On the definition and interpretation of a static quark anti-quark potential in the colour-adjoint channel

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[O. Philipsen and M. Wagner, arXiv:1305.5957 [hep-lat]]

Original motivation (1)

- Determination of $\Lambda_{\overline{\mathrm{MS}}}$ from the (singlet) static potential for $n_f = 0$ (Yang-Mills theory) and $n_f = 2, 3$ (QCD) and gauge group $SU(3)$:
  - Fit the perturbative result, which depends on the perturbative scale $\Lambda_{\overline{\mathrm{MS}}}$, to the corresponding lattice result, where the scale has been set e.g. by the typical non-perturbative scale $r_0 \approx 0.45\,\mathrm{fm} \ldots 0.50\,\mathrm{fm}$ (or by other hadronic quantities, e.g. m_π and f_π).
  - Similar problem: relate the perturbative scale $\Lambda_{\overline{\mathrm{MS}}}$ and the non-perturbative scale r_0 by determining the dimensionless quantity $\Lambda_{\overline{\mathrm{MS}}} r_0$.
  - Instead of $\Lambda_{\overline{\mathrm{MS}}}$ one can also determine α_s at some fixed scale.

[N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo, Phys. Rev. Lett. **105**, 212001 (2010) [arXiv:1006.2066 [hep-ph]]]

[K. Jansen, F. Karbstein, A. Nagy and M. Wagner *et al.* [ETM Collaboration], JHEP **1201**, 025 (2012) [arXiv:1110.6859 [hep-ph]]]

[A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D **86**, 114031 (2012) [arXiv:1205.6155 [hep-ph]]]

Original motivation (2)

- Perturbative calculation of the color adjoint static potential up to 2 loops, recently also up to 3 loops (the "octet static potential" for gauge group $SU(3)$).

  [T. Collet and M. Steinhauser, Phys. Lett. B **704**, 163 (2011) [arXiv:1107.0530 [hep-ph]]]
  [C. Anzai, M. Prausa, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, arXiv:1308.1202 [hep-ph]]

- Plan:
  - Compute the octet static potential using lattice QCD.
  - Determine $\Lambda_{\overline{\mathrm{MS}}}$ using perturbative and lattice results for the octet static potential.

- We encountered some conceptual problems, lattice results and perturbative results show strong qualitative differences ...

Original motivation (3)

- This work is concerned with the interpretation of the colour adjoint static potential from Wilson loops with generator insertions (using different gauges).
- We discuss both non-perturbative (lattice) and perturbative calculations; the focus, however, will be on the non-perturbative side.

Lattice Yang Mills theory/QCD (1)

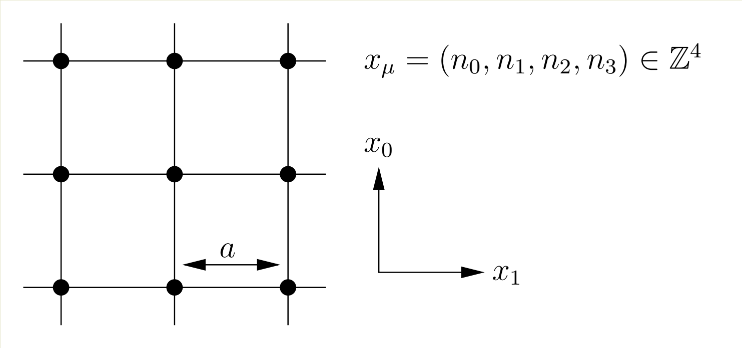
- Lattice gauge theory is based on the path integral formulation of Yang Mills theory/QCD,

$$\langle\Omega|\mathcal{O}[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]|\Omega\rangle = \\ = \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,\mathcal{O}[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $\mathcal{O}[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]$: functional of the quark and gluon fields.
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x},t)$ and $A_\mu(\mathbf{x},t)$.
  - $e^{-S[x]}$: weight factor containing the Yang-Mills/QCD action.

Lattice Yang Mills theory/QCD (2)

- Numerical implementation of the path integral formalism in Yang Mills theory/QCD:
  - Discretise spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite size effects".



Lattice Yang Mills theory/QCD (3)

- After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \bigg) \ldots ,$$

where

$$U_\nu(x_\mu) \quad = \quad P\bigg(\exp\bigg(ig \int_{x_\mu}^{x_\mu + a e_\mu^{(\nu)}} dz_\rho\, A_\rho(z)\bigg)\bigg),$$

i.e. the lattice gauge field is stored in parallel transporters connecting neighbouring lattice sites (so-called links).

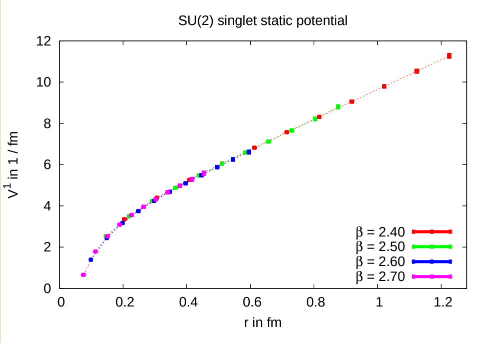
- Advantages/disadvantages of lattice Yang Mills theory/QCD:

(+) Exact Yang Mills/QCD results (no approximations, no model assumptions, etc.).

(−) Only numerical results, i.e. numbers, no analytical functions, etc.

Introduction: singlet static potential (1)

- The (singlet) static potential V^1 is a very common and important observable in lattice gauge theory.
- It is the energy of a static antiquark $\bar{Q}(\mathbf{x})$ and a static quark $Q(\mathbf{y})$ in a colour singlet (i.e. a gauge invariant) orientation as a function of the separation $r \equiv |\mathbf{x} - \mathbf{y}|$.
- The spin of a static quark is irrelevant, i.e. in the following
  - no spin indices or γ matrices,
  - only spinless colour charges,
    $\bar{Q}^a_A(\mathbf{x}) = (Q^{a,\dagger}(\mathbf{x})\gamma_0)_A \to Q^{a,\dagger}(\mathbf{x})$,
    $Q^a_A(\mathbf{y}) \to Q^a(\mathbf{y})$,
    where a denotes a colour index and A a spin index.

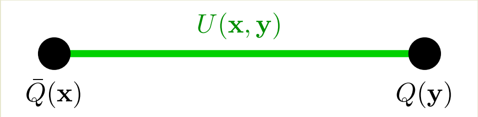


Introduction: singlet static potential (2)

- The singlet static potential for gauge group $SU(N)$ can be obtained as follows:

  (1) Define a trial state

  $$|\Phi^1\rangle \equiv \bar{Q}(\mathbf{x})U(\mathbf{x},\mathbf{y})Q(\mathbf{y})|0\rangle.$$

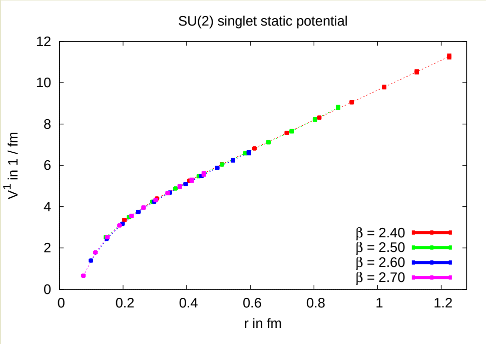


  (2) The temporal correlation function of this trial state simplifies to the well known Wilson loop,

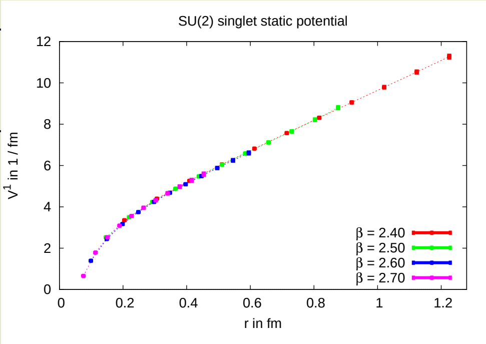
  $$\langle\Phi^1(t_2)|\Phi^1(t_1)\rangle = e^{-2M\Delta t}N\Big\langle W_1(r,\Delta t)\Big\rangle \quad , \quad \Delta t \equiv t_2 - t_1 > 0,$$

  where

  $$W_1(r,\Delta t) = \\ = \frac{1}{N}\mathrm{Tr}\bigg(P\bigg(\exp\bigg(ig\oint dz_\mu\,A_\mu(z)\bigg)\bigg)\bigg).$$



Introduction: singlet ...



- The singlet static potential for gauge group $SU(N)$ can be obtained as follows:

  (3) The singlet static potential $V^1 \equiv V_0^1$ can be obtained from the asymptotic exponential behaviour of the Wilson loop,

$$\Big\langle W_1(r,\Delta t)\Big\rangle \propto \langle \Phi^1(t_2)|\Phi^1(t_1)\rangle = e^{+E_0\Delta t}\langle \Phi^1(t_1)|e^{-H\Delta t}|\Phi^1(t_1)\rangle =$$

$$= \sum_{n=0}^{\infty} \langle \Phi^1|n\rangle e^{-V_n^1(r)\Delta t}\langle n|\Phi^1\rangle =$$

$$= \sum_{n=0}^{\infty} \underbrace{\Big|\langle \Phi^1|n\rangle\Big|^2}_{=c_n} e^{-V_n^1(r)\Delta t} \stackrel{\Delta t\to\infty}{\propto} \exp\Big(-V^1(r)\Delta t\Big)$$

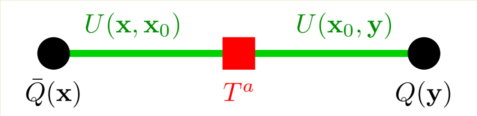
$$V^1(r) = -\lim_{\Delta t\to\infty} \frac{\langle \dot{W}_1(r,\Delta t)\rangle}{\langle W_1(r,\Delta t)\rangle}$$

($\sum_n$ is the sum over eigenstates of the Hamiltonian, which have the quantum numbers of $|\Phi^1\rangle$, in particular a static $Q\bar{Q}$ pair at $\mathbf{x}$ and $\mathbf{y}$).

Colour adjoint static potential (1)

- **Goal of this work:** compute and interpret the potential of a static antiquark $\bar{Q}(\mathbf{x})$ and a static quark $Q(\mathbf{y})$ in a colour adjoint (i.e. a gauge variant) orientation in various gauges as a function of the separation $r \equiv |\mathbf{x} - \mathbf{y}|$.
- A colour adjoint orientation of a static antiquark and a static quark can be obtained by inserting the generators of the colour group T^a (e.g. for $SU(3)$, $T^a = \lambda^a/2$), i.e. $\bar{Q} T^a Q|0\rangle$.
- If the static antiquark and the static quark are separated in space, a straightforward generalisation is

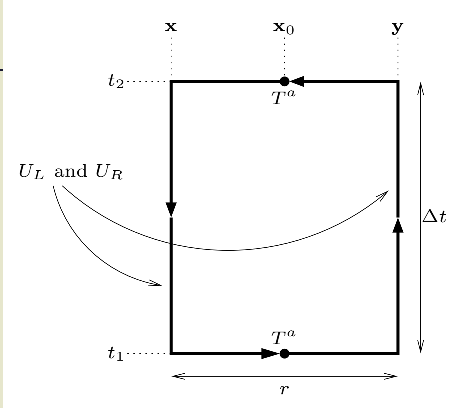
$$|\Phi^{T^a}\rangle \equiv \bar{Q}(\mathbf{x}) U(\mathbf{x}, \mathbf{x}_0) T^a U(\mathbf{x}_0, \mathbf{y}) Q(\mathbf{y})|0\rangle.$$



- A corresponding definition of the colour adjoint static potential has been proposed and used in pNRQCD (a framework based on perturbation theory).

[N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77**, 1423 (2005) [hep-ph/0410047]]

Colour adjoint static ... (2)



- We discuss non-perturbative calculations analogous as for the singlet static potential in various gauges,

$$\langle \Phi^{T^a}(t_2) | \Phi^{T^a}(t_1) \rangle = e^{-2M\Delta t} N \Big\langle W_{T^a}(r, \Delta t) \Big\rangle \quad ,$$

$$W_{T^a}(r, \Delta t) \equiv \frac{1}{N} \mathrm{Tr}\Big(T^a U_R T^{a,\dagger} U_L\Big)$$

$$\Big\langle W_{T^a}(r, \Delta t) \Big\rangle = \sum_{n=0}^{\infty} c_n \exp\Big(-V_n^{T^a}(r) \Delta t\Big) \overset{\Delta t \rightarrow \infty}{\propto} \exp\Big(-V^{T^a}(r) \Delta t\Big).$$

- In particular we are interested,
  - whether the colour adjoint static potential $V^{T^a} \equiv V_0^{T^a}$ is gauge invariant (i.e. whether the obvious gauge dependence of the correlation function $\langle W_{T^a}(r, \Delta t) \rangle$ only appears in the matrix elements c_n),
  - whether V^{T^a} indeed corresponds to the potential of a static antiquark and a static quark in a colour adjoint orientation, or whether it has to be interpreted differently.

V^{T^a} without gauge fixing

- Without gauge fixing

$$\Big\langle W_{T^a}(r, \Delta t) \Big\rangle \;\; = \;\; 0,$$

because this correlation function is gauge variant (and does not contain any gauge invariant contribution).

$\rightarrow$ Without gauge fixing the calculation of a colour adjoint static potential fails.

V^{T^a} in Coulomb gauge

- Coulomb gauge: $\nabla \mathbf{A}^g(x) = 0$, which amounts to an independent condition on every time slice t.

- The remaining residual gauge symmetry corresponds to global independent colour rotations $h^{\mathrm{res}}(t) \in SU(N)$ on every time slice t; with respect to this residual gauge symmetry the colour adjoint Wilson loop transforms as

$$\Big\langle W_{T^a}(r, \Delta t) \Big\rangle \quad = \quad \frac{1}{N} \mathrm{Tr}\Big(T^a U_R T^{a,\dagger} U_L\Big) \quad \rightarrow_{h^{\mathrm{res}}}$$
$$\rightarrow_{h^{\mathrm{res}}} \quad \frac{1}{N} \mathrm{Tr}\Big(h^{\mathrm{res},\dagger}(t_1) T^a h^{\mathrm{res}}(t_1) U_R h^{\mathrm{res}}(t_2) T^{a,\dagger} h^{\mathrm{res},\dagger}(t_2) U_L\Big).$$

- Since $h^{\mathrm{res}}(t_1)$ and $h^{\mathrm{res}}(t_2)$ are independent, the situation is analogous to that without gauge fixing, i.e.

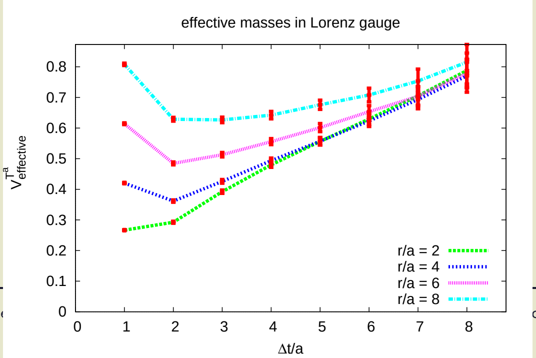
$$\Big\langle W_{T^a}(r, \Delta t) \Big\rangle_{\mathrm{Coulomb\ gauge}} \quad = \quad 0.$$

$\rightarrow$ In Coulomb gauge the calculation of a colour adjoint static potential fails.

V^{T^a} in Lorenz gauge

- Lorenz gauge: $\partial_\mu A^g_\mu(x) = 0$.
- In Lorenz gauge a Hamiltonian or a transfer matrix does not exist.
- Only gauge invariant correlation functions like the ordinary Wilson loop $\langle W_1(r, \Delta t)\rangle$ exhibit an asymptotic exponential behaviour and, therefore, allow the determination of energy eigenvalues.
- The colour adjoint Wilson loop $\langle W_{T^a}(r, \Delta t)\rangle_{\textrm{Lorenz gauge}}$ does not decay exponentially in the limit of large Δt.

→ The physical meaning of a colour adjoint static potential determined from $\langle W_{T^a}(r, \Delta t)\rangle_{\textrm{Lorenz gauge}}$ is unclear.



V^{T^a} in temporal gauge (1)

- Temporal gauge: $\partial_\mu A_0^g(x) = 0$ or equivalently $U_0^g(x) = 1$.
- Temporal links gauge transform as

$$U_0^g(t,\mathbf{x}) \quad = \quad g(t,\mathbf{x}) U_0(t,\mathbf{x}) g^\dagger(t+a,\mathbf{x}) \quad , \quad g(t,\mathbf{x}) \in SU(N).$$

- A possible choice to implement temporal gauge is

$$\begin{aligned}
g(t=2a,\mathbf{x}) &= U_0(t=a,\mathbf{x}), \\
g(t=3a,\mathbf{x}) &= g(t=2a,\mathbf{x}) U_0(t=2a,\mathbf{x}) \quad = \quad U_0(t=a,\mathbf{x}) U_0(t=2a,\mathbf{x}), \\
g(t=4a,\mathbf{x}) &= g(t=3a,\mathbf{x}) U_0(t=3a,\mathbf{x}) \quad = \quad \ldots, \\
\ldots &= \ldots
\end{aligned}$$

V^{T^a} in temporal gauge (2)

- By inserting the transformation to temporal gauge $g(t, \mathbf{x})$, the gauge variant colour adjoint Wilson loop turns into a gauge invariant observable:

$$\Big\langle W_{T^a}(r, \Delta t) \Big\rangle_{\textrm{temporal gauge}} =$$
$$= \frac{1}{N} \Big\langle \textrm{Tr}\Big(U^{T^a,g}(t_1; \mathbf{x}, \mathbf{y}) U^{T^{a,\dagger},g}(t_2; \mathbf{y}, \mathbf{x})\Big) \Big\rangle_{\textrm{temporal gauge}} = \ldots =$$
$$= \frac{2}{N(N^2-1)} \sum_a \sum_b \Big\langle \textrm{Tr}\Big(T^a U_R T^b U_L\Big) \textrm{Tr}\Big(T^a U(t_1, t_2; \mathbf{x}_0) T^b U(t_2, t_1; \mathbf{x}_0)\Big) \Big\rangle$$

$(U^{T^a}(\mathbf{x}, \mathbf{y}) = U(\mathbf{x}, \mathbf{x}_0) T^a U(\mathbf{x}_0, \mathbf{y}))$.

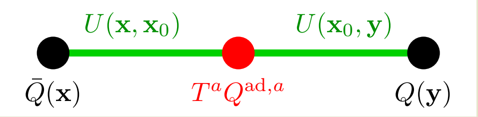
- $\textrm{Tr}(T^a U_R T^b U_L)$: Wilson loop with generator insertions.
- $\textrm{Tr}(T^a U(t_1, t_2; \mathbf{x}_0) T^b U(t_2, t_1; \mathbf{x}_0))$: propagator of a static adjoint quark.

$\rightarrow$ The colour adjoint Wilson loop in temporal gauge is a correlation function of a gauge invariant three-quark state, one fundamental static quark, one fundamental static anti-quark, one adjoint static quark.

V^{T^a} in temporal gauge (3)

- Equivalently, after defining

$$|\Phi^{Q\bar{Q}Q^{\mathrm{ad}}}\rangle \quad \equiv \quad Q^{\mathrm{ad},a}(\mathbf{x}_0)(\bar{Q}(\mathbf{x})U^{T^a}(\mathbf{x},\mathbf{y})Q(\mathbf{y}))|0\rangle,$$



one can verify

$$\langle\Phi^{Q\bar{Q}Q^{\mathrm{ad}}}(t_2)|\Phi^{Q\bar{Q}Q^{\mathrm{ad}}}(t_1)\rangle \quad \propto \quad \Big\langle W_{T^a}(r,\Delta t)\Big\rangle_{\text{temporal gauge}}.$$

→ V^{T^a} in temporal gauge should not be interpreted as the potential of a static quark and a static anti-quark, which form a colour-adjoint state.

→ V^{T^a} in temporal gauge is the potential of a colour-singlet three-quark state.

→ V^{T^a} in temporal gauge does not only depend on the $Q\bar{Q}$ separation $r = |\mathbf{x}-\mathbf{y}|$, but also on the position $s = |\mathbf{x}-\mathbf{x}_0|/2 - |\mathbf{y}-\mathbf{x}_0|/2$ of the static adjoint quark Q^{ad}, i.e. $V^{T^a}(r,s)$ (in the following we work with the symmetric alignment $\mathbf{x}_0 = (\mathbf{x}+\mathbf{y})/2$).

V^{T^a} in temporal gauge (4)

- A different approach, leading to the same result, is the transfer matrix formalism.

  [O. Jahn and O. Philipsen, Phys. Rev. D **70**, 074504 (2004) [hep-lat/0407042]]
  [O. Philipsen, Nucl. Phys. B **628**, 167 (2002) [hep-lat/0112047]]

- One can perform a spectral analysis of the colour adjoint Wilson loop:

$$\Big\langle W_{T^a}(r,\Delta t)\Big\rangle_{\text{temporal gauge}} = \frac{1}{N}\sum_k e^{-(V_k^{T^a}(r)-\mathcal{E}_0)\Delta t}\sum_{\alpha,\beta}\Big|\langle k^a_{\alpha\beta}|U^{T^a}_{\alpha\beta}(\mathbf{x},\mathbf{y})|0\rangle\Big|^2,$$

  where $|k^a_{\alpha\beta}\rangle$ denotes states containing three static quarks (one fundamental static quark, one fundamental static anti-quark, one adjoint static quark).

$\rightarrow$ Again the conclusion is that V^{T^a} in temporal gauge is the potential of a colour-singlet three-quark state.

A gauge invariant definition via $\mathbb{B}$ fields?

- In the literature one can also find a proposal of a gauge invariant quantity to determine a colour adjoint static potential,

$$W_B(r,\Delta t) \quad \equiv \quad \frac{1}{N}\mathrm{Tr}\Big(T^a U_R T^{b,\dagger} U_L\Big)\mathbf{B}^a(\mathbf{x}_0,t_1)\mathbf{B}^b(\mathbf{x}_0,t_2),$$

i.e. open colour indices are saturated by colour magnetic fields.

[N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77**, 1423 (2005) [hep-ph/0410047]]

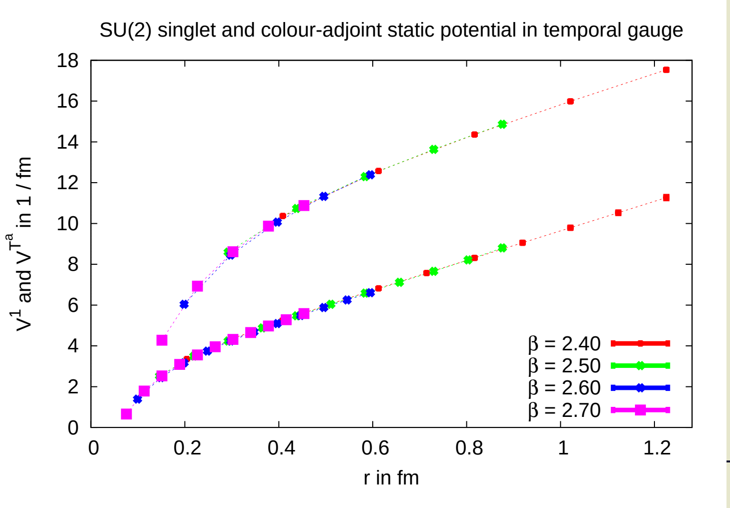
- Using the transfer matrix formalism one can again perform a spectral analysis and show that only states with a fundamental quark and a fundamental antiquark $|k_{\alpha\beta}\rangle$ (i.e. singlet static potentials) contribute:

$$\Big\langle W_B(r,\Delta t)\Big\rangle \quad = \quad \sum_k e^{-(V_k^{1,-}(r)-\mathcal{E}_0)\Delta t}\sum_{\alpha,\beta}\Big|\langle k_{\alpha\beta}|U_{\alpha\beta}^{T^a\mathbf{B}^a}(\mathbf{x},\mathbf{y})|0\rangle\Big|^2.$$

$\rightarrow$ $\langle W_B(r,\Delta t)\rangle$ is suited to extract colour singlet static potentials only (quantum numbers "parity" $[PC$, $P_x]$ and angular momentum may differ from the ordinary singlet static potential $\rightarrow$ hybrid potentials).

Numerical lattice results for $SU(2)$

- $SU(2)$ colour group, four different lattice spacings $a = 0.038\,\text{fm} \ldots 0.102\,\text{fm}$.
- In temporal gauge the colour adjoint (or rather $Q\bar{Q}Q^{\text{ad}}$) static potential V^{T^a} is attractive,
  - for small separations stronger than the singlet static potential V^1,
  - for large separations the slope is the same as for the singlet static potential V^1 (indicates flux tube formation between QQ^{ad} and $\bar{Q}Q^{\text{ad}}$).



LO perturbative calculations (1)

- Perturbation theory for static potentials is a good approximation for small quark separations and should agree in that region with corresponding non-perturbative results.
- **Singlet static potential** (gauge invariant, i.e. the gauge is not important):

$$V^1(r) = -\frac{(N^2-1)g^2}{8N\pi r} + \text{const} + \mathcal{O}(g^4).$$

- **Colour adjoint static potential** (in **Lorenz gauge**):

$$V^{T^a}(r) = +\frac{g^2}{8N\pi r} + \text{const} + \mathcal{O}(g^4).$$

  - In Lorenz gauge a Hamiltonian or a transfer matrix does not exist, i.e. the physical meaning is unclear; appears frequently in the literature.
  - The repulsive behaviour is not reproduced by any of the presented non-perturbative considerations or computations.

LO perturbative calculations (2)

- **Colour adjoint static potential** ("in **temporal gauge**"; more precisely: perturbative calculation in Lorenz gauge of the gauge invariant observable, which is equivalent to the colour adjoint Wilson loop in temporal gauge):

$$V^{T^a}(r, s = 0) \quad = \quad V^{Q\bar{Q}Q^{\mathrm{ad}}}(r, s = 0) \quad = \quad -\frac{(4N^2 - 1)g^2}{8N\pi r} + \mathrm{const} + \mathcal{O}(g^4).$$

  - Attractive and stronger by a factor $4 \ldots 5$ than the singlet static potential (depending on N).
  - Qualitative agreement with numerical lattice results for $SU(2)$.

Matching lattice/perturbative results (1)

- Lattice results for the static potential exhibit large discretisation errors for $r < 2a$ (for our ensembles $2a \approx 0.08\,\text{fm} \ldots 0.20\,\text{fm}$).
- Perturbative results for the static potential are only trustworthy for separations $\lesssim 0.2\,\text{fm}$.
  $\rightarrow$ Small region of overlap between lattice and perturbative results.
- The leading order of perturbation theory, which we will use in the following,

$$V^{1,\text{LO}}(r) \quad = \quad -\frac{3g^2}{16\pi r} + \text{const} \quad , \quad V^{T^a,\text{LO}}(r, s = 0) \quad = \quad -\frac{15g^2}{16\pi r} + \text{const}$$

(here specialized to gauge group $SU(2)$) is known to be a rather poor approximation.
$\rightarrow$ Only qualitative agreement expected, when comparing to lattice results.

Matching lattice/perturbative results (2)

- We determine $\alpha_s \equiv g^2/4\pi$ from the corresponding static forces $F^X(r) = dV^X(r)/dr$, $X \in \{1, T^a\}$; one the lattice the derivative is defined by a finite difference,

$$\frac{V^{1,\text{lattice}}(3a) - V^{1,\text{lattice}}(2a)}{a} = \frac{3\alpha_s^1}{4(2.5 \times a)^2}$$

$$\frac{V^{T^a,\text{lattice}}(6a) - V^{T^a,\text{lattice}}(4a)}{2a} = \frac{15\alpha_s^{T^a}}{4(5 \times a)^2}$$

(static colour charges are separated by at least $2a$, while at the same time their separation is still quite small).

- $\Delta\alpha_s^{\text{rel}}$ is quite small.
  $\rightarrow$ A clear sign of agreement between lattice and perturbative results.

- $\alpha_s < 0.5$ for $\beta = 2.60\,,\ 2.70$.
  $\rightarrow$ Perturbation theory "valid".

β	a in fm	α_s^1	$\alpha_s^{T^a}$	$\Delta\alpha_s^{\text{rel}}$
2.40	0.102	0.89	0.75	17%
2.50	0.073	0.59	0.52	13%
2.60	0.050	0.43	0.40	9%
2.70	0.038	0.36	0.33	6%

Conclusions

- We have discussed the non-perturbative definition of a static potential V^{T^a} for a quark antiquark pair in a colour adjoint orientation, based on Wilson loops with generator insertions $\langle W_{T^a}(r, \Delta t) \rangle$ in various gauges:
  - **Without gauge fixing/Coulomb gauge:** $\langle W_{T^a}(r, \Delta t) \rangle = 0$, i.e. the calculation of a potential V^{T^a} fails.
  - **Lorenz gauge:** a Hamiltonian or a transfer matrix does not exist, the physical meaning of a corresponding potential V^{T^a} is unclear.
  - **Temporal gauge:** a strongly attractive potential V^{T^a}, which should be interpreted as the potential of three quarks, i.e. $V^{T^a} = V^{Q\bar{Q}Q^{\mathrm{ad}}}$.
- Saturating open colour indices with $\mathbf{B}^a$, yields a singlet static potential (a hybrid potential).
- LO perturbation theory in Lorenz gauge has long predicted V^{T^a} to be repulsive; it appears impossible, to reproduce this repulsive behaviour by a non-perturbative computation based on Wilson loops.