Search for $\bar{b}b\bar{u}u$ and $\bar{b}\bar{c}c\bar{u}d$ tetraquark bound states using lattice QCD

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Introduction

- We and several other independent groups study the existence and properties of $\bar{b}b\bar{u}s$ and $\bar{b}\bar{c}ud$ tetraquarks (discussed in this talk) as well as $\bar{b}bud$ tetraquarks (not discussed in this talk) with lattice QCD using NRQCD for the $\bar{b}$ quarks.

- Summary of main results and current status:
  - $\bar{b}b\bar{u}s$ with $J^P = 1^+$:
    A strong-interaction-stable tetraquark around 90 MeV below the $BB^*_s$ threshold.
  - $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$ and with $I(J^P) = 0(1^+)$:
    No indication for strong-interaction-stable tetraquarks, but shallow bound states cannot be excluded. Existence of resonances not yet investigated.
  - $\bar{b}bud$ with $I(J^P) = 0(1^+)$:
    A strong-interaction-stable tetraquark around 130 MeV below the $BB^*$ threshold.
  - $\bar{b}bud$ with $I(J^P) = 0(1^-)$:
    No strong-interaction-stable tetraquark. Existence of a resonance, explored with lattice QCD static potentials and the Born-Oppenheimer approximation, seems unlikely.
    [J. Hoffmann, poster at QWG 2022]

- No experimental results for these tetraquarks yet, but for a charm-charm counterpart, $T^{++}_{cc}(\bar{c}\bar{c}ud)$ recently discovered by LHCb.
    [R. Aaij et al. [LHCb], Nature Commun. 13, 3351 (2022) [arXiv:2109.01056]]
Existing work and references

- This talk is mainly a summary of our recent work
  [S. Meinel, M. Pflaumer, M. Wagner, Phys. Rev. D 106, 034507 (2022) [arXiv:2205.13982]] ($\bar{b}b\bar{s}$, $\bar{b}\bar{c}ud$)

- Previous related lattice QCD work on $\bar{Q}\bar{Q}qq$ tetraquarks using NRQCD for $\bar{b}$ quarks:
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. 118, 142001 (2017) [arXiv:1607.05214]] ($\bar{b}bud$, $\bar{b}bus$)
  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D 99, 034507 (2019) [arXiv:1810.12285]] ($\bar{b}bud$, $\bar{b}bus$)
  [N. Mathur, M. Padmanath, PoS LATTICE2021, 443 (2022) [arXiv:2111.01147 [hep-lat]] ($\bar{b}cud$)
Motivation / focus of this work (1)

- Lattice QCD = full QCD (numerically with high performance computers) ... i.e. no assumptions, no approximations, etc.

- A lattice QCD result, if generated in a technically sound and solid way, is a full QCD result and can be confronted with experiment in a direct and meaningful way.

- However, lattice QCD is technically difficult, in particular, when studying exotic hadrons, e.g. $\bar{Q}Qqq$ tetraquarks.
  → Quite often lattice QCD studies are not (yet) fully rigorous, i.e. certain assumptions are made, quark masses are unphysical, no continuum and/or infinite volume limit, no convincing separation and extraction of low-lying energy eigenstates, etc.
  → Important to read (at least some) technical details of lattice QCD papers, to be able to judge their quality.

- Hadron masses (e.g. the mass of a $\bar{b}b\bar{s}u$s tetraquark), more precisely low-lying energy eigenvalues $E_n$, are determined from the exponential decays of temporal correlation functions $C_{jk}(t)$ of (hadron creation) operators $O_j$:

\[
C_{jk}(t) = \langle \Omega | O_j(t) O_k(0) | \Omega \rangle = \langle \Omega | O_j | 0 \rangle \langle 0 | O_k | \Omega \rangle e^{-E_0 t} + \langle \Omega | O_j | 1 \rangle \langle 1 | O_k | \Omega \rangle e^{-E_1 t} + \ldots
\]

- $C_{jk}(t)$ can be computed with lattice QCD.
- The analytical expression on the right hand side is used to determine $E_0$, $E_1$, ...
Motivation / focus of this work (2)

\[ C_{jk}(t) = \langle \Omega | O_j^\dagger(t) O_k(0) | \Omega \rangle = \langle \Omega | O_j^\dagger | 0 \rangle \langle 0 | O_k | \Omega \rangle e^{-E_0 t} + \langle \Omega | O_j^\dagger | 1 \rangle \langle 1 | O_k | \Omega \rangle e^{-E_1 t} + \ldots \]

• In principle one can use any operator \( O_j \), which “has the same quantum numbers” as the hadron of interest (if one would be able to compute \( C_{jk}(t) \) very precisely for very large \( t \)).

• In practice one needs operators with the following properties:
  – The operators have to generate large overlap to the low-lying energy eigenstates states (not only the hadron of interest, but also multi-particle states of similar mass).
  – There must be at least one operator for each low-lying state.
  – The operators must not be too similar (ideally “they are almost orthogonal”).

Otherwise it is questionable, whether the corresponding analysis correctly extracts \( E_0, E_1, \ldots \) from the correlation function \( C_{jk}(t) \).

A major problem is that such analyses always provide numbers, but these might be wrong ... e.g. one could obtain \( \approx (E_0 + E_1)/2 \) instead of \( E_0 \), if one does not use both bound state and scattering operators.

• We improve on existing lattice QCD studies by considering both local and scattering operators for \( \bar{Q}Qqq \) systems. This allows a more trustworthy and precise extraction of energy eigenvalues as well as to carry out scattering analyses.
Lattice setup

- Five ensembles of gauge link configurations generated with 2+1 quark flavors by the RBC and UKQCD collaboration. These have different volumes, different lattice spacings and different light quark masses.

<table>
<thead>
<tr>
<th>ensemble</th>
<th>$N_s^3 \times N_t$</th>
<th>$a$ [fm]</th>
<th>$m_{\pi}$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C00078</td>
<td>$48^3 \times 96$</td>
<td>0.1141(3)</td>
<td>139(1)</td>
</tr>
<tr>
<td>C005</td>
<td>$24^3 \times 64$</td>
<td>0.1106(3)</td>
<td>340(1)</td>
</tr>
<tr>
<td>C01</td>
<td>$24^3 \times 64$</td>
<td>0.1106(3)</td>
<td>431(1)</td>
</tr>
<tr>
<td>F004</td>
<td>$32^3 \times 64$</td>
<td>0.0828(3)</td>
<td>303(1)</td>
</tr>
<tr>
<td>F006</td>
<td>$32^3 \times 64$</td>
<td>0.0828(3)</td>
<td>360(1)</td>
</tr>
</tbody>
</table>

[T. Blum et al. [RBC and UKQCD], Phys. Rev. D 93, 074505 (2016) [arXiv:1411.7017]]

- Domain-wall action for $u$, $d$ and $s$ quarks.

- NRQCD action for valence $b$ quarks, anisotropic clover action for valence $c$ quarks.

- Local operators (representing bound states) and scattering operators (representing meson-meson states).

- Scattering operators at the moment only at one end of the correlation functions, because we are using point-to-all-operators. (Scattering operators at both ends is work in progress.)
\( \bar{b}\bar{b}u\bar{s} \) with \( J^P = 1^+ \): operators

- **Local operators** (at the source and at the sink):

  \[ O_1 = O_{[BB^*_s](0)} = \sum_x \bar{b}\gamma_5 u(x) \bar{b}\gamma_j s(x) \quad (BB^*_s \text{ bound state}) \]

  \[ O_2 = O_{[B^*B_s](0)} = \sum_x \bar{b}\gamma_j u(x) \bar{b}\gamma_5 s(x) \quad (B^*B_s \text{ bound state}) \]

  \[ O_3 = O_{[B^*B^*_s](0)} = \epsilon_{jkl} \sum_x \bar{b}\gamma_k u(x) \bar{b}\gamma_l s(x) \quad (B^*B^*_s \text{ bound state}) \]

  \[ O_4 = O_{[Dd](0)} = \sum_x \bar{b}^a\gamma_j \bar{C}\bar{b}^b,^T (x) u^{a,T} C\gamma_5 s^b (x) \quad \text{(diquark-antidiquark)}. \]

- **Scattering operators** (only at the sink):

  \[ O_5 = O_{B(0)B^*_s(0)} = \left( \sum_x \bar{b}\gamma_5 u(x) \right) \left( \sum_y \bar{b}\gamma_j s(y) \right) \quad (BB^*_s \text{ 2-particle state}) \]

  \[ O_6 = O_{B^*(0)B_s(0)} = \left( \sum_x \bar{b}\gamma_j u(x) \right) \left( \sum_y \bar{b}\gamma_5 s(y) \right) \quad (B^*B_s \text{ 2-particle state}) \]

  \[ O_7 = O_{B^*(0)B^*_s(0)} = \epsilon_{jkl} \left( \sum_x \bar{b}\gamma_k u(x) \right) \left( \sum_y \bar{b}\gamma_l s(y) \right) \quad (B^*B^*_s \text{ 2-particle state}). \]
$\bar{b}b\bar{u}s$ with $J^P = 1^+$: energy levels

- Plot: Energy levels $\Delta E_n = E_n - E_B - E_{B_s}$ for ensemble C01 obtained with various operator subsets and temporal fitting ranges.

- Only local operators $\rightarrow \Delta E_0 \approx 0$ MeV.

- Local and scattering operators $\rightarrow \Delta E_0 \approx -100$ MeV, $\Delta E_1 \approx 0$ MeV.

$\rightarrow$ Ground state corresponds to a strong-interaction-stable tetraquark.
**$\bar{b}\bar{b}us$ with $J^P = 1^+$: our final results**

- **Bottom plot**: Overlaps of each operator to the lowest three energy eigenstates ($O_1'$ to $O_3'$ are linear combinations of $O_1$ to $O_4$, $O_4'$ to $O_6'$ correspond to $O_5$ to $O_7$).
  - Roughly equal contributions to the ground state from a local $BB_s^* / B^*B_s$ operator (“$I = 0$”) ...
  - ... and a local $B^*B_s^*$ operator, ...
  - ... a smaller but still sizable contribution from a diquark-antidiquark operator.

- **Right plot**: Almost no light quark mass dependence.
  \[ \Delta E_0 (m_{\pi,\text{phys}}) = (-86 \pm 22 \pm 10) \text{ MeV}, \]
  \[ m_{\bar{b}\bar{b}us \text{ tetraquark}} (m_{\pi,\text{phys}}) = (10609 \pm 22 \pm 10) \text{ MeV}. \]

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**Categorization:**

- **Types of data:** Quantum mechanics, experimental results, graphs, and tables.
- **Domain:** Particle physics, specifically lattice QCD studies.
- **Themes:** Quantum states, operator overlaps, mass dependence, and energy levels.
- **Tools/techniques:** Lattice QCD calculations, statistical analysis.

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**Graphical Elements:**

- **Charts and Graphs:** Overlap plots for different operators, energy difference graph, mass dependence graph.
- **Diagrams:** Schematic representation of quantum states and operator overlaps.

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**References:**

- Marc Wagner, "Search for $\bar{b}\bar{b}us$ and $\bar{b}\bar{cud}$ tetraquark bound states using lattice QCD", September 28, 2022.
\( \bar{b}b\bar{u}s \) with \( J^P = 1^+ \): existing results

- Lattice QCD results from three independent groups (Francis et al., Junnarkar et al., our work) consistent within statistical errors.

- Strong discrepancies between non-lattice QCD results.
\( \bar{b} \bar{c} u d \) with \( I(J^P) = 0(0^+) \): operators

- **Local operators** (at the source and at the sink):

  \[
  O_1 = O_{[BD](0)} = \sum_x \bar{b} \gamma_5 u(x) \bar{c} \gamma_5 d(x) - (u \leftrightarrow d) \quad (BD \text{ bound state})
  \]

  \[
  O_2 = O_{[Dd](0)} = \sum_x \bar{b}^a \gamma_5 \bar{c}^{b,T}(x) u^{a,T} \gamma_5 d^b(x) - (u \leftrightarrow d) \quad (\text{diquark-antidiquark}).
  \]

- **Scattering operators** (only at the sink):

  \[
  O_3 = O_{B(0)D(0)} = \left( \sum_x \bar{b} \gamma_5 u(x) \right) \left( \sum_y \bar{c} \gamma_5 d(y) \right) - (u \leftrightarrow d) \quad (BD \text{ 2-particle state}).
  \]
\[ \bar{b}cud \textbf{ with } I(J^P) = 0(1^+) : \text{operators} \]

- **Local operators** (at the source and at the sink):

  \[ O_1 = O_{[B^*D](0)} = \sum_x \bar{b}\gamma_j u(x) \bar{c}\gamma_5 d(x) - (u \leftrightarrow d) \quad (B^*D \text{ bound state}) \]

  \[ O_2 = O_{[BD^*](0)} = \sum_x \bar{b}\gamma_5 u(x) \bar{c}\gamma_j d(x) - (u \leftrightarrow d) \quad (BD^* \text{ bound state}) \]

  \[ O_3 = O_{[Dd](0)} = \sum_x \bar{b}^a \gamma_j C\bar{c}^b,^T(x) u^{a,^T} C\gamma_5 d^b(x) - (u \leftrightarrow d) \quad (\text{diquark-antidiquark}), \]

- **Scattering operators** (only at the sink):

  \[ O_4 = O_{B^*(0)D(0)} = \left( \sum_x \bar{b}\gamma_j u(x) \right) \left( \sum_y \bar{c}\gamma_5 d(y) \right) - (u \leftrightarrow d) \quad (B^*D \text{ 2-particle state}) \]

  \[ O_5 = O_{B(0)D^*(0)} = \left( \sum_x \bar{b}\gamma_5 u(x) \right) \left( \sum_y \bar{c}\gamma_j d(y) \right) - (u \leftrightarrow d) \quad (BD^* \text{ 2-particle state}). \]
\( \bar{b}c\bar{u}d: \) energy levels

- **Left plot:** \( I(J^P) = 0(0^+) \), energy levels \( \Delta E_j = E_j - E_B - E_D \) for ensemble C01 obtained with various operator subsets and temporal fitting ranges.

- **Right plot:** \( I(J^P) = 0(1^+) \), energy levels \( \Delta E_j = E_j - E_B^* - E_D \) for ensemble C01 obtained with various operator subsets and temporal fitting ranges.

- Ground states always consistent with or above the lowest meson-meson thresholds.
  - No indication for the existence of a strong-interaction-stable tetraquark.
  - Operator overlaps support this, i.e. suggest that the ground states are meson-meson scattering states.
$ar{b}cud$: final results

- **Left plot**: $I(J^P) = 0(0^+)$, ensemble dependence of ground state energy.
- **Right plot**: $I(J^P) = 0(1^+)$, ensemble dependence of ground state energy.
- To exclude the existence of a shallow bound state with binding energy of only a few MeV, more precise data and an infinite volume extrapolation is needed.
- Results from previous lattice QCD studies are mostly consistent with our results.
- Results from previous non-lattice studies exhibit strong discrepancies (some predict the existence of a stable tetraquark, others claim the opposite).

![Graph 1](image1.png)

![Graph 2](image2.png)
Ongoing work, outlook

- Currently we are including scattering operators at both ends of the correlation functions (technically difficult).

- First results for $\bar{b}b\bar{u}d$, $I(J^P) = 0(1^+)$:
  
  [M. Pflaumer, talk at Lattice 2022]
  [M. Wagner, poster at Lattice 2022]
  
  – Clear separation of the ground state (the strong-interaction-stable tetraquark) and the first excitation (a meson-meson scattering state).
  – Finite volume extrapolation via a scattering analysis (Lüscher’s method).
  – Resulting binding energy slightly smaller, $\Delta E_0(m_{\pi,\text{phys}}) = (-103 \pm 8)$ MeV, but still consistent with our previous result (arXiv:1904.04197), where scattering operators were used only at one end of the correlation functions.

- The main motivation is, however, to prepare a setup, which allows to study tetraquark resonances, e.g. $\bar{b}b\bar{u}d$ with $I(J^P) = 0(1^-)$.
  
  [J. Hoffmann, poster at QWG 2022]