Numerical computation of phase diagrams of QCD-inspired models in the large-N limit

Jonas Stoll, Marc Wagner, Marc Winstel, Niklas Zorbach

Goethe-Universität Frankfurt am Main

Introduction, goals

- Low T and high μ region of QCD phase diagram not yet accessible to lattice QCD, understanding mostly based on QCD-inspired models.
- Study the phase diagram of various QCD-inspired models with particular focus on inhomogeneous phases.
- Determine the spatial modulation of the condensates without using specific ansätze (e.g. no restriction to a chiral density wave).
- Inhomogeneous phases with 2- or 3-dimensional modulations?
- Phase structure not only with respect to μ and T but also isospin and strangeness chemical potential μ_I, μ_S.

GN model in 1+1 dimensions

- At the moment we explore numerical methods in the GN model.

$$S \;=\; \int d^2x \left(\sum_{j=1}^{N} \bar{\psi}_j \Big(\gamma_0(\partial_0+\mu)+\gamma_1\partial_1\Big)\psi_j - \frac{g^2}{2}\left(\sum_{j=1}^{N}\bar{\psi}_j\psi_j\right)^2\right).$$

- Phase diagram analytically known.
  [O. Schnetz, M. Thies and K. Urlichs, Annals Phys. **314**, 425 (2004) [hep-th/0402014]]
- After introducing a scalar field σ (= condensate) and performing the integration over fermionic fields

$$S_{\text{eff}} \;=\; N\left(\frac{1}{2\lambda}\int d^2x\,\sigma^2 - \ln\Big(\det(\underbrace{\gamma_0(\partial_0+\mu)+\gamma_1\partial_1+\sigma}_{=Q})\Big)\right)$$

$$Z \;=\; \int D\sigma\, e^{-S_{\text{eff}}} \quad (\lambda = Ng^2).$$

- $N\rightarrow\infty$: only one field configuration important (minimum of S_{eff}/N).
- For numerical treatment finite volume and discretization needed.
  - For example lattice field theory.
  - There are other possibilities, e.g. finite mode discretization, discretization by piecewise polynomial functions, etc.
- Challenges, problems:
  - Discretization of the fermionic determinant (cf. right column).
  - Efficient computation of $\det(Q)$ and minimization of S_{eff}/N.
  - Inhomogeneous phases and finite volume.

Discretization of the fermionic determinant

- Various discretizations tested.
- Expansion in a set of basis functions, e.g. plane waves,

$$\psi(x,t) \;\rightarrow\; \sum_{m_t,m_x} c_{m_t,m_x} e^{i(p_{m_t}t+p_{m_x}x)} \quad , \quad \sigma(x) \;\rightarrow\; \sum_{m_x} d_{m_x} e^{ip_{m_x}x}.$$

  [M. Wagner, Phys. Rev. D **76**, 076002 (2007) [arXiv:0704.3023 [hep-lat]]]

  (−) Requires $\det(Q)=\det(Q^\dagger)$, not the case e.g. for $\mu_I\neq 0$ or $\mu_s\neq 0$.
  - ∗ $\det(Q)\rightarrow\det(\langle f_n|Q|f_{n'}\rangle)$, where f_n are basis functions.
  - ∗ Problem: $\text{span}\{f_n\}\neq\text{span}\{Qf_n\}$
    $\rightarrow$ artificially small eigenvalues or zero modes in $\langle f_n|Q|f_{n'}\rangle$
    $\rightarrow$ wrong and weird results.
  - ∗ A larger set of basis functions does not cure the problem.
  - ∗ Solution: $\ln(\det(Q))\rightarrow(1/2)\ln(\det(Q^\dagger Q))$.

  (−) Number of modes in $\psi(x,t)$ should be larger than in $\sigma(x)$.
  (+) No fermion doubling.
  (+) Resulting condensates $\sigma(x)$ are continuous functions.
- Lattice discretization:
  - Naively discretized fermions.

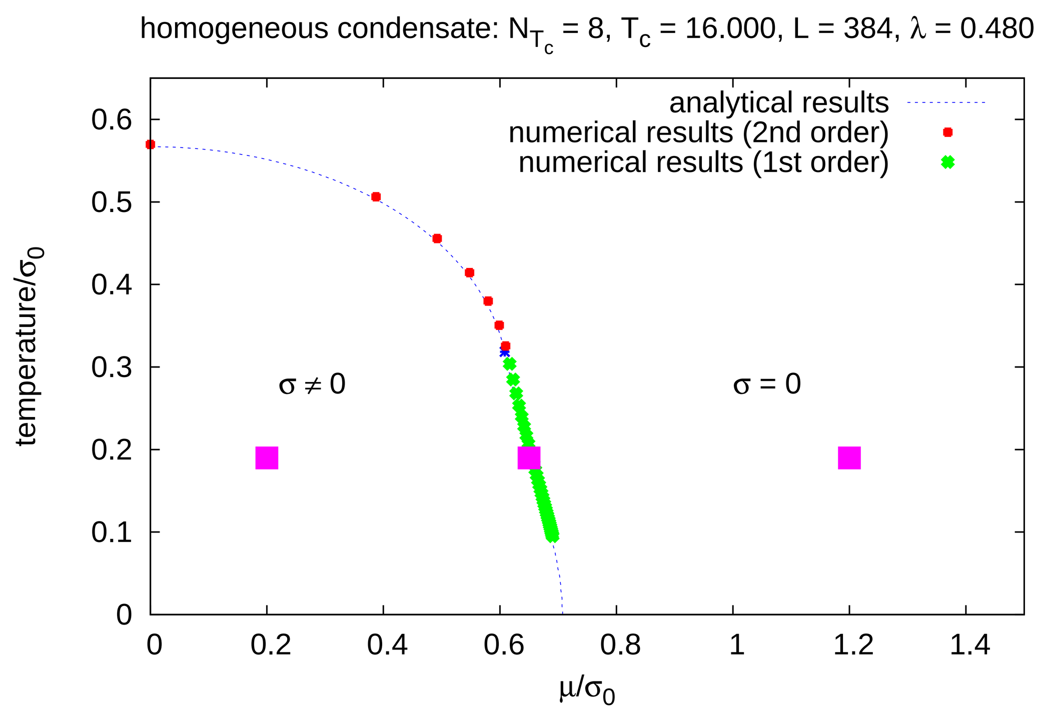
$$\psi(x,t) \;\rightarrow\; \psi_{x,t} \quad , \quad \partial_x\psi(x,t) \;\rightarrow\; \frac{\psi_{x+a,t}-\psi_{x-a,t}}{2a} \quad , \quad \ldots$$

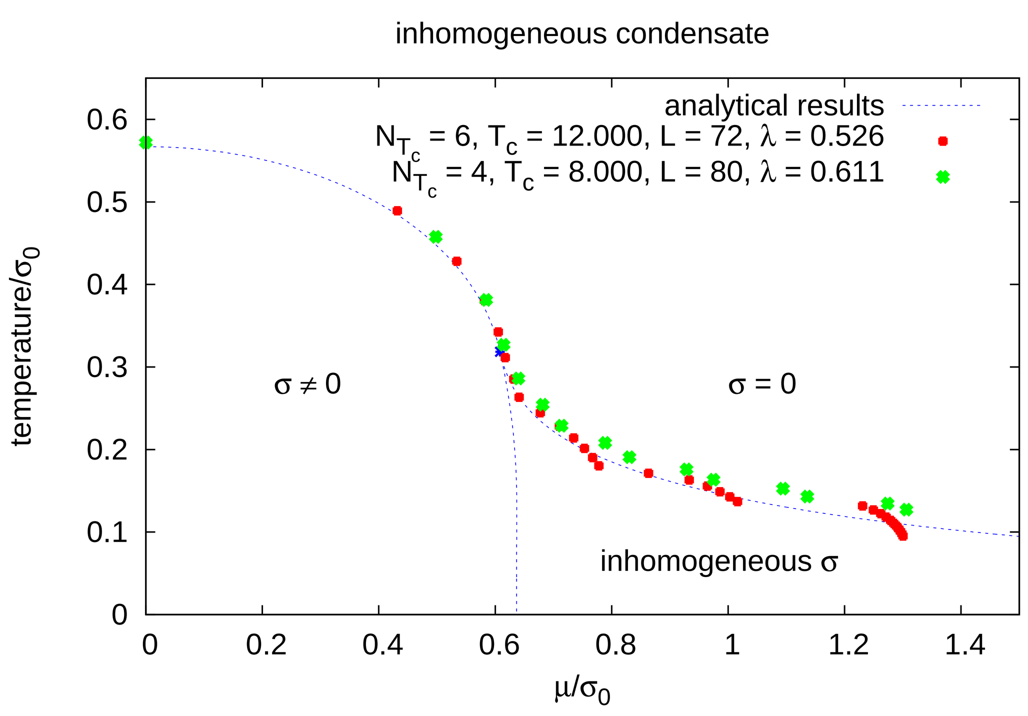
    (−) Fermion doubling.
  - Naively discretized with non-symmetric derivatives.
    (−) No fermion doubling, but other severe problems.
  - Staggered fermions.
    [P. de Forcrand and U. Wenger, PoS LATTICE **2006**, 152 (2006) [hep-lat/0610117]]
    (−) Fermion doubling still present.
- Most promising seems to be a combination of two approaches:
  - Plane wave expansion in t direction.
    (+) Easy analytical simplifications, e.g. $\det(Q)$ factorizes.
  - Naive lattice discretization in x direction.
    (+) $\det(Q)=\det(Q^\dagger)$ not required $\rightarrow \mu_I\neq 0$, $\mu_S\neq 0$ possible.
    (+) Fermion doubling not a problem (large-N: "$2\times\infty=\infty$").

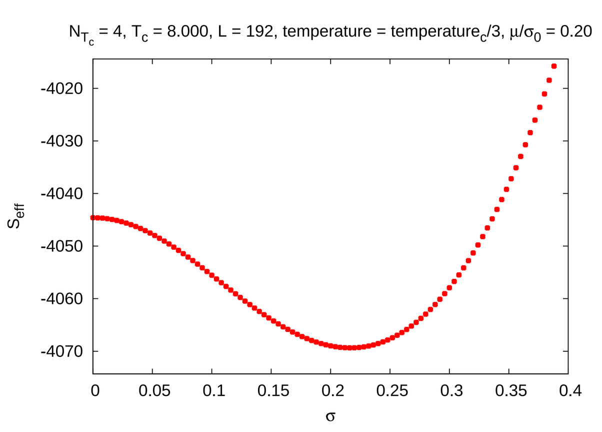
$$\psi(x,t) \;\rightarrow\; \psi_x(t) \;=\; \sum_m \psi_{x,m}e^{ip_mt} \quad , \quad \sigma(x) \;\rightarrow\; \sigma_x.$$

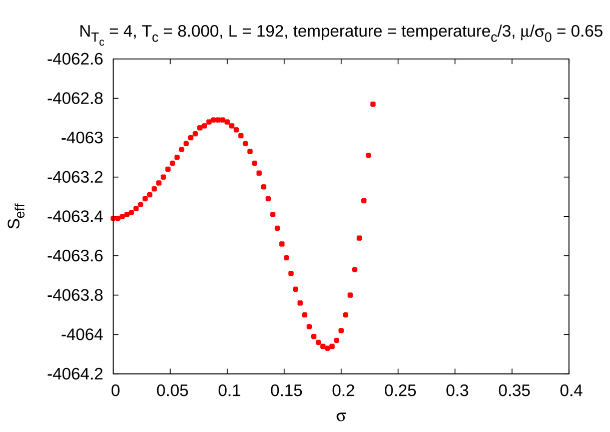
Numerical results

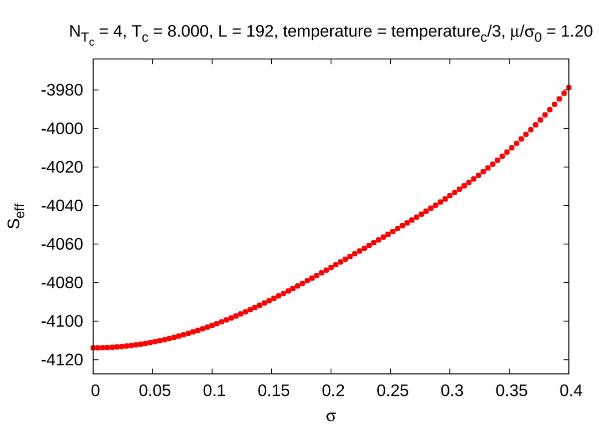
- Plane wave expansion in t direction, lattice discretization in x direction.











- Piecewise polynomial expansion both in t and in x direction.