BB, $B\bar{B}$ and hybrid static potentials from lattice QCD

Effective Field Theory Seminar – Technische Universität München, Germany

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Goals, motivation (1)

- **Study exotic mesons (tetraquarks/mesonic molecules, hybrid mesons) by combining lattice QCD and phenomenology/model calculations.**
- Compute the potential of two heavy valence quarks
  - in the presence of two additional light valence quarks (tetraquarks/mesonic molecules),
  - in the presence of gluonic excitations (hybrid mesons)

  using lattice QCD.
- Explore, whether the potentials are sufficiently attractive to generate a bound state (a rather stable exotic meson) using phenomenology/model calculations.

Goals, motivation (2)

- Why are such investigations important?
  Quite a number of mesons are only poorly understood.
  - Example $X(3872)$ ($\bar{c}c$ state): mass not as expected from quark models; could be a D-D^* molecule, a bound diquark-antidiquark, ...
  - Example $D_{s0}^*(2317)$, $D_{s1}(2460)$: masses significantly lower than expected from quark models, almost equal or even lower than the corresponding D mesons; could be tetraquarks, ...
  - Charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states.
  - Charged charmonium states, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$ ... must be four quark states.
  - Mesons with non-quark model quantum numbers, e.g. $\pi_1(1400)$, $\pi_1(1600)$ ... candidates for hybrid mesons.

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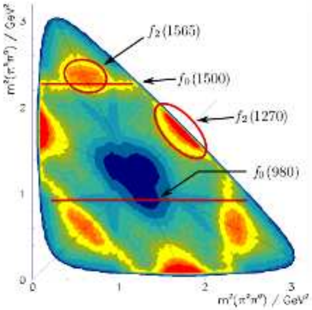
Physics - Hadron Spectroscopy

Search for Gluonic Excitations



One of the main challenges of hadron physics is the search for gluonic excitations, i.e. hadrons in which the gluons can act as principal components. These gluonic hadrons fall into two main categories: glueballs, i.e. states where only gluons contribute to the overall quantum numbers, and hybrids, which consist of valence quarks and antiquarks as hadrons plus one or more excited gluons which contribute to the overall quantum numbers.

The additional degrees of freedom carried by gluons allow these hybrids and glueballs to have J^{PC} exotic quantum numbers. In this case mixing effects with nearby $q\bar{q}$ states are excluded and this makes their experimental identification easier. The properties of glueballs and hybrids are determined by the long-distance features of QCD and their study will yield fundamental insight into the structure of the QCD vacuum. Antiproton-proton annihilations provide a very favourable environment to search for gluonic hadrons.

Charmonium Spectroscopy

The charmonium spectrum can be calculated within the framework of non-relativistic potential models, EFT and LQCD. All 8 charmonium states below open charm threshold are known, but the measurements of their parameters and decays is far from complete (e.g. width and decay modes of h_c and $\eta_c(2S)$). Above threshold very little is known: on one hand the expected D- and F- wave states have not been identified (with the possible exception of the $\psi(3770)$, mostly $3D_1$), on the other hand the nature of the recently discovered X, Y, Z states is not known.

At full luminosity PANDA will collect several thousand $c\bar{c}$ states per day. By means of fine scans it will be possible to measure masses with an accuracy of the order of 100 keV and widths to 10% or better. PANDA will explore the entire energy region below and above the open charm threshold, to find the missing D- and F- wave states and unravel the nature of the newly discovered X, Y, Z states.

D Meson Spectroscopy

The recent discoveries of new open charm mesons at the BaBar, Belle and CLEO has attracted much interest both in the theoretical and

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Outline

- A brief introduction to lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Hadron spectroscopy.
  - Lattice QCD.
- Ongoing lattice projects:

  (1) BB static potentials.

  (2) $B\bar{B}$ static potentials.

  (3) Hybrid static potentials.

QCD (quantum chromodynamics)

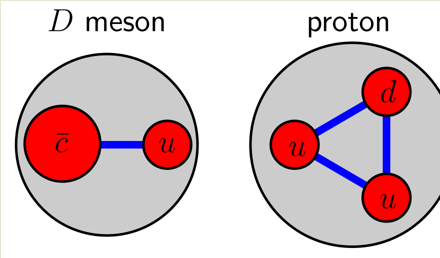
- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (usually mesons $= q\bar{q}$ and baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD simple:

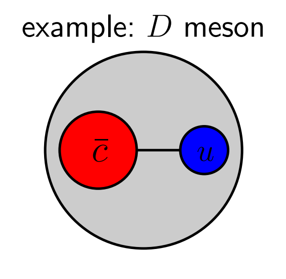
$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu \Big) + m^{(f)} \Big) \psi^{(f)} + \frac{1}{2g^2} \text{Tr} \Big(F_{\mu\nu} F_{\mu\nu} \Big) \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$



- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).
  $\rightarrow$ Solve QCD numerically by means of lattice QCD.

Hadron spectroscopy



- Proceed as follows:

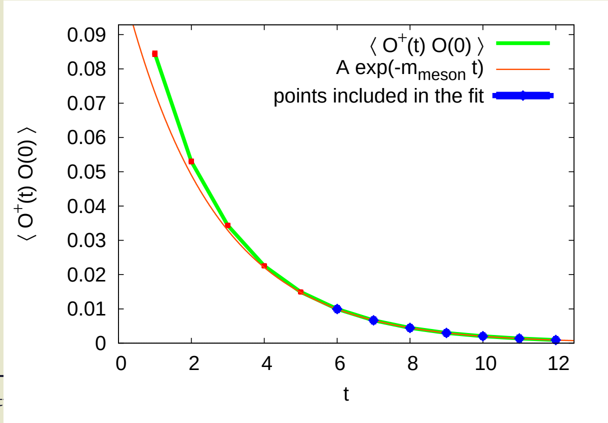
  (1) Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator O (an operator O, which generates the quantum numbers of the hadron of interest, when applied to the vacuum $|\Omega\rangle$).

  (2) Determine the corresponding hadron mass from the asymptotic exponential decay in time.

- Example: D meson mass m_D (valence quarks $\bar{c}$ and u, $J^P = 0^-$),

$$O \equiv \int d^3r\, \bar{c}(\mathbf{r})\gamma_5 u(\mathbf{r})$$

$$C(t) \equiv \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle \overset{t\to\infty}{\propto} \exp\Big(-m_D t\Big).$$



Lattice QCD (1)

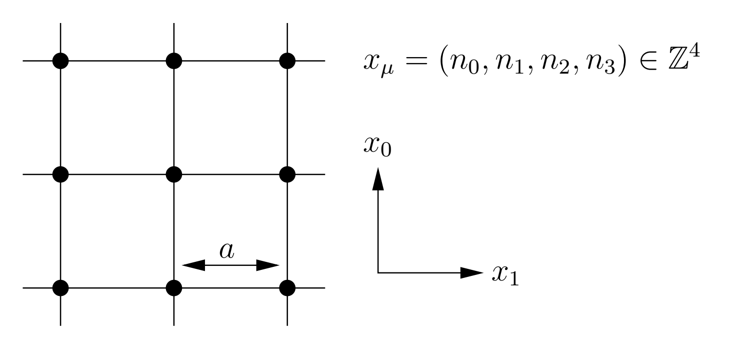
- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle = \\ = \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,O^\dagger(t)O(0)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slides).
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x},t)$ and $A_\mu(\mathbf{x},t)$.
  - $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite size effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \Big(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \Big) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    * x_μ: $32^4 \approx 10^6$ lattice sites.
    * $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    * $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    * In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

  $\rightarrow$ standard approaches for numerical integration not applicable

  $\rightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

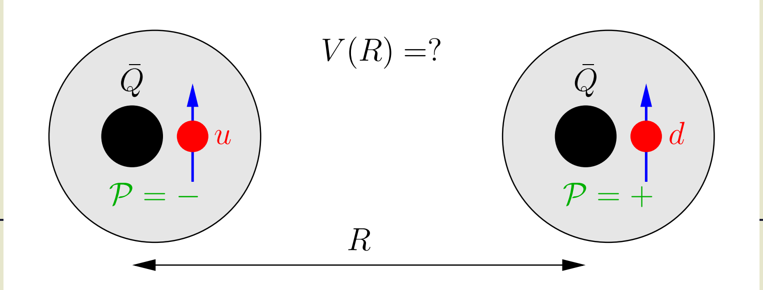
Heavy-heavy-light-light tetraquarks (1)

- Study possibly existing $\bar{Q}\bar{Q}qq$ and $\bar{Q}Q\bar{q}q$ tetraquark states ($q \in \{u, d, s, c\}$):
  - Use the static approximation for the heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$ (reduces the necessary computation time significantly).
  - Most appropriate for $\bar{Q}\bar{Q} \equiv \bar{b}\bar{b}$ and $\bar{Q}Q \equiv \bar{b}b$, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$.
  - Could also yield information about $\bar{Q}\bar{Q} \equiv \bar{c}\bar{c}$ and $\bar{Q}Q \equiv \bar{c}c$, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$.
- Proceed in two steps:
  (1) Compute the potential of two heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$ in the background of two light quarks qq and $\bar{q}q$ by means of lattice QCD.
  (2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$; a bound state would indicate a tetraquark state.

Heavy-heavy-light-light tetraquarks (2)

- Since heavy b quarks are treated in the static approximation, their spins are irrelevant (mesons are labeled by the spin of the light degrees of freedom j).
- Consider only pseudoscalar/vector mesons ($j^\mathcal{P} = (1/2)^-$, PDG: B, B^*) and scalar/pseudovector mesons ($j^\mathcal{P} = (1/2)^+$, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons.
- Study the dependence of the mesonic potential $V(R)$ on
  - the "light" quark flavors u, d, s and/or c (isospin),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson S and/or P_-.

$\rightarrow$ Many different channels/quantum numbers ... attractive, repulsive ...

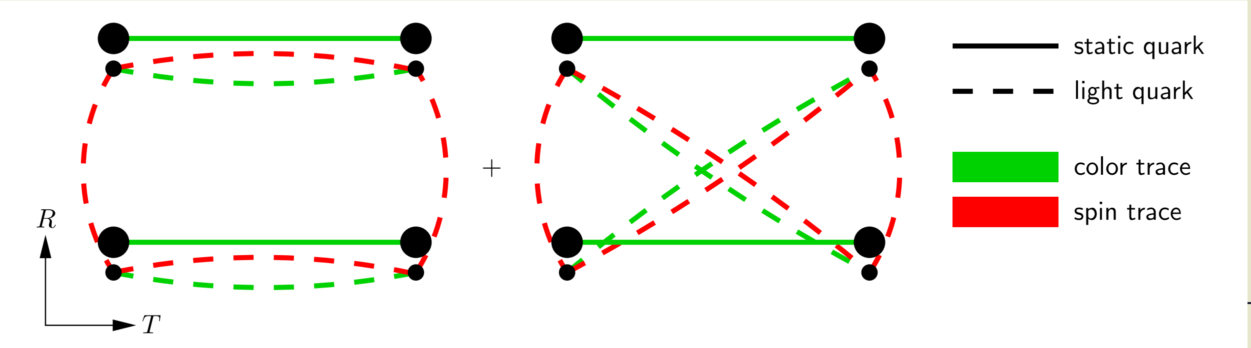


BB static potentials/tetraquarks (1)

- In the following $\bar{Q}\bar{Q}qq$, i.e. "BB" (not $\bar{Q}Q\bar{q}q$, i.e. "$B\bar{B}$").
- To extract the potential(s) of a given sector $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$, compute the temporal correlation function of the trial state

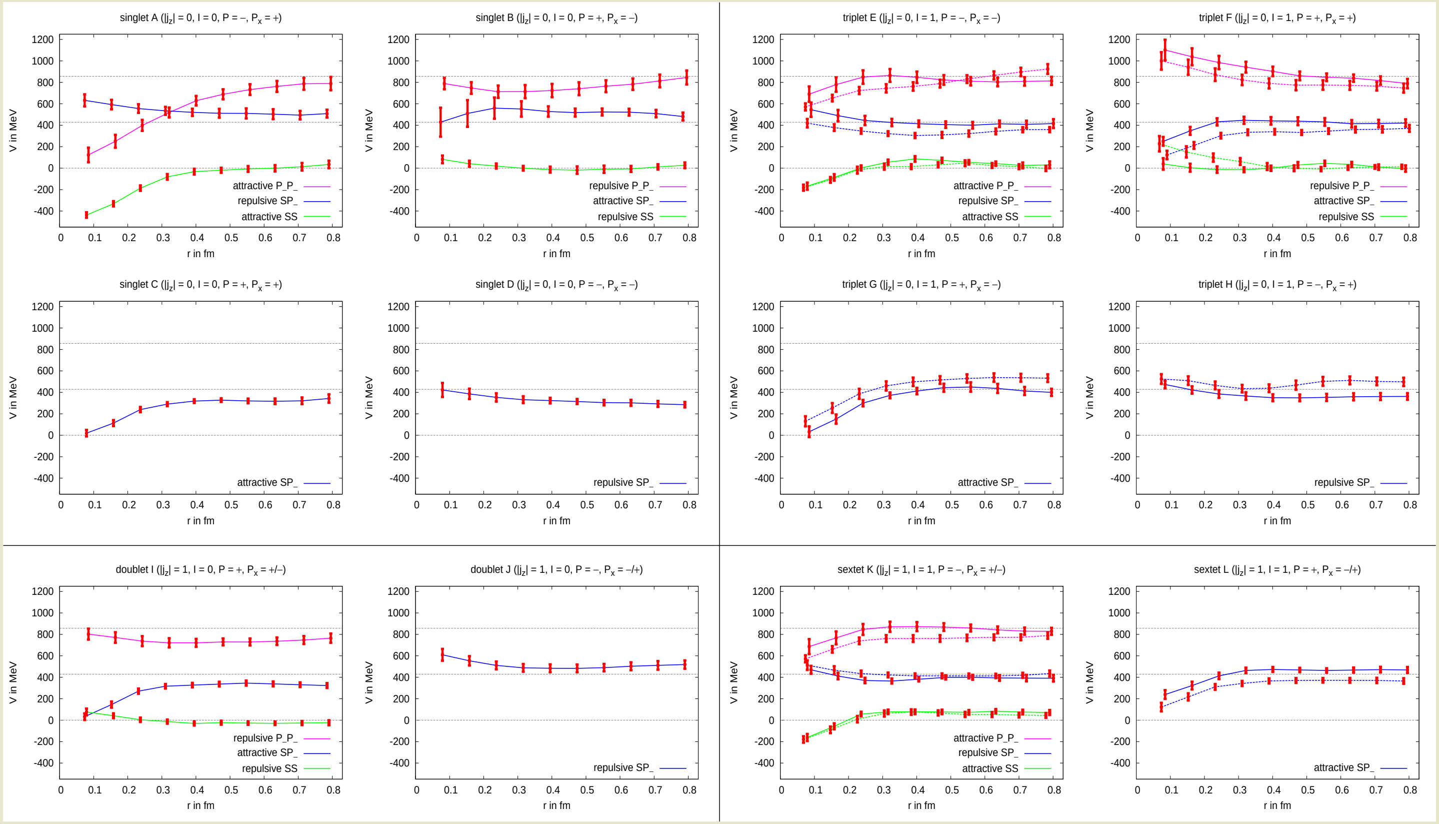
$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(1)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(2)}(+R/2)\Big)|\Omega\rangle.$$

  - $\mathcal{C} = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $q^{(1)}q^{(2)} \in \{ud - du\,,\ uu\,,\ dd\,,\ ud + du\,,\ ss\,,\ cc\}$ (isospin I, I_z).
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}$, $\mathcal{P}_x$).



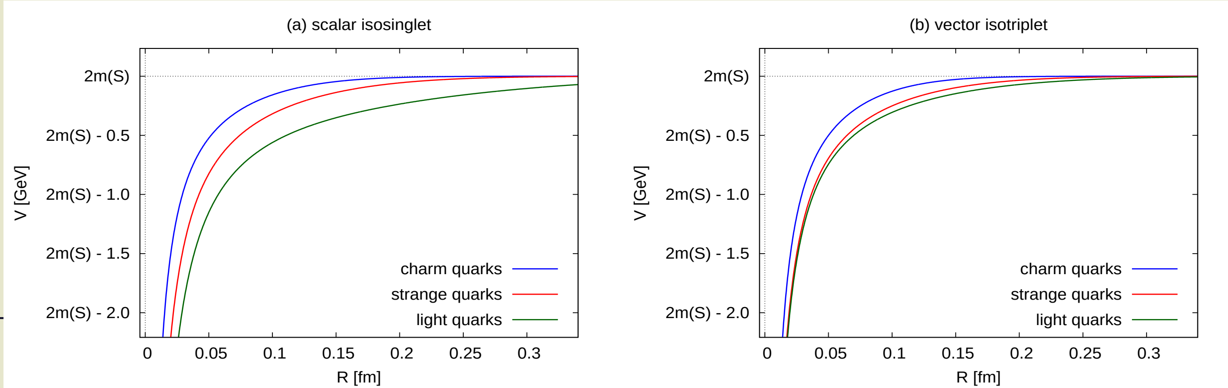
BB static potentials/tetraquarks (2)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



BB static potentials/tetraquarks (3)

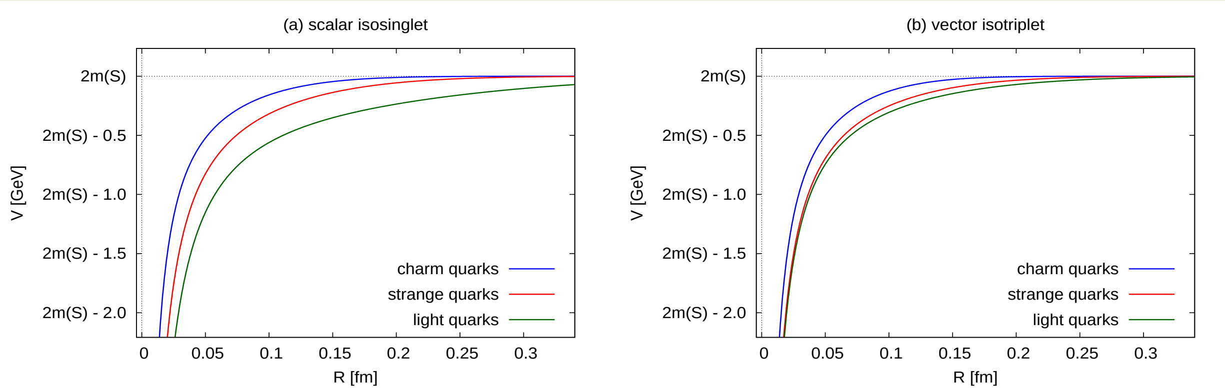
- Focus on the two attractive channels between ground state static-light mesons "B and/or B^*" (probably the best candidates to find a tetraquark):
  - Scalar isosinglet (more attractive):
    $qq = (ud - du)/\sqrt{2}$, $\Gamma = \gamma_5 + \gamma_0\gamma_5$,
    quantum numbers $(I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (0, 0, -, +)$.
  - Vector isotriplet (less attractive):
    $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$, $\Gamma = \gamma_j + \gamma_0\gamma_j$,
    quantum numbers $(I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (1, \{0, 1\}, -, \pm)$.
- Computations for $qq = ll, ss, cc$ ($l \in \{u, d\}$) to study the mass dependence.



BB static potentials/tetraquarks (4)

- Two competing effects:
  - The potential for light quarks is wider/deeper, i.e. favors the existence of a bound state (a tetraquark).
  - Heavier quarks correspond to heavier mesons ($m(B) < m(B_s) < m(B_c)$), which form more readily a bound state (a tetraquark), i.e. require a less wide/deep potential for a bound state.

[M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538]]
[M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147]]
[B. Wagenbach, P. Bicudo, M.W., arXiv:1411.2453]



BB static potentials/tetraquarks (5)

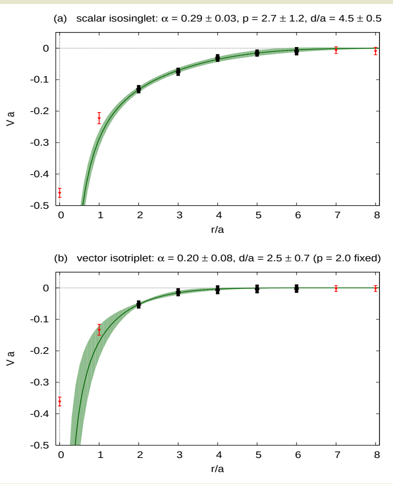
- Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}\bar{Q}$,

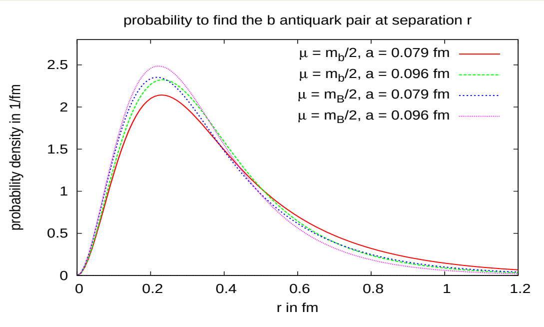
$$\left(-\frac{1}{2\mu}\triangle + V(r)\right)\underbrace{\psi(\mathbf{r})}_{=R(r)/r} = E\psi(\mathbf{r}) \quad , \quad \mu = m(B_{(s,c)})/2;$$

a bound state, i.e. $E_0 < 0$, would be an indication for a tetraquark state.

- Clear indication for a bound state for the scalar isosinglet and $qq = ll$ (i.e. BB), binding energy $E \approx -50\,\mathrm{MeV}$, confidence level $\approx 2\,\sigma$.

- No binding for the vector isotriplet or for $qq = ss, cc$ (i.e. $B_s B_s$, $B_c B_c$).





BB static potentials/tetraquarks (6)

- To quantify "no binding", we list for each channel the factor, by which the effective mass μ in Schrödinger's equation has to be multiplied, to obtain binding with confidence level $1\,\sigma$ and $2\,\sigma$ (the potential is not changed).

flavor	light		strange		charm	
confidence level for binding	$1\,\sigma$	$2\,\sigma$	$1\,\sigma$	$2\,\sigma$	$1\,\sigma$	$2\,\sigma$
scalar isosinglet	0.8	1.0	1.9	2.2	3.1	3.2
vector isotriplet	1.9	2.1	2.5	2.7	3.4	3.5

  - Factors ≤ 1.0 indicate binding.
  - Light quarks (u/d) are unphysically heavy (correspond to $m_\pi \approx 340\,\text{MeV}$); physically light u/d quarks are expected to yield stronger binding for the scalar isosinglet, might lead to binding also for the vector isotriplet (computations in progress).
  - Mass splitting $m(B^*) - m(B) \approx 50\,\text{MeV}$, neglected at the moment, is expected to weaken binding (coupled channel analysis in progress).

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
[B. Wagenbach, P. Bicudo, M.W., arXiv:1411.2453]

$B\bar{B}$ static potentials/tetraquarks (1)

- Experimentally more interesting case: $\bar{Q}Q\bar{q}q$, i.e. "$B\bar{B}$", trial states

$$\gamma_{5,AB}\Gamma_{CD}\Big(\bar{Q}_A(-R/2)q_D^{(1)}(-R/2)\Big)\Big(\bar{q}_C(+R/2)Q_B^{(2)}(+R/2)\Big)|\Omega\rangle.$$

- At the moment only preliminary results for $\bar{q}q = \bar{c}c$, "$I = 1$".
- Qualitative difference to $\bar{Q}\bar{Q}qq$: all channels are attractive (for $\bar{Q}\bar{Q}qq$ half of them are attractive, half of them are repulsive).
  - Can be understood by comparing the potential of $\bar{Q}Q$ and of $\bar{Q}\bar{Q}$ generated by one-gluon exchange.
  - For $\bar{Q}\bar{Q}$ the Pauli principle applied to qq implies either a symmetric (sextet) or an antisymmetric (triplet) color orientation of the static quarks corresponding to a repulsive or attractive interaction, respectively.
  - For $\bar{Q}Q$ no such restriction is present, i.e. all channels contain contributions of the attractive color singlet, which dominates the repulsive color octet.

Heavy-heavy-light-light tetraquarks (3)

- Future plans for BB and $B\bar{B}$:
  - Computations with light u/d quarks of physical mass ($m_\pi \approx 140\,\text{MeV}$ instead of $m_\pi \approx 340\,\text{MeV}$).
  - Light quarks of different mass: BB_s, BB_c and $B_s B_c$ potentials.
  - Refined model calculations with the resulting static-static-light-light potentials: take mass splitting $m(B^*) - m(B) \approx 50\,\text{MeV}$ into account (coupled channel analysis).

Heavy-heavy-light-light tetraquarks (4)

- Future plans for BB and $B\bar{B}$:
  - Study the structure of the states corresponding to the computed potentials:
    - In a lattice computation two different creation operators generating the same quantum numbers yield the same potential.
    - At the moment exclusively creation operators of mesonic molecule type.
    - For BB use also
      - creation operators of diquark-antidiquark type.
    - For $B\bar{B}$ use also
      - creation operators of diquark-antidiquark type,
      - creation operators of bottomonium + pion type ($Q\bar{Q}$ string + π),
      - for $I = 0$ creation operators of bottomonium type ($Q\bar{Q}$ string).
    - Resulting correlation matrices provide information about the structure.

Hybrid static potentials (1)

- Hybrid mesons:
  - Quark antiquark states with excited gluonic fields.
  - Not restricted to quark model quantum numbers J^{PC}, where $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$ (L: angular momentum, S: spin).
  - Exotic states with $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, \ldots$ can be realized by excited gluonic fields.
  - Examples for $J^{PC} = 1^{-+}$ states: $\pi_1(1400)$, $\pi_1(1600)$.

Hybrid static potentials (2)

- Quantum numbers of states with a static quark and a static antiquark:
  - Angular momentum j_z with respect to the axis of separation; states with $j_z = 0, \pm 1, \pm 2, \ldots$ are also labeled by $\Sigma, \Pi, \Delta, \ldots$.
  - The combination of parity and charge conjugation $P \circ C$; states with $P \circ C = +, -$ are also labeled by g, u.
  - Rotational invariant Σ states are either symmetric or antisymmetric with respect to spatial reflections along an axis perpendicular to the axis of separation denoted by $P_x = +, -$.
- Example: the ordinary static potential has quantum numbers $J^{P_x}_{P \circ C} = \Sigma^+_g$.
- Hybrid static potentials: quantum numbers different from Σ^+_g.

Hybrid static potentials (3)

- Hybrid creation operators:

$$O \equiv \bar{Q}(-R/2)U(-R/2;0)\ \text{insertion}\ U(0;+R/2)Q(+R/2).$$

  - $Q(+R/2)$, $\bar{Q}(-R/2)$: static quark antiquark pair at separation R.
  - $U(z_1, z_2)$: gluonic parallel transporter along the axis of separation,

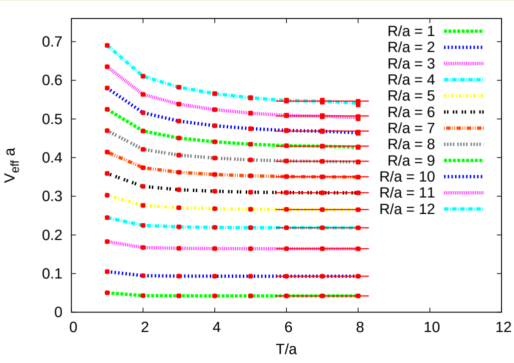
$$U(z_1, z_2) \equiv P\bigg(\exp\bigg(i\int_{z_1}^{z_2} dz\, A_z(z)\bigg)\bigg).$$

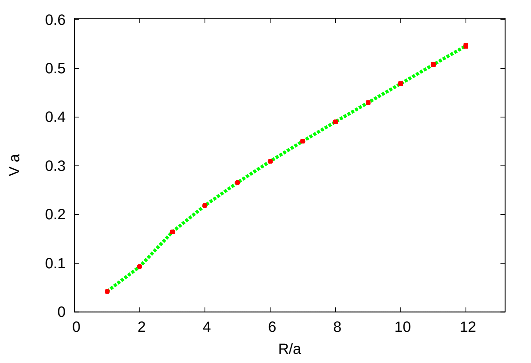
  - "insertion": cf. table.

quantum numbers $J^{P_x}_{P\circ C}$	operator insertions
Σ_g^+	1 , $\mathbf{R}\cdot\mathbf{E}$, $\mathbf{R}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	$\mathbf{R}\times\mathbf{E}$, $\mathbf{R}\times(\mathbf{D}\times\mathbf{B})$
Σ_u^-	$\mathbf{R}\cdot\mathbf{B}$, $\mathbf{R}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	$\mathbf{R}\times\mathbf{B}$, $\mathbf{R}\times(\mathbf{D}\times\mathbf{E})$
Σ_g^-	$(\mathbf{R}\cdot\mathbf{D})(\mathbf{R}\cdot\mathbf{B})$

Hybrid static potentials (4)

- Preliminary SU(2) results.
- Lattice setup:
  - More than 700 essentially independent gauge link configurations.
  - 24^4 lattice sites.
  - Lattice spacing $a \approx 0.073\,\mathrm{fm}$ (when identifying r_0 with $0.46\,\mathrm{fm}$).
- Extract a potential value $V(R)$ from the plateau of the corresponding effective mass $V(R) = \ln(C(R, t + a)/C(R, t))/a$, where $C(R, t)$ are Wilson loops with the previously discussed insertions.



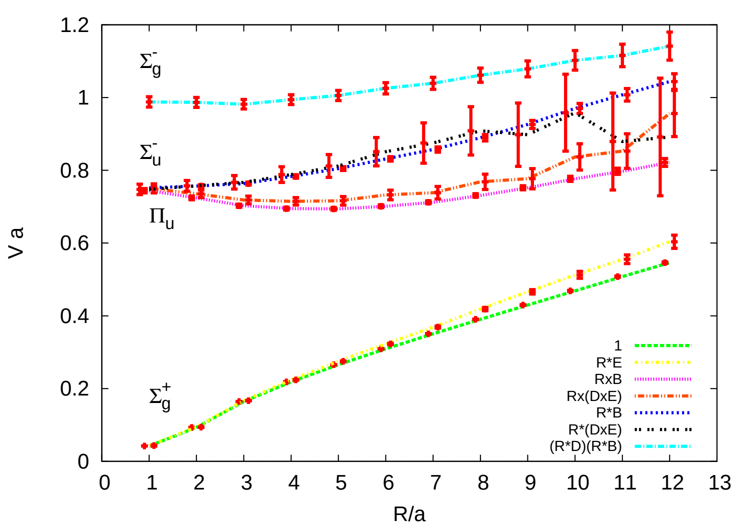


Hybrid static potentials (5)

- Quantum numbers Σ_g^+, Π_u, Σ_u^- and Σ_g^- (two different hybrid creation operators for Σ_g^+, Π_u and Σ_u^-):
  - Resulting potentials identical within statistical errors.
  - Σ_g^+ (ordinary static potential): Wilson loops (green) superior to $\mathbf{R} \cdot \mathbf{E}$ (yellow).
  - Π_u: $\mathbf{R} \times \mathbf{B}$ (magenta) superior to $\mathbf{R} \times (\mathbf{D} \times \mathbf{E})$ (orange).
  - Σ_u^-: $\mathbf{R} \cdot \mathbf{B}$ (blue) superior to $\mathbf{R} \cdot (\mathbf{D} \times \mathbf{E})$ (black).

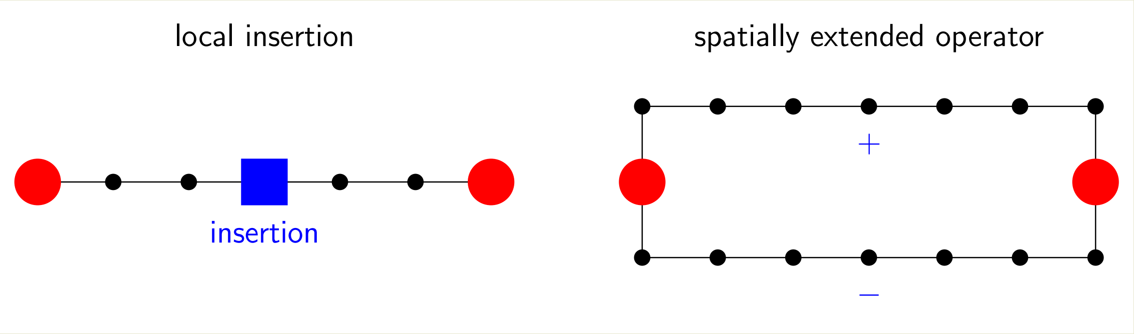
$\rightarrow$ Certain information about the gluonic string.

[Philipp Wolf, M.W., arXiv:1410.7578]



Hybrid static potentials (6)

- Statistical errors of hybrid static potentials quite large
  $\rightarrow$ local insertions might generate structures rather different from those of the corresponding physical states.
- Implement spatially extended creation operators generating the same quantum numbers (Σ_g^+, Π_u, Σ_u^-, Σ_g^-, ...)
  $\rightarrow$ corresponding correlation functions could be dominated by the ground state already at small temporal separations
  $\rightarrow$ smaller statistical errors expected.



Hybrid static potentials (7)

- Goals:
  - Precise results for hybrid static potentials for SU(3) Yang-Mills theory and QCD.
  - Use these results to estimate masses of hybrid mesons by solving a Schrödinger-like equation with the computed hybrid static potentials.
  - In the context of effective field theories like pNRQCD there might be interest in the short distance behavior of hybrid static potentials, which is related to gluelump masses ...?

    [N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77**, 1423 (2005) [hep-ph/0410047]]

Conclusions

- **Lattice QCD computations with static quarks combined with model calculations could provide interesting qualitative and to some extent also quantitative insights.**