

Interaction energies between static-light mesons

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Marc Wagner

Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik

mwagner@th.physik.uni-frankfurt.de

<http://people.physik.hu-berlin.de/~mcwagner/>

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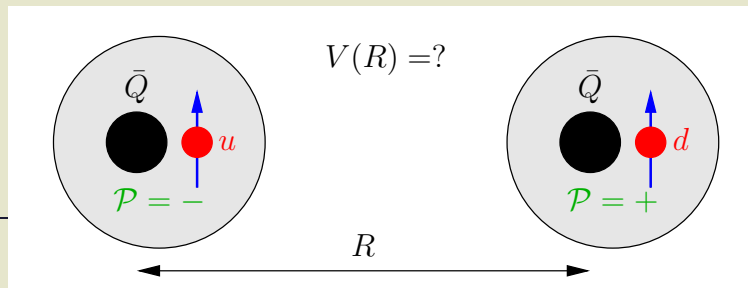
[M. Wagner, PoS [LATTICE2010](#), 162 (2010)]

[M. Wagner, [arXiv:1103.5147](#) [hep-lat]]



Introduction (1)

- Goal: compute the potential of (or equivalently the force between) two B mesons from first principles by means of lattice QCD:
 - Treat the b quark in the static approximation.
 - Consider only pseudoscalar/vector mesons ($j^{\mathcal{P}} = (1/2)^-$, denoted by S , PDG: B, B^*) and scalar/pseudovector mesons ($j^{\mathcal{P}} = (1/2)^+$, denoted by P_- , PDG: B_0^*, B_1^*), which are among the lightest static-light mesons.
 - Study the dependence of the mesonic potential $V(R)$ on
 - * the light quark flavor u and/or d (isospin),
 - * the light quark spin (the static quark spin is irrelevant),
 - * the type of the meson S and/or P_- .



Introduction (2)

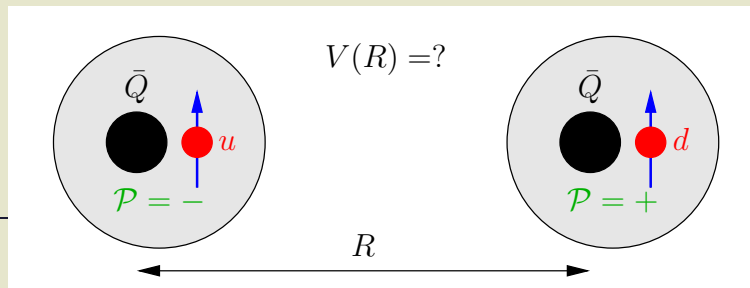
- Motivation:

- First principles computation of a hadronic force.
- Possible application: determine, whether two B mesons may form bound states (tetraquarks).
- Until now
 - * it has mainly been studied in the quenched approximation,
 - * only pseudoscalar (S), but no scalar (P_-) B mesons have been considered.

[C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]

[W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

[G. Bali and M. Hetzenegger, PoS **LATTICE2010**, 142 (2010)]



Outline

- Symmetries and quantum numbers of B mesons and BB systems.
- Lattice setup.
- Results and their interpretation.

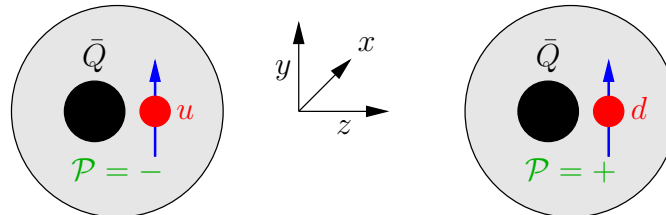
(Pseudo)scalar B mesons

- Symmetries and quantum numbers of static-light mesons:
 - Isospin: $I = 1/2$, $I_z = \pm 1/2$, i.e. $B \equiv \bar{Q}u$ or $B \equiv \bar{Q}d$.
 - Parity: $\mathcal{P} = \pm$,
 - * $\mathcal{P} = - \equiv S$ (wave),
 - * $\mathcal{P} = + \equiv P_-$ (wave).
 - Rotations:
 - * Light cloud angular momentum $j = 1/2$ (for S and P_-), $j_z = \pm 1/2$.
 - * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
- Examples of static-light meson creation operators:
 - $\bar{Q}\gamma_5 q$ (pseudoscalar, i.e. S), $q \in \{u, d\}$,
 - $\bar{Q}q$ (scalar, i.e. P_-)

(j_z is not well-defined, when using these operators).

BB systems (1)

- Symmetries and quantum numbers of a pair of static-light mesons (separated along the z -axis):
 - Isospin: $I = 0, 1, I_z = -1, 0, +1$.
 - Rotations around the z -axis:
 - * Angular momentum of the light degrees of freedom $j_z = -1, 0, +1$.
 - * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
 - Parity: $\mathcal{P} = \pm$.
 - If $j_z = 0$, reflection along the x -axis: $\mathcal{P}_x = \pm$.
 - Instead of using $j_z = \pm 1$ one can also label states by $|j_z| = 1, \mathcal{P}_x = \pm$.
- Label BB states by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$.



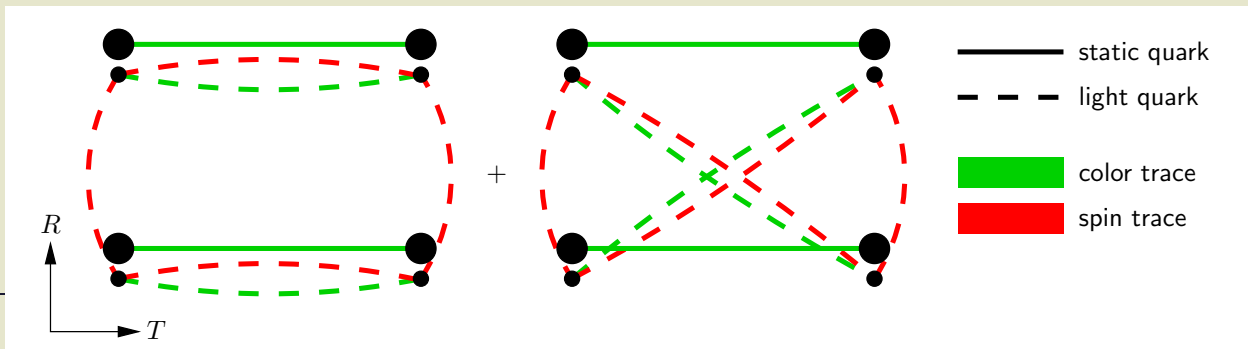
BB systems (2)

- To extract the potential(s) of a given sector (characterized by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$), compute the temporal correlation function of the trial state

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(1)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(2)}(+R/2) \right) |\Omega\rangle,$$

where

- $\mathcal{C} = \gamma_0 \gamma_2$ (charge conjugation matrix),
- $q^{(1)} q^{(2)} \in \{ud - du, uu, dd, ud + du\}$ (isospin I, I_z),
- Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}, \mathcal{P}_x$).



BB systems (3)

- *BB* creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(u)}(+R/2) \right).$$

Γ	$ j_z , \mathcal{P}, \mathcal{P}_x$
1	0, -, -
$\gamma_0 \gamma_5$	0, +, +
γ_5	0, +, +
γ_0	0, +, -
γ_3	0, -, -
$\gamma_0 \gamma_3 \gamma_5$	0, +, +
$\gamma_3 \gamma_5$	0, -, +
$\gamma_0 \gamma_3$	0, -, -
γ_1	1, -, +
$\gamma_0 \gamma_1 \gamma_5$	1, +, -
$\gamma_1 \gamma_5$	1, -, -
$\gamma_0 \gamma_1$	1, -, +
...	...

BB systems (4)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(d)}(+R/2) \right) \pm (u \leftrightarrow d).$$

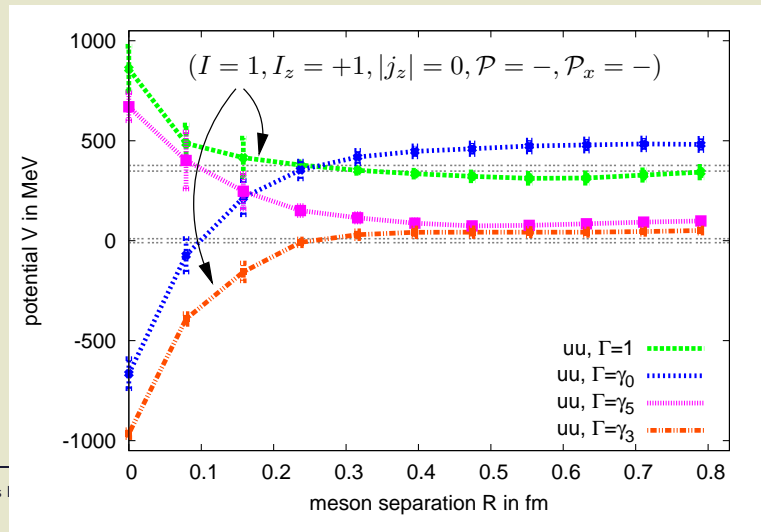
Γ, \pm	$ j_z , I, \mathcal{P}, \mathcal{P}$
$\gamma_5, -$	0, 0, -, +
$\gamma_0, -$	0, 0, -, -
1, -	0, 0, +, -
$\gamma_0\gamma_5, -$	0, 0, -, +
$\gamma_3\gamma_5, -$	0, 0, +, +
$\gamma_0\gamma_3, -$	0, 0, +, -
$\gamma_3, -$	0, 0, +, -
$\gamma_0\gamma_3\gamma_5, -$	0, 0, -, +
$\gamma_5, +$	0, 1, +, +
$\gamma_0, +$	0, 1, +, -
1, +	0, 1, -, -
$\gamma_0\gamma_5, +$	0, 1, +, +
...	...

Lattice setup

- Lattice spacing: $a \approx 0.079$ fm.
- Lattice extension: $L \approx 1.90$ fm (periodic boundary conditions).
- Pion mass: $m_{\text{PS}} \approx 340$ MeV.

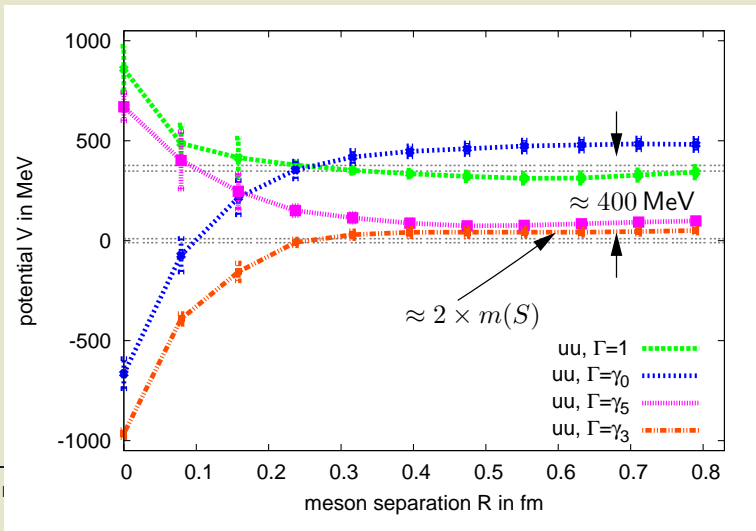
Discussion of results (1)

- Four “types of potentials”:
 - Two attractive, two repulsive.
 - Two have asymptotic values, which are larger by ≈ 400 MeV.
- There are cases, where two potentials with identical quantum numbers are completely different (i.e. of different type)
 - at least one of the corresponding trial states must have very small ground state overlap
 - physical understanding, i.e. interpretation of trial states needed.



Discussion of results (2)

- Expectation at large meson separation R : $V(R) \approx 2 \times \text{meson mass}$.
 - Potentials with smaller asymptotic value at $\approx 2 \times m(S)$.
 - $m(P_-) - m(S) \approx 400 \text{ MeV}$: approximately the observed difference between different types of potentials.
- Two types correspond to $S \leftrightarrow S$ potentials.
- Two types correspond to $S \leftrightarrow P_-$ potentials.
- Can this be understood in detail on the level of the used BB creation operators?



Discussion of results (3)

- Express the BB creation operators in terms of static-light meson creation operators (use suitable spin and parity projectors for the light quarks).

– Examples:

$$* uu, \Gamma = 1 \quad \rightarrow \quad \mathcal{P} = -, \mathcal{P}_x = -:$$

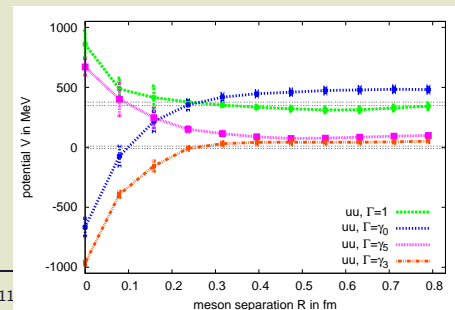
$$(\mathcal{C}1)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(u)}(+R/2) \right) \propto \\ \propto S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow.$$

$$* uu, \Gamma = \gamma_3 \quad \rightarrow \quad \mathcal{P} = -, \mathcal{P}_x = -:$$

$$(\mathcal{C}\gamma_3)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(u)}(+R/2) \right) \propto \\ \propto S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow.$$

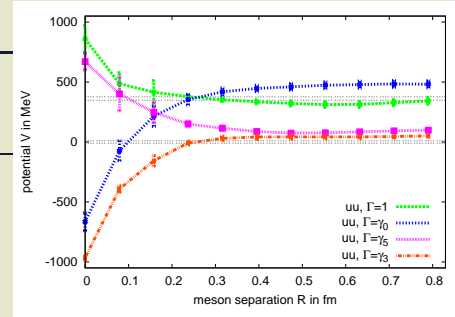
– SS/SP_- content and asymptotic values in agreement for all 64 correlation functions/
potentials

→ asymptotic differences understood.



Discussion of results (4)

- Is there a general rule, about when a potential is repulsive and when attractive?



– $S \leftrightarrow S$ potentials:

- * $(I = 0, s = 0)$ or $(I = 1, s = 1)$, i.e. $I = s \rightarrow$ attractive
 - * $(I = 0, s = 1)$ or $(I = 1, s = 0)$, i.e. $I \neq s \rightarrow$ repulsive
- (s : combined angular momentum of the two mesons).

- * **Example:** $uu, \Gamma = \gamma_3 \rightarrow \mathcal{P} = -, \mathcal{P}_x = -:$

$$\begin{aligned}
 (\mathcal{C}\gamma_3)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(u)}(+R/2) \right) &\propto \\
 \propto S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow.
 \end{aligned}$$

i.e. $I = 1, s = 1$; the numerically obtained potential is attractive, i.e. in agreement with the above stated rule.

- * All 32 $S \leftrightarrow S$ correlation functions/potentials fulfill the rule.
- * Agreement with similar quenched lattice studies.

[C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]

[W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

Discussion of results (5)

– $S \leftrightarrow P_-$ potentials:

- * Do not obey the above stated rule.

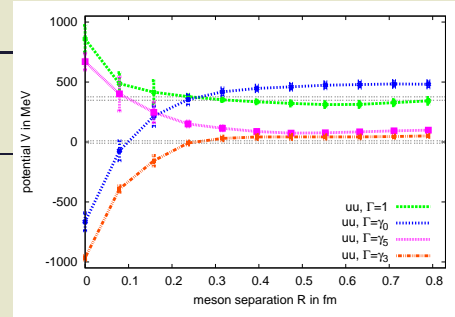
- * It can, however, easily be generalized by including parity, i.e. symmetry or antisymmetry under exchange of S and P_- :
 - trial state symmetric under meson exchange \rightarrow attractive
 - trial state antisymmetric under meson exchange \rightarrow repulsive (meson exchange \equiv exchange of flavor, spin and parity).

- * **Example:** $uu, \Gamma = \gamma_0 \rightarrow \mathcal{P} = +, \mathcal{P}_x = -$:

$$\begin{aligned}
 (\mathcal{C}\gamma_0)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(u)}(+R/2) \right) &\propto \\
 \propto S_\uparrow P_\downarrow - S_\downarrow P_\uparrow - P_\uparrow S_\downarrow + P_\downarrow S_\uparrow, &
 \end{aligned}$$

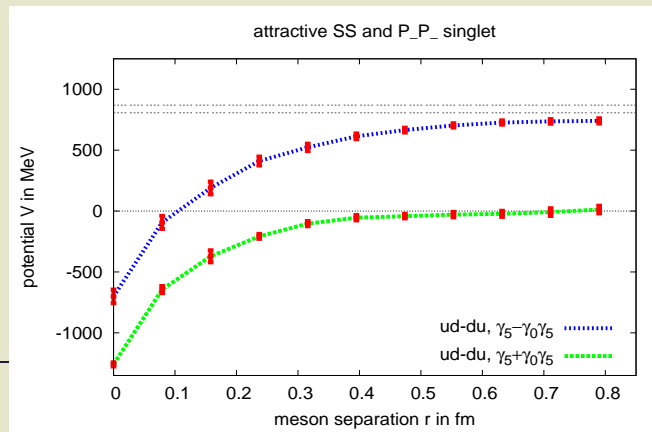
i.e. $I = 1$ (symmetric), $s = 0$ (antisymmetric), antisymmetric with respect to $S \leftrightarrow P_-$; the numerically obtained potential is attractive, i.e. in agreement with the above stated general rule.

- * All 32 $S \leftrightarrow P_-$ correlation functions/potentials (and all 32 $S \leftrightarrow S$ correlation functions/potentials) fulfill the generalized rule.



Discussion of results (6)

- Improvements after having understood the extraction and interpretation of BB potentials from single correlation functions:
 - Linearly combine BB operators to either eliminate $P_- \leftrightarrow P_-$ or $S \leftrightarrow S$ combinations.
 - Example:
 - $ud - du, \Gamma = \gamma_5 \rightarrow -S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow + P_\downarrow P_\uparrow$
 - $ud - du, \Gamma = \gamma_0 \gamma_5 \rightarrow -S_\uparrow S_\downarrow + S_\downarrow S_\uparrow + P_\uparrow P_\downarrow - P_\downarrow P_\uparrow$
 - use $\gamma_5 + \gamma_0 \gamma_5$ to obtain a better signal for the $S \leftrightarrow S$ potential
 - use $\gamma_5 - \gamma_0 \gamma_5$ to extract the $P_- \leftrightarrow P_-$ potential.



Discussion of results (7)

- Improvements after having understood the extraction and interpretation of BB potentials from single correlation functions:
 - Use correlation matrices instead of single correlation functions to avoid mixing with BB states of lower energy, which is present, because
 - * although the product of two specific B meson creation operators closely resembles the corresponding BB state, it will still have a non-vanishing overlap to BB states corresponding to B mesons with different isospin, spin and/or parity,
 - * twisted mass lattice QCD explicitly breaks isospin and parity (the breaking is proportional to the lattice spacing a ; isospin and parity will be restored in the continuum limit).

Summary of BB states and degeneracies

- Two B mesons, each can have $I_z = \pm 1/2$, $j_z = \pm 1/2$, $\mathcal{P} = \pm$
 $\rightarrow 8 \times 8 = 64$ states.

- $S \leftrightarrow S$ potentials:

– Attractive: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{6}_{I=1,|j_z|=1}$ (10 states).

– Repulsive: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{2}_{I=0,|j_z|=1}$ (6 states).

- $S \leftrightarrow P_-$ potentials:

– Attractive: $\underbrace{1 \oplus 1 \oplus 3 \oplus 3}_{|j_z|=0} \oplus \underbrace{2 \oplus 6}_{|j_z|=1}$ (16 states).

– Repulsive: $\underbrace{1 \oplus 1 \oplus 3 \oplus 3}_{|j_z|=0} \oplus \underbrace{2 \oplus 6}_{|j_z|=1}$ (16 states).

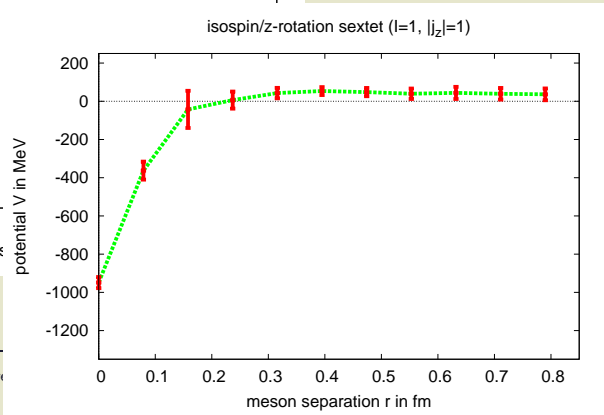
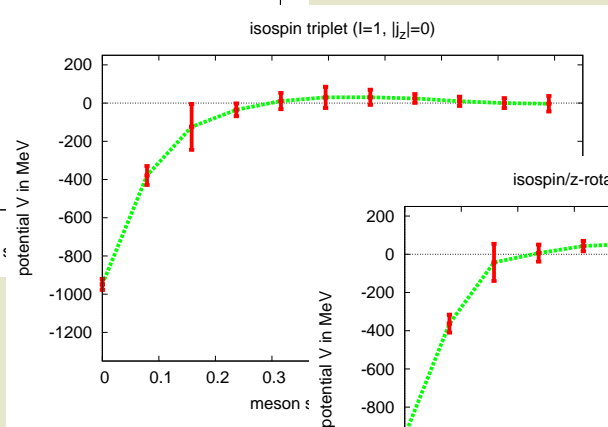
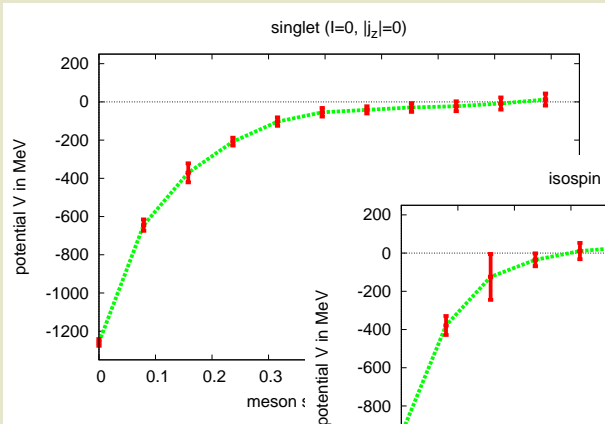
- $P_- \leftrightarrow P_-$ potentials: identical to $S \leftrightarrow S$ potentials.

- In total 24 different potentials.

Attractive $S \leftrightarrow S$ potentials

- Attractive $S \leftrightarrow S$ potentials are relevant, when trying to determine, whether BB may form a bound state.

- Three different attractive $S \leftrightarrow S$ potentials: $\underbrace{1}_{I=0, |j_z|=0} \oplus \underbrace{3}_{I=1, |j_z|=0} \oplus \underbrace{6}_{I=1, |j_z|=1}$.



Summary, conclusions, future plans (1)

- Computation of BB potentials (arbitrary flavor, spin and parity) with “light” dynamical quarks ($m_{\text{PS}} \approx 340 \text{ MeV}$).
 - Qualitative agreement with existing quenched results for $S \leftrightarrow S$ potentials.
 - First lattice computation of $S \leftrightarrow P_-$ and $P_- \leftrightarrow P_-$ potentials.
 - Clear statements about whether a potential of a given channel is attractive or repulsive.
- Statistical accuracy problematic (exponentially decaying correlation functions are quickly lost in statistical noise):
 - Reasonable accuracy for attractive $S \leftrightarrow S$ potentials (interesting, when trying to determine, whether BB may form a bound state).
 - Other (higher) potentials:
 - BB potentials are extracted at rather small temporal separations
 - slight contamination from excited states cannot be excluded.

Summary, conclusions, future plans (2)

- Further plans and possibilities:
 - Other values of the lattice spacing, the spacetime volume and/or the u/d quark mass.
 - Partially quenched computations, to obtain $B_s B_s$ and/or $B_s B$ potentials.
 - Improve lattice meson potentials at small separations (where the suppression of UV fluctuations due to the lattice cutoff yields wrong results) with corresponding perturbative potentials.

BB systems (A)

- Wilson twisted mass action:

$$S_F[\chi, \bar{\chi}, U] = a^4 \sum_x \bar{\chi}(x) \left(D_W + i\mu_q \gamma_5 \tau_3 \right) \chi(x) \quad , \quad \psi(x) = e^{i\gamma_5 \tau_3 \omega/2} \chi(x).$$

- Symmetries of Wilson twisted mass lattice QCD compared to QCD:
 - SU(2) isospin breaks down to U(1): I_z is still a good quantum number, I is not.
 - Parity \mathcal{P} is replaced by $\mathcal{P}^{(\text{tm})}$, which is parity combined with light flavor exchange.
 - Twisted mass BB sectors:

$$* I_z = \pm 1: (I_z, |j_z|, \underbrace{\mathcal{P}^{(\text{tm})} \mathcal{P}_x^{(\text{tm})}}_{=\mathcal{P} \mathcal{P}_x}),$$

$$* I_z = 0: (I_z, |j_z|, \underbrace{\mathcal{P}^{(\text{tm})}}_{=\mathcal{P} \times (2I-1)}, \underbrace{\mathcal{P}_x^{(\text{tm})}}_{=\mathcal{P}_x \times (2I-1)}).$$

→ QCD sectors $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$ are pairwise combined.

BB systems (B)

- BB creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) \chi_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) \chi_B^{(u)}(+R/2) \right).$$

Γ twisted	$ j_z , \mathcal{P}^{(\text{tm}, \text{light})} \mathcal{P}_x^{(\text{tm}, \text{light})}$	Γ pseudo physical	$ j_z , \mathcal{P}^{(\text{light})}, \mathcal{P}_x^{(\text{light})}$
γ_5	0, +	$\mp i$	0, -, -
$\gamma_0 \gamma_5$	0, +	$+\gamma_0 \gamma_5$	0, +, +
1	0, +	$\mp i \gamma_5$	0, +, +
γ_0	0, -	$+\gamma_0$	0, +, -
γ_3	0, +	$+\gamma_3$	0, -, -
$\gamma_0 \gamma_3$	0, +	$\mp i \gamma_0 \gamma_3 \gamma_5$	0, +, +
$\gamma_3 \gamma_5$	0, -	$+\gamma_3 \gamma_5$	0, -, +
$\gamma_0 \gamma_3 \gamma_5$	0, +	$\mp i \gamma_0 \gamma_3$	0, -, -
γ_1	1, -	$+\gamma_1$	1, -, +
$\gamma_0 \gamma_1$	1, -	$\mp i \gamma_0 \gamma_1 \gamma_5$	1, +, -
$\gamma_1 \gamma_5$	1, +	$+\gamma_1 \gamma_5$	1, -, -
$\gamma_0 \gamma_1 \gamma_5$	1, -	$\mp i \gamma_0 \gamma_1$	1, -, +
...

BB systems (C)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) \chi_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) \chi_B^{(d)}(+R/2) \right) \pm (u \leftrightarrow d).$$

Γ twisted, \pm	$ j_z , \mathcal{P}^{(\text{tm}, \text{light})}, \mathcal{P}_x^{(\text{tm}, \text{light})}$	Γ pseudo physical, \pm	$ j_z , I, \mathcal{P}^{(\text{light})}, \mathcal{P}_x^{(\text{light})}$
$\gamma_5, +$	0, +, +	$+\gamma_5, +$	0, 1, +, +
$\gamma_0\gamma_5, +$	0, +, +	$+i\gamma_0, -$	0, 0, -, -
1, -	0, -, +	$+1, -$	0, 0, +, -
$\gamma_0, -$	0, +, +	$+i\gamma_0\gamma_5, +$	0, 1, +, +
$\gamma_5, -$	0, +, -	$+\gamma_5, -$	0, 0, -, +
$\gamma_0\gamma_5, -$	0, +, -	$+i\gamma_0, +$	0, 1, +, -
1, +	0, -, -	$+1, +$	0, 1, -, -
$\gamma_0, +$	0, +, -	$+i\gamma_0\gamma_5, -$	0, 0, -, +
$\gamma_3, +$	1, -, -	$+i\gamma_3\gamma_5, -$	0, 0, +, +
$\gamma_0\gamma_3, +$	1, -, -	$+\gamma_0\gamma_3, +$	0, 1, -, -
$\gamma_3\gamma_5, -$	1, -, -	$+i\gamma_3, +$	0, 1, -, -
$\gamma_0\gamma_3\gamma_5, -$	1, +, -	$+\gamma_0\gamma_3\gamma_5, -$	0, 0, -, +
...

Simulation setup (A)

- Fermionic action: Wilson twisted mass, $N_f = 2$ degenerate flavors,

$$S_F[\chi, \bar{\chi}, U] = a^4 \sum_x \bar{\chi}(x) \left(D_W + i\mu_q \gamma_5 \tau_3 \right) \chi(x)$$

$$D_W = \frac{1}{2} \left(\gamma_\mu (\nabla_\mu + \nabla_\mu^*) - a \nabla_\mu^* \nabla_\mu \right) + m_0$$

(m_0 : untwisted mass; μ_q : twisted mass; τ_3 : third Pauli matrix acting in flavor space).

- Relation between the physical basis ψ and the twisted basis χ (in the continuum):

$$\psi = \frac{1}{\sqrt{2}} \left(\cos(\omega/2) + i \sin(\omega/2) \gamma_5 \tau_3 \right) \chi$$

$$\bar{\psi} = \frac{1}{\sqrt{2}} \bar{\chi} \left(\cos(\omega/2) + i \sin(\omega/2) \gamma_5 \tau_3 \right)$$

(ω : twist angle; $\omega = \pi/2$: maximal twist).

Simulation setup (B)

- $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0040$
 - lattice spacing $a \approx 0.079$ fm
 - lattice extension $L \approx 1.90$ fm
 - pion mass $m_{\text{PS}} \approx 340$ MeV.
- Inversions/contractions on 210 gauge configurations for light u/d quarks.
- 12 u and 12 d inversions per gauge configuration (stochastic timeslice sources located on the same timeslice).
- APE smearing of spatial links and Gaussian smearing of light quark fields to “optimize” the ground state overlap of trial states.
- Wilson lines of static quarks are discretized by path ordered products of ordinary links (small separations) and HYP2 smeared links (large separations).

