Search for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$ tetraquark bound states using lattice QCD

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Introduction

- We and several other independent groups study the existence and properties of $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$ tetraquarks (discussed in this talk) as well as $\bar{b}\bar{b}ud$ tetraquarks (not discussed in this talk) with lattice QCD using NRQCD for the $\bar{b}$ quarks.

- Summary of main results and current status:
  - $\bar{b}\bar{b}us$ with $J^P = 1^+$:
    A strong-interaction-stable tetraquark around 90 MeV below the BB_s^* threshold.
  - $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$ and with $I(J^P) = 0(1^+)$:
    No indication for strong-interaction-stable tetraquarks, but shallow bound states cannot be excluded. Existence of resonances not yet investigated.
  - $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$:
    A strong-interaction-stable tetraquark around 130 MeV below the BB^* threshold.
  - $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$:
    No strong-interaction-stable tetraquark. Existence of a resonance, explored with lattice QCD static potentials and the Born-Oppenheinmer approximation, seems unlikely.
    [J. Hoffmann, poster at QWG 2022]

- No experimental results for these tetraquarks yet, but for a charm-charm counterpart, $T_{cc}^+(\bar{c}\bar{c}ud)$ recently discovered by LHCb.
  [R. Aaij *et al.* [LHCb], Nature Commun. **13**, 3351 (2022) [arXiv:2109.01056]]

Existing work and references

- This talk is mainly a summary of our recent work
  [S. Meinel, M. Pflaumer, M. Wagner, Phys. Rev. D **106**, 034507 (2022) [arXiv:2205.13982]] ($\bar{b}\bar{b}us$, $\bar{b}\bar{c}ud$)

- Previous related lattice QCD work on $\bar{Q}\bar{Q}qq$ tetraquarks using NRQCD for $\bar{b}$ quarks:
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)

  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. D **99**, 054505 (2019) [arXiv:1810.10550]] ($\bar{b}\bar{c}ud$)

  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)

  [L. Leskovec, S. Meinel, M. Pflaumer, M. Wagner, Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197]] ($\bar{b}\bar{b}ud$)

  [R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis, K. Maltman, Phys. Rev. D **102**, 114506 (2020) [arXiv:2006.14294]] ($\bar{b}\bar{c}ud$)

  [P. Mohanta, S. Basak, Phys. Rev. D **102**, 094516 (2020) [arXiv:2008.11146]] ($\bar{b}\bar{b}ud$)

  [N. Mathur, M. Padmanath, PoS **LATTICE2021**, 443 (2022) [arXiv:2111.01147 [hep-lat]] ($\bar{b}\bar{c}ud$)

Motivation / focus of this work (1)

- Lattice QCD $=$ full QCD (numerically with high performance computers) ... i.e. no assumptions, no approximations, etc.

- A lattice QCD result, if generated in a technically sound and solid way, is a full QCD result and can be confronted with experiment in a direct and meaningful way.

- However, lattice QCD is technically difficult, in particular, when studying exotic hadrons, e.g. $\bar{Q}\bar{Q}qq$ tetraquarks.
  $\rightarrow$ Quite often lattice QCD studies are not (yet) fully rigorous, i.e. certain assumptions are made, quark masses are unphysical, no continuum and/or infinite volume limit, no convincing separation and extraction of low-lying energy eigenstates, etc.
  $\rightarrow$ Important to read (at least some) technical details of lattice QCD papers, to be able to judge their quality.

- Hadron masses (e.g. the mass of a $\bar{b}\bar{b}us$ tetraquark), more precisely low-lying energy eigenvalues E_n, are determined from the exponential decays of temporal correlation functions $C_{jk}(t)$ of (hadron creation) operators O_j:

$$C_{jk}(t) = \langle \Omega | O_j^\dagger(t) O_k(0) | \Omega \rangle = \langle \Omega | O_j^\dagger | 0 \rangle \langle 0 | O_k | \Omega \rangle e^{-E_0 t} + \langle \Omega | O_j^\dagger | 1 \rangle \langle 1 | O_k | \Omega \rangle e^{-E_1 t} + \ldots$$

  - $C_{jk}(t)$ can be computed with lattice QCD.
  - The analytical expression on the right hand side is used to determine E_0, E_1, ...

Motivation / focus of this work (2)

- $C_{jk}(t) = \langle \Omega | O_j^\dagger(t) O_k(0) | \Omega \rangle = \langle \Omega | O_j^\dagger | 0 \rangle \langle 0 | O_k | \Omega \rangle e^{-E_0 t} + \langle \Omega | O_j^\dagger | 1 \rangle \langle 1 | O_k | \Omega \rangle e^{-E_1 t} + \ldots$
- In principle one can use any operator O_j, which "has the same quantum numbers" as the hadron of interest (if one would be able to compute $C_{jk}(t)$ very precisely for very large t).
- In practice one needs operators with the following properties:
  - The operators have to generate large overlap to the low-lying energy eigenstates states (not only the hadron of interest, but also multi-particle states of similar mass).
  - There must be at least one operator for each low-lying state.
  - The operators must not be too similar (ideally "they are almost orthogonal").

  Otherwise it is questionable, whether the corresponding analysis correctly extracts E_0, E_1, ... from the correlation function $C_{jk}(t)$.
  A major problem is that such analyses always provide numbers, but these might be wrong ... e.g. one could obtain $\approx (E_0 + E_1)/2$ instead of E_0, if one does not use both bound state and scattering operators.
- **We improve on existing lattice QCD studies by considering both local and scattering operators for $\bar{Q}\bar{Q}qq$ systems. This allows a more trustworthy and precise extraction of energy eigenvalues as well as to carry out scattering analyses.**

Lattice setup

- Five ensembles of gauge link configurations generated with 2+1 quark flavors by the **RBC and UKQCD collaboration**. These have different volumes, different lattice spacings and different light quark masses.

ensemble	$N_s^3 \times N_t$	a [fm]	m_π [MeV]
C00078	$48^3 \times 96$	$0.1141(3)$	$139(1)$
C005	$24^3 \times 64$	$0.1106(3)$	$340(1)$
C01	$24^3 \times 64$	$0.1106(3)$	$431(1)$
F004	$32^3 \times 64$	$0.0828(3)$	$303(1)$
F006	$32^3 \times 64$	$0.0828(3)$	$360(1)$

[Y. Aoki *et al.* [RBC and UKQCD], Phys. Rev. D **83**, 074508 (2011) [arXiv:1011.0892]]
[T. Blum *et al.* [RBC and UKQCD], Phys. Rev. D **93**, 074505 (2016) [arXiv:1411.7017]]

- Domain-wall action for u, d and s quarks.

- NRQCD action for valence b quarks, anisotropic clover action for valence c quarks.

- Local operators (representing bound states) and scattering operators (representing meson-meson states).

- Scattering operators at the moment only at one end of the correlation functions, because we are using point-to-all-operators. (Scattering operators at both ends is work in progress.)

$\bar{b}\bar{b}us$ with $J^P = 1^+$: operators

- Local operators (at the source and at the sink):

$$O_1 = O_{[BB_s^*](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_5 u(\mathbf{x})\, \bar{b}\gamma_j s(\mathbf{x}) \quad (BB_s^* \text{ bound state})$$

$$O_2 = O_{[B^*B_s](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_j u(\mathbf{x})\, \bar{b}\gamma_5 s(\mathbf{x}) \quad (B^*B_s \text{ bound state})$$

$$O_3 = O_{[B^*B_s^*](0)} = \epsilon_{jkl} \sum_{\mathbf{x}} \bar{b}\gamma_k u(\mathbf{x})\, \bar{b}\gamma_l s(\mathbf{x}) \quad (B^*B_s^* \text{ bound state})$$

$$O_4 = O_{[Dd](0)} = \sum_{\mathbf{x}} \bar{b}^a \gamma_j \mathcal{C} \bar{b}^{b,T}(\mathbf{x})\, u^{a,T} \mathcal{C}\gamma_5 s^b(\mathbf{x}) \quad (\text{diquark-antidiquark}).$$

- Scattering operators (only at the sink):

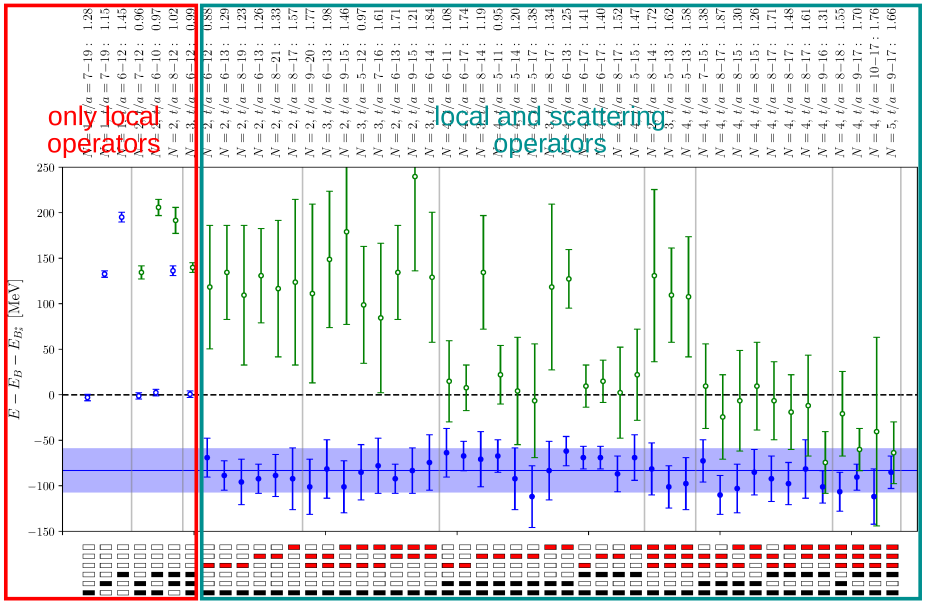
$$O_5 = O_{B(0)B_s^*(0)} = \Big(\sum_{\mathbf{x}} \bar{b}\gamma_5 u(\mathbf{x})\Big)\Big(\sum_{\mathbf{y}} \bar{b}\gamma_j s(\mathbf{y})\Big) \quad (BB_s^* \text{ 2-particle state})$$

$$O_6 = O_{B^*(0)B_s(0)} = \Big(\sum_{\mathbf{x}} \bar{b}\gamma_j u(\mathbf{x})\Big)\Big(\sum_{\mathbf{y}} \bar{b}\gamma_5 s(\mathbf{y})\Big) \quad (B^*B_s \text{ 2-particle state})$$

$$O_7 = O_{B^*(0)B_s^*(0)} = \epsilon_{jkl}\Big(\sum_{\mathbf{x}} \bar{b}\gamma_k u(\mathbf{x})\Big)\Big(\sum_{\mathbf{y}} \bar{b}\gamma_l s(\mathbf{y})\Big) \quad (B^*B_s^* \text{ 2-particle state}).$$

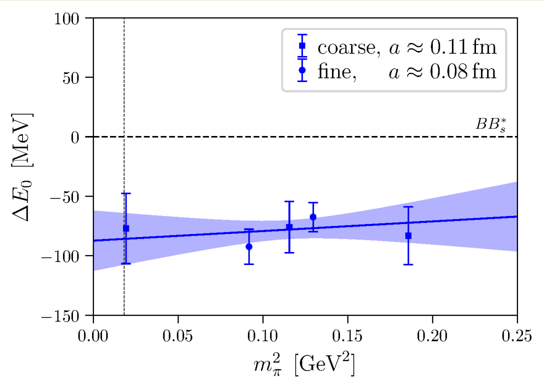
$\bar{b}\bar{b}us$ with $J^P = 1^+$: energy levels

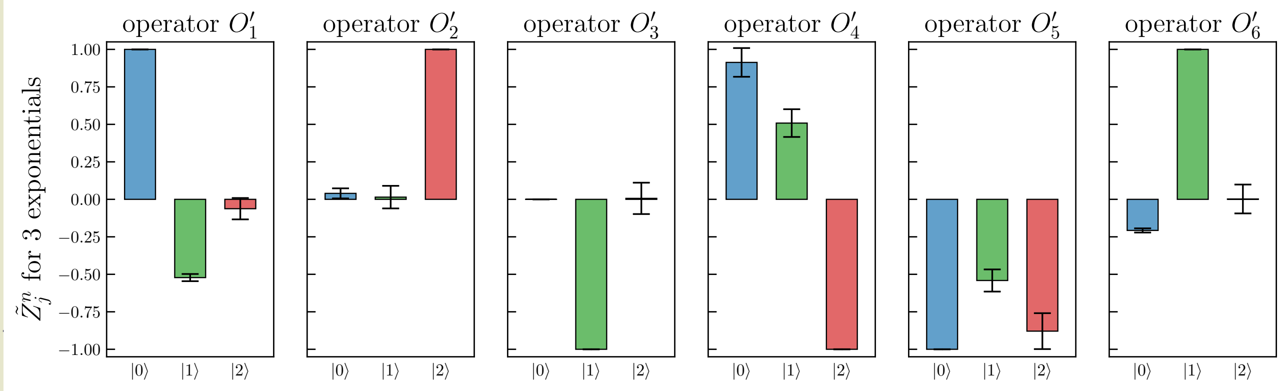
- Plot: Energy levels $\Delta E_n = E_n - E_B - E_{B_s^*}$ for ensemble C01 obtained with various operator subsets and temporal fitting ranges.
- Only local operators $\quad\rightarrow\quad \Delta E_0 \approx 0\,\text{MeV}$.
- Local and scattering operators $\quad\rightarrow\quad \Delta E_0 \approx -100\,\text{MeV}$, $\Delta E_1 \approx 0\,\text{MeV}$.
  $\rightarrow$ Ground state corresponds to a strong-interaction-stable tetraquark.



$\bar{b}\bar{b}us$ with $J^P = 1^+$: our final results

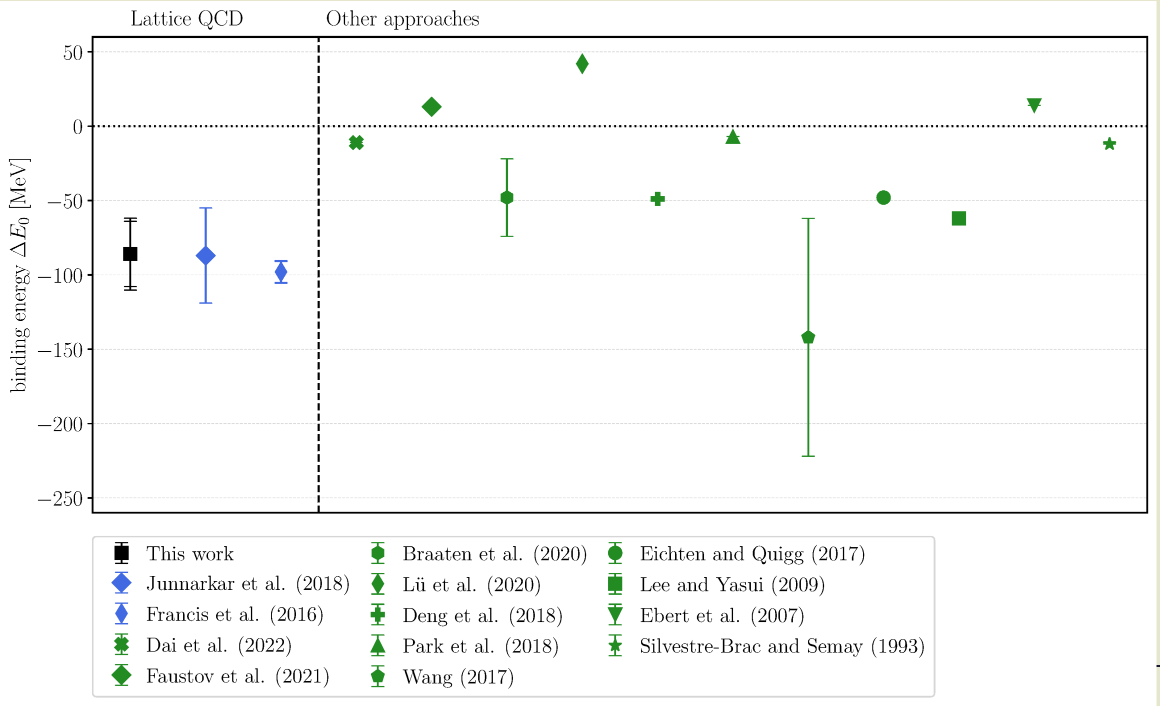
- Bottom plot: Overlaps of each operator to the lowest three energy eigenstates (O'_1 to O'_3 are linear combinations of O_1 to O_4, O'_4 to O'_6 correspond to O_5 to O_7).
  - Roughly equal contributions to the ground state from a local BB^*_s / B^*B_s operator ("$I = 0$") ...
  - ... and a local $B^*B^*_s$ operator, ...
  - ... a smaller but still sizable contribution from a diquark-antidiquark operator.
- Right plot: Almost no light quark mass dependence.
  $\rightarrow \Delta E_0(m_{\pi,\mathrm{phys}}) = (-86 \pm 22 \pm 10)$ MeV,
  $m_{\bar{b}\bar{b}us\ \text{tetraquark}}(m_{\pi,\mathrm{phys}}) = (10609 \pm 22 \pm 10)$ MeV.





$\bar{b}\bar{b}us$ with $J^P = 1^+$: existing results

- Lattice QCD results from three independent groups (Francis et al., Junnarkar et al., our work) consistent within statistical errors.
- Strong discrepancies between non-lattice QCD results.



$\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$: operators

- Local operators (at the source and at the sink):

$$O_1 = O_{[BD](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_5 u(\mathbf{x})\, \bar{c}\gamma_5 d(\mathbf{x}) - (u \leftrightarrow d) \quad (BD \text{ bound state})$$

$$O_2 = O_{[Dd](0)} = \sum_{\mathbf{x}} \bar{b}^a \gamma_5 \mathcal{C} \bar{c}^{b,T}(\mathbf{x})\, u^{a,T} \mathcal{C} \gamma_5 d^b(\mathbf{x}) - (u \leftrightarrow d) \quad (\text{diquark-antidiquark}).$$

- Scattering operators (only at the sink):

$$O_3 = O_{B(0)D(0)} = \Big(\sum_{\mathbf{x}} \bar{b}\gamma_5 u(\mathbf{x})\Big) \Big(\sum_{\mathbf{y}} \bar{c}\gamma_5 d(\mathbf{y})\Big) - (u \leftrightarrow d) \quad (BD \text{ 2-particle state}).$$

$\bar{b}\bar{c}ud$ with $I(J^P) = 0(1^+)$: operators

- Local operators (at the source and at the sink):

$$O_1 = O_{[B^* D](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_j u(\mathbf{x})\, \bar{c}\gamma_5 d(\mathbf{x}) - (u \leftrightarrow d) \quad (B^* D \text{ bound state})$$

$$O_2 = O_{[B D^*](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_5 u(\mathbf{x})\, \bar{c}\gamma_j d(\mathbf{x}) - (u \leftrightarrow d) \quad (B D^* \text{ bound state})$$

$$O_3 = O_{[Dd](0)} = \sum_{\mathbf{x}} \bar{b}^a \gamma_j \mathcal{C} \bar{c}^{b,T}(\mathbf{x})\, u^{a,T} \mathcal{C} \gamma_5 d^b(\mathbf{x}) - (u \leftrightarrow d) \quad (\text{diquark-antidiquark}),$$

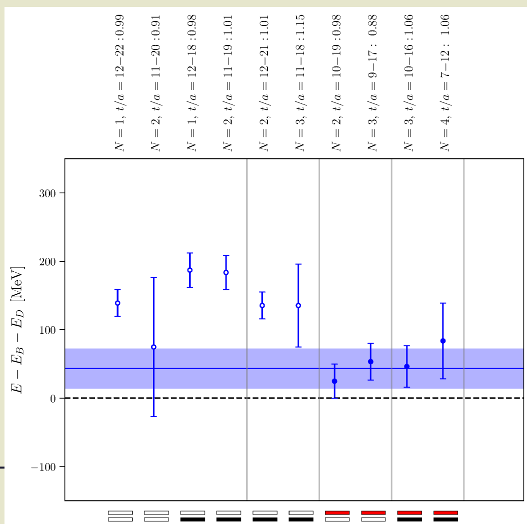
- Scattering operators (only at the sink):

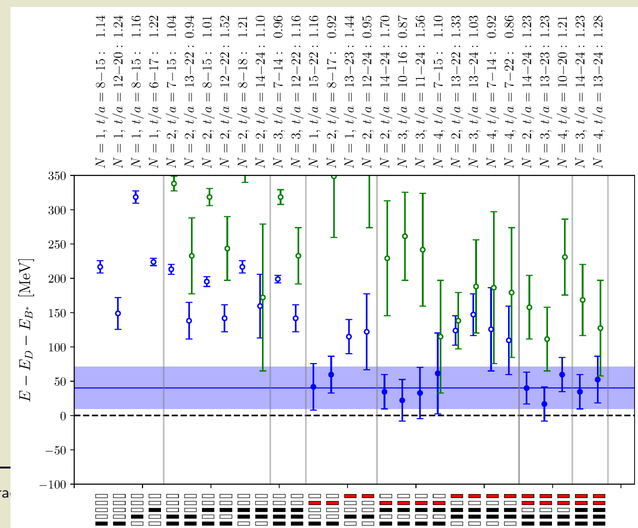
$$O_4 = O_{B^*(0) D(0)} = \Big(\sum_{\mathbf{x}} \bar{b}\gamma_j u(\mathbf{x})\Big) \Big(\sum_{\mathbf{y}} \bar{c}\gamma_5 d(\mathbf{y})\Big) - (u \leftrightarrow d) \quad (B^* D \text{ 2-particle state})$$

$$O_5 = O_{B(0) D^*(0)} = \Big(\sum_{\mathbf{x}} \bar{b}\gamma_5 u(\mathbf{x})\Big) \Big(\sum_{\mathbf{y}} \bar{c}\gamma_j d(\mathbf{y})\Big) - (u \leftrightarrow d) \quad (B D^* \text{ 2-particle state}).$$

$\bar{b}\bar{c}ud$: energy levels

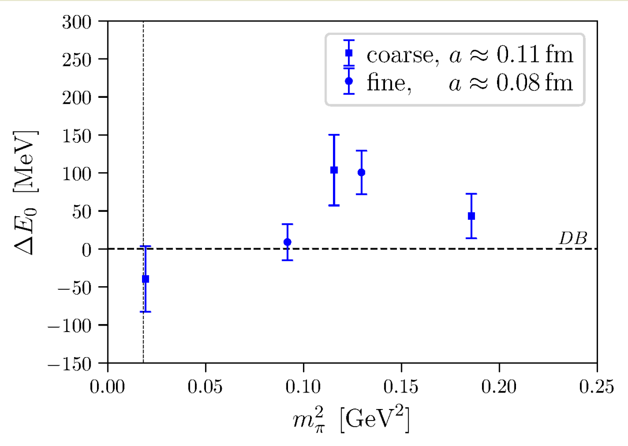
- Left plot: $I(J^P) = 0(0^+)$, energy levels $\Delta E_j = E_j - E_B - E_D$ for ensemble C01 obtained with various operator subsets and temporal fitting ranges.
- Right plot: $I(J^P) = 0(1^+)$, energy levels $\Delta E_j = E_j - E_{B^*} - E_D$ for ensemble C01 obtained with various operator subsets and temporal fitting ranges.
- Ground states always consistent with or above the lowest meson-meson thresholds.
  $\to$ No indication for the existence of a strong-interaction-stable tetraquark.
  $\to$ Operator overlaps support this, i.e. suggest that the ground states are meson-meson scattering states.

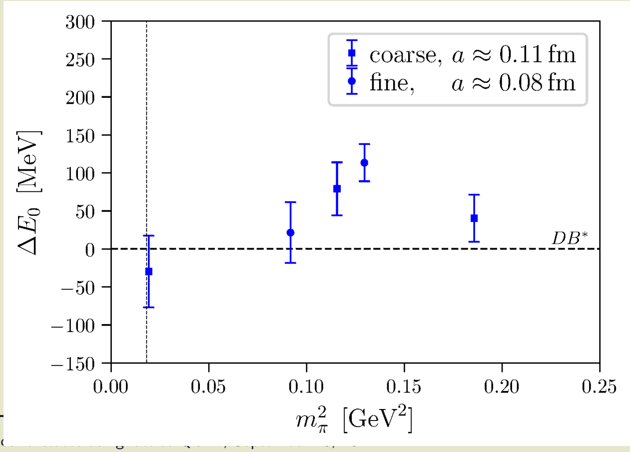




$\bar{b}\bar{c}ud$: final results

- Left plot: $I(J^P) = 0(0^+)$, ensemble dependence of ground state energy.
- Right plot: $I(J^P) = 0(1^+)$, ensemble dependence of ground state energy.
- To exclude the existence of a shallow bound state with binding energy of only a few MeV, more precise data and an infinite volume extrapolation is needed.
- Results from previous lattice QCD studies are mostly consistent with our results.
- Results from previous non-lattice studies exhibit strong discrepancies (some predict the existence of a stable tetraquark, others claim the opposite).





Ongoing work, outlook

- Currently we are including scattering operators at both ends of the correlation functions (technically difficult).

- First results for $\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$:
  [M. Pflaumer, talk at Lattice 2022]
  [M. Wagner, poster at Lattice 2022]

  - Clear separation of the ground state (the strong-interaction-stable tetraquark) and the first excitation (a meson-meson scattering state).
  - Finite volume extrapolation via a scattering analysis (Lüscher's method).
  - Resulting binding energy slightly smaller, $\Delta E_0(m_{\pi,\text{phys}}) = (-103 \pm 8)$ MeV, but still consistent with our previous result (arXiv:1904.04197), where scattering operators were used only at one end of the correlation functions.

- The main motivation is, however, to prepare a setup, which allows to study tetraquark resonances, e.g. $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$.
  [J. Hoffmann, poster at QWG 2022]