat]]]
[M.W. *et al.*, J. Phys. Conf. Ser. **503**, 012031 (2014) [arXiv:1310.6905 [hep-lat]]]
[J. Schneider, bachelor thesis, Goethe University Frankfurt am Main (2014)]
[A. Abdel-Rehim, J. Berlin *et al.*, PoS **LATTICE2014**, 104 (2014) [arXiv:1410.8757 [hep-lat]]]

Why are Z_c's difficult? (1)

- Lattice QCD = QCD ... i.e. a quantum field theory.
  - E.g. quark numbers are not fixed (quark-antiquark creation and annihilation possible).
  - **The only input to determine a meson state, e.g. the mass, are its QCD quantum numbers** (flavor, J^{PC})**.**
  - No additional input or ansatz possible.
  - **Quark numbers and structure of a state** (like spin S, angular momentum L, spatial width, ...) **are part of the lattice QCD result, i.e. dictated by QCD dynamics.**
- In lattice QCD one provides quantum numbers (flavor, J^{PC}) ... and obtains the n lowest states from that sector (typically $n = 1$, sometimes $n = 2, 3, 4$ ... the larger n, the more difficult the computation).
- Z_c states are close in mass to 2-meson states.
  - E.g. $Z_c(3900)^+$, quantum numbers $J^P = 1^+$ (needs confirmation)

    $m_{Z_c(3900)^+} = 3889\,\text{MeV}$
    $m_{J/\psi} + m_\pi = (3097 + 139)\,\text{MeV} = 3236\,\text{MeV}$
    $m_{\eta_c} + m_\rho = (2984 + 775)\,\text{MeV} = 3759\,\text{MeV}$
    $m_D + m_{D^*} = (1870 + 2007)\,\text{MeV} = 3877\,\text{MeV}$
    ... further 2-meson states ... additionally states with relative momentum ...

$\rightarrow$ **These states must be determined with a single computation at the same time.**

Why are Z_c's difficult? (2)

- Such a computation requires a sizable number of hadron creation operators.
  - **Each of the states, which one intents to compute, has to be crudely approximated by one of these operators.**
    - E.g. if you are interested to study tetraquarks, you need 4-quark operators ...
    - ... if you cannot exclude a 2-quark structure, you also need 2-quark operators ...
    - ... for the 2-meson states of similar mass you need additional 4-quark operators of 2-meson structure.
  - E.g. for $Z_c(3900)^+$ one should/could consider the following operators:

$$O_{\text{diquark}} = \int d^3r \underbrace{\Big(\epsilon^{abc}\bar{d}^b(\mathbf{r})C\gamma_5\bar{c}^{c,T}(\mathbf{r})\Big)}_{\equiv\text{antidiquark}} \underbrace{\Big(\epsilon^{ade}c^{d,T}(\mathbf{r})C\gamma_j u^e(\mathbf{r})\Big)}_{\equiv\text{diquark}} \quad , \quad \ldots$$

$$O_{\text{mesonic molecule}} = \int d^3r \underbrace{\Big(\bar{d}(\mathbf{r})\gamma_5 c(\mathbf{r})\Big)}_{\equiv D} \underbrace{\Big(\bar{c}(\mathbf{r})\gamma_j u(\mathbf{r})\Big)}_{\equiv D^*} \quad , \quad \ldots$$

$$O_{J/\psi+\pi} = \underbrace{\int d^3r \Big(\bar{c}(\mathbf{r})\gamma_j c(\mathbf{r})\Big)}_{\equiv J/\psi} \underbrace{\int d^3\tilde{r} \Big(\bar{d}(\tilde{\mathbf{r}})\gamma_5 u(\tilde{\mathbf{r}})\Big)}_{\equiv\pi} \quad , \quad \ldots$$

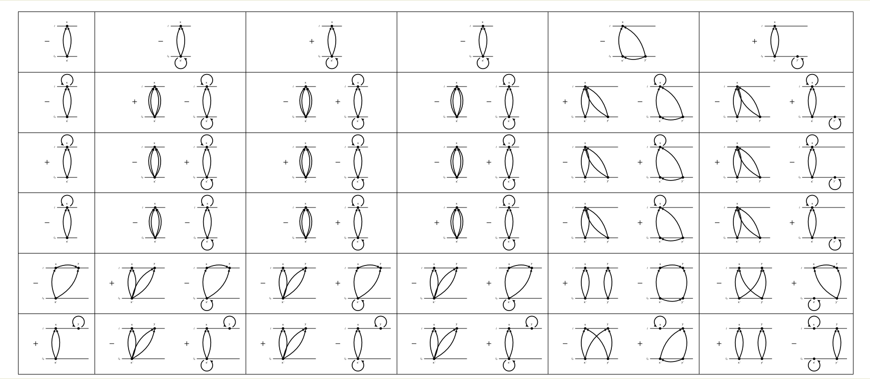
Why are Z_c's difficult? (3)

- **Then one has to compute the temporal correlation matrix from these operators,**

$$C_{jk}(t) \quad = \quad \langle \Omega | O_j^\dagger(t) O_k(0) | \Omega \rangle .$$

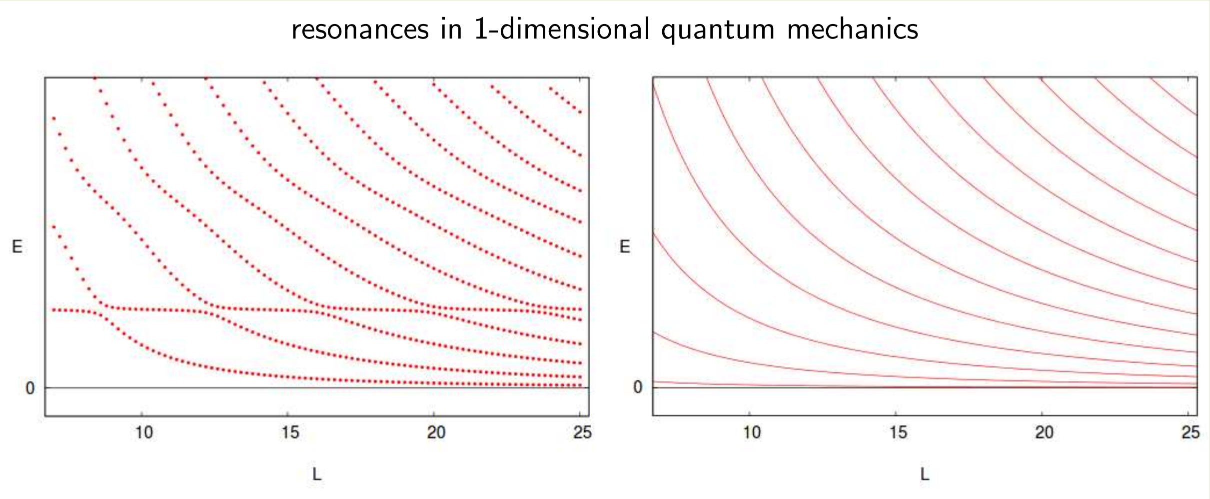
The extraction of several states (the Z_c state and the 2-meson states of similar mass) requires very high precision.

  - 4-quark operators tend to increase statistical errors drastically.
  - $\bar{c}c$ pairs in operators ($\rightarrow$ quark-antiquark creation and annihilation, closed quark loops, disconnected diagrams) drastically increase statistical errors.



Why are Z_c's difficult? (4)

- Until now we assumed that the Z_c state of interest is a stable state (i.e. corresponds to an eigenstate of the QCD Hamiltonian).
- If the Z_c state of interest is unstable, i.e. readily decays into a lighter meson-meson pair, the computation is even more difficult.
  $\rightarrow$ One needs to compute the masses of 2-meson states as functions of the spatial volume ... distortions compared to the spectrum of non-interacting mesons provide information about resonance parameters (mass, width).



Current status of Z_c's

- A very nice recent lattice QCD paper about charged Z_c states:
  [S. Prelovsek, C. B. Lang, L. Leskovec and D. Mohler, Phys. Rev. D **91**, 014504 (2015) [arXiv:1405.7623 [hep-lat]]]

- Current status of our computations:
  - Developing and testing numerical methods to compute the corresponding correlation matrix sufficiently precise.
  - No physics results yet. (*"At unphysically heavy u/d quark masses, and when neglecting $s\bar{s}$ creation and annihilation, $a_0(980)$ is not a stable tetraquark."*)

Study of the Z_c^+ channel using lattice QCD

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(Dated: January 21, 2015)

Recently experimentalists have discovered several charged charmonium-like hadrons Z_c^+ with unconventional quark content $\bar{c}c\bar{d}u$. We perform a search for Z_c^+ with mass below 4.2 GeV in the channel $I^G(J^{PC}) = 1^+(1^{+-})$ using lattice QCD. The major challenge is presented by the two-meson states $J/\psi\,\pi$, $\psi_{2S}\pi$, $\psi_{1D}\pi$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\eta_c\rho$ that are inevitably present in this channel. The spectrum of eigenstates is extracted using a number of meson-meson and diquark-antidiquark interpolating fields. For our pion mass of 266 MeV we find all the expected two-meson states but no additional candidate for Z_c^+ below 4.2 GeV. Possible reasons for not seeing an additional eigenstate related to Z_c^+ are discussed. We also illustrate how a simulation incorporating interpolators with a structure resembling low-lying two-mesons states seems to render a Z_c^+ candidate, which is however

Part 2b: Creation operators with 2 quarks of finite mass and 2 static quarks (Z_b states)

[M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538]]
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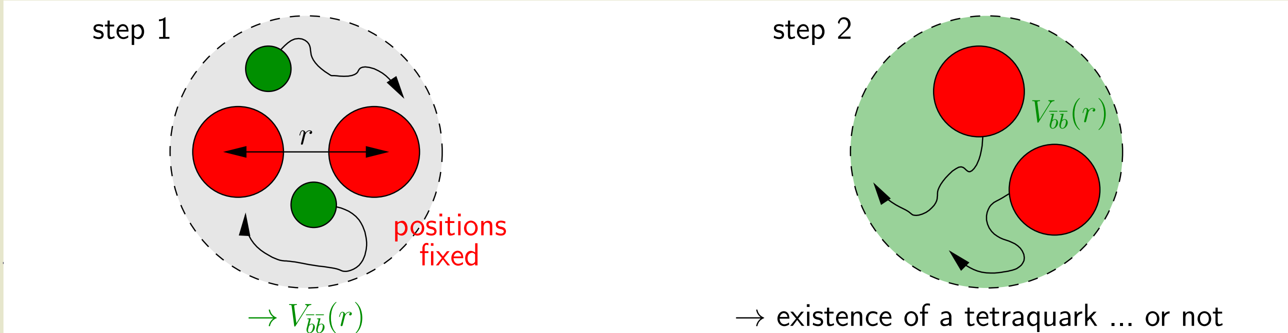
[B. Wagenbach, P. Bicudo, M.W., J. Phys.: Conf. Ser. 599, 012006 (2015) [arXiv:1411.2453]]
[P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., arXiv:1505.00613]
[J. Scheunert, P. Bicudo, A. Uenver, M.W., arXiv:1505.03496]

Z_b's ($\bar{b}\bar{b}qq$ tetraquarks) (1)

- **Basic idea**: Investigate the existence of heavy tetraquarks $\bar{b}\bar{b}qq$ in two steps.
  (1) **Compute potentials of two heavy antiquarks ($\bar{b}\bar{b}$) in the presence of two lighter quarks ($qq \in \{ud, ss, cc\}$) using lattice QCD.**
  (2) **Check, whether these potentials are sufficiently attractive, to host a bound state by solving a corresponding Schrödinger equation.** ($\rightarrow$ This would indicate a stable $\bar{b}\bar{b}qq$ tetraquark.)
- (1) + (2) $\rightarrow$ **Born-Oppenheimer approximation**:
  - Proposed in 1927 for molecular and solid state calculations.
    [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Annalen der Physik 389, Nr. 20, 1927]
  - In our computations step (1) not quantum mechanics, but lattice QCD.
  - Approximation valid, if $m_q \ll m_b$ (most appropriate for $qq = ud$).



Z_b's ($\bar{b}\bar{b}qq$ tetraquarks) (2)

- **B.-O., step (1)**: Lattice QCD computation of $\bar{b}\bar{b}$ potentials $V_{\bar{b}\bar{b}}(r)$.

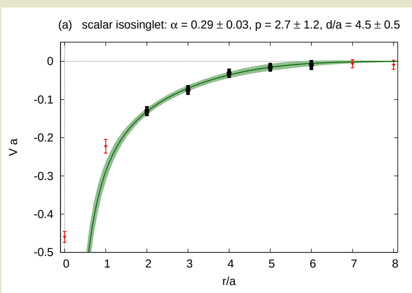
  (1) Use $\bar{b}\bar{b}qq$ creation operators

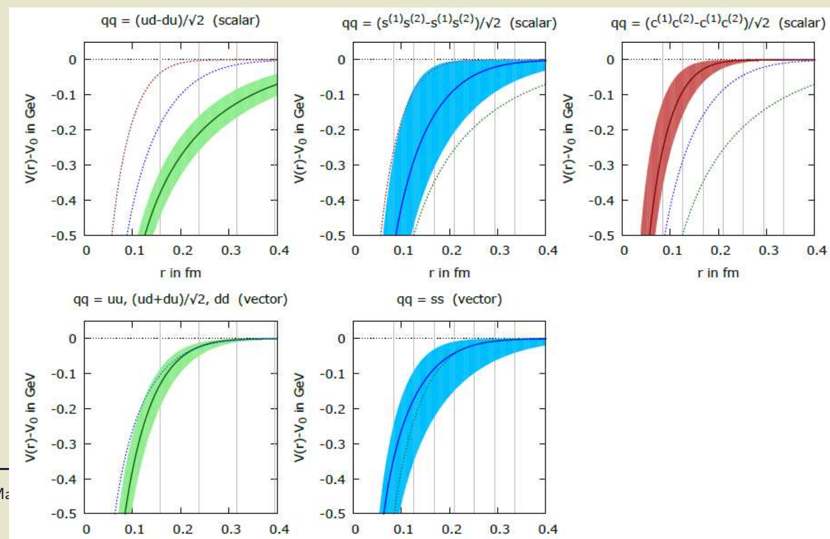
  $$O_{\bar{b}\bar{b}qq} \equiv (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{b}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{b}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big)$$

  (different light quark flavors $qq \in \{ud, ss, cc\}$ and quark spin/parity).

  (2) Compute temporal correlation functions.

  (3) Determine $V_{\bar{b}\bar{b}}(r)$ from the exponential decays of the correlation functions.





Z_b's ($\bar{b}\bar{b}qq$ tetraquarks) (3)

- **B.-O., step (2)**: Solve the Schrödinger equation for the relative coordinate $\mathbf{r}$ of the two $\bar{b}$ quarks,

$$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\right)\underbrace{\psi(\mathbf{r})}_{=R(r)/r} \quad = \quad E\psi(\mathbf{r}) \quad , \quad \mu \quad = \quad m_b/2;$$

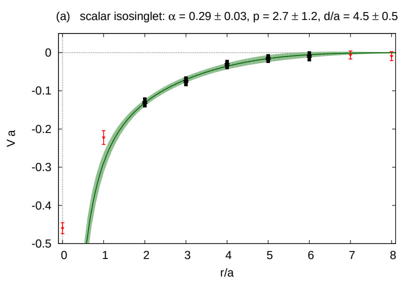
possibly existing bound states, i.e. $E < 0$, indicate $\bar{b}\bar{b}qq$ tetraquarks.

- A single bound state for one specific potential $V_{\bar{b}\bar{b}}(r)$ and light quarks $qq = ud$:
  - Binding energy $E = -93^{+47}_{-43}\,\mathrm{MeV}$, i.e. confidence level $\approx 2\,\sigma$.
  - Quantum numbers of the $\bar{b}\bar{b}ud$ tetraquark: $I(J^P) = 0(1^+)$.

$\rightarrow$ **Prediction of a tetraquarks.** (No experimental results for $\bar{b}\bar{b}qq$ available.)

- No further bound states, in particular not for $qq = ss$ or $qq = cc$.

- Work in Progress:
  - Including effects due to the $\bar{b}\bar{b}$ spins
    $\rightarrow$ binding energy reduced, $\bar{b}\bar{b}ud$ persists (preliminary).
  - The experimetally more relevant case $\bar{b}b\bar{q}q$ (e.g. $Z_b(10610)^+$, $Z_b(10650)^+$).



Conclusions

- The study of tetraquarks, resonances, etc. is currently a very popular, but also one of the most challenging problems in lattice QCD.
- In my opinion solid Z state results from lattice QCD, i.e. results
  - at physical quark masses,
  - including a continuum extrapolation,
  - without approximations, e.g. neglect of closed quark loops, disconnected diagrams, etc.,

  will require more time ... (Perhaps $\mathcal{O}$(5 years) ...?)
- Not meant as a negative statement ... lattice QCD has made continuous and significant progress over the last decades:

∼1980 ... 1990	Only gluons (Yang-Mills theory with infinitely heavy quarks).
∼1990 ... 2000	First dynamical, but unphysically heavy u/d quarks.
∼2000 ... 2015	Decreasing $m_{u/d}$ to its physical value.
∼2000	First crude results for simple $\bar{q}q$ mesons (uncontrolled systematic errors).
∼2010	First solid results for simple $\bar{q}q$ mesons.
∼2020	First solid results for mesons of more complicated structure, e.g. Z's ...?