The adjoint potential in the pseudoparticle approach: string breaking and Casimir scaling

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Outline

Part I: introduction to the pseudoparticle approach

- Basic principle, pseudoparticle ensembles and their building blocks.

Part II: adjoint string breaking in the pseudoparticle approach

- Pure Wilson loop static potentials, Casimir scaling.
- The static adjoint potential, string breaking.

Part I: introduction to the pseudoparticle approach

Basic principle (1)

- Pseudoparticle approach (PP approach; F. Lenz, M.W., 2005):
  - A numerical technique to approximate Euclidean path integrals.
  - In this talk: application to SU(2) Yang-Mills theory,

$$\Big\langle \mathcal{O} \Big\rangle = \frac{1}{Z} \int DA\, \mathcal{O}[A] e^{-S[A]}$$

$$S[A] = \frac{1}{4g^2} \int d^4x\, F^a_{\mu\nu} F^a_{\mu\nu} \quad , \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu .$$

  - Goals:
    - ∗ Build a model for SU(2) Yang-Mills theory with a small number of physically relevant degrees of freedom.
    - ∗ Analyze the importance of certain classes of gauge field configurations with respect to confinement and other essential properties of SU(2) Yang-Mills theory.

Basic principle (2)

- Related work:
  - Ensembles of regular gauge instantons and merons (F. Lenz, J. W. Negele, M. Thies, 2003).
  - Ensembles of calorons with non-trivial holonomy (P. Gerhold, E.-M. Ilgenfritz, M. Müller-Preussker, 2006).
  - Ensembles of dyons (D. Diakonov, V. Petrov, 2007).

Basic principle (3)

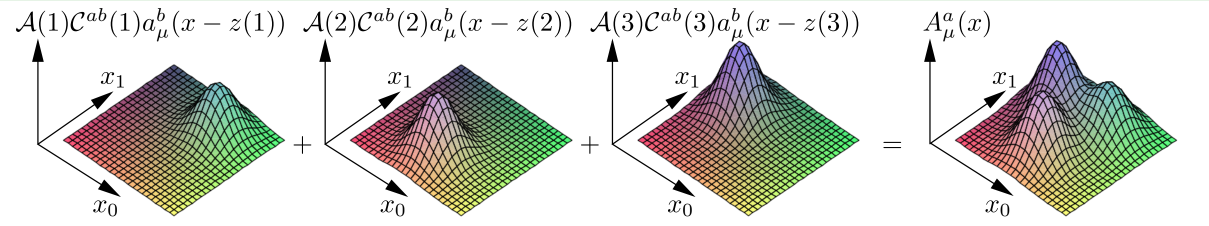
- PP: any gauge field configuration a_μ^a, which is localized in space and in time.
- Consider only those gauge field configurations, which can be written as a sum of a fixed number of PPs:

$$A_\mu^a(x) \quad = \quad \sum_j \mathcal{A}(j)\mathcal{C}^{ab}(j)a_\mu^b(x - z(j))$$

(j: PP index; $\mathcal{A}(j) \in \mathbb{R}$: amplitude of the j-th PP; $\mathcal{C}^{ab}(j) \in$ SO(3): color orientation of the j-th PP; $z(j) \in \mathbb{R}^4$: position of the j-th PP).

- Define the functional integration as an integration over PP amplitudes and color orientations:

$$\int DA \ldots \quad \rightarrow \quad \int \left(\prod_j d\mathcal{A}(j)d\mathcal{C}(j)\right) \ldots$$



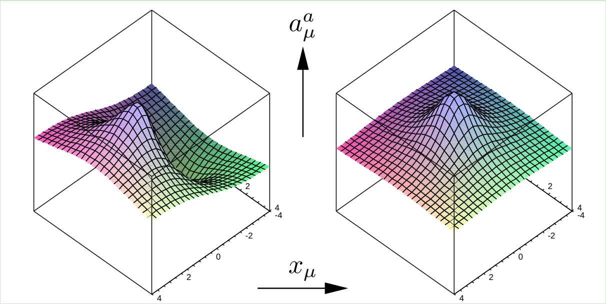
Basic principle (4)

- PP ensemble: 625 long range PPs ("instantons", "antiinstantons", akyrons) inside a periodic spacetime hypercube (extension 5.0^4):

$$a^a_{\mu,\mathrm{instanton}}(x) = \eta^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2}$$

$$a^a_{\mu,\mathrm{antiinstanton}}(x) = \bar{\eta}^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2}$$

$$a^a_{\mu,\mathrm{akyron}}(x) = \delta^{a1} \frac{x_\mu}{x^2 + \lambda^2},$$



i.e.

$$A^a_\mu(x) = \sum_i \mathcal{A}(i) \mathcal{C}^{ab}(i) a^b_{\mu,\mathrm{instanton}}(x - z(i)) + \sum_j \mathcal{A}(j) \mathcal{C}^{ab}(j) a^b_{\mu,\mathrm{antiinstanton}}(x - z(j)) + \sum_k \mathcal{A}(k) \mathcal{C}^{ab}(k) a^b_{\mu,\mathrm{akyron}}(x - z(k)).$$

Basic principle (5)

- **A_μ^a is not a classical solution (not even close to a classical solution), i.e. the PP approach is not a semiclassical method.**
  - Long range interactions between PPs.
  - Variable amplitudes $\mathcal{A}(i)$.

Part II: adjoint string breaking in the pseudoparticle approach

String breaking

- **Static potential $V(R)$:** the energy of the lowest state containing two static charges ϕ and $\phi^\dagger$ ("two infinitely heavy quarks") at separation R (+ light particles [gluons, dynamical quarks, ...]).
- **String breaking in QCD:** when two static charges are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by dynamical quarks; a pair of (essentially) non-interacting static-light mesons is formed.
- **String breaking in pure SU(2) Yang-Mills theory:**
  - Fundamental representation ($\phi^{(1/2)} = (\phi_1, \phi_2)$): no string breaking, since gluons are not able to screen charges in the fundamental representation.
  - Adjoint representation ($\phi^{(1)} = (\phi_1, \phi_2, \phi_3)$ or $\phi^{(1)} = \phi_a \sigma_a / 2$): when two static charges are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by gluons; a pair of (essentially) non-interacting gluelumps is formed.

Pure Wilson loop static potentials (1)

- The starting point to extract the energy of the lowest state containing two static charges $\phi^{(J)}$ and $(\phi^{(J)})^\dagger$ in spin-J-representation are "string trial states"

$$S^{(J)}(\mathbf{x},\mathbf{y})|\Omega\rangle \quad = \quad (\phi^{(J)}(\mathbf{x}))^\dagger U^{(J)}(\mathbf{x};\mathbf{y})\phi^{(J)}(\mathbf{y})|\Omega\rangle \quad , \quad |\mathbf{x}-\mathbf{y}| \quad = \quad R.$$

- We consider temporal correlations

$$\mathcal{C}^{(J)}_{\mathrm{string}}(T) \quad = \quad \langle\Omega|\Big(S^{(J)}(\mathbf{x},\mathbf{y},T)\Big)^\dagger S^{(J)}(\mathbf{x},\mathbf{y},0)|\Omega\rangle \quad = \quad \Big\langle W^{(J)}_{(R,T)}\Big\rangle$$

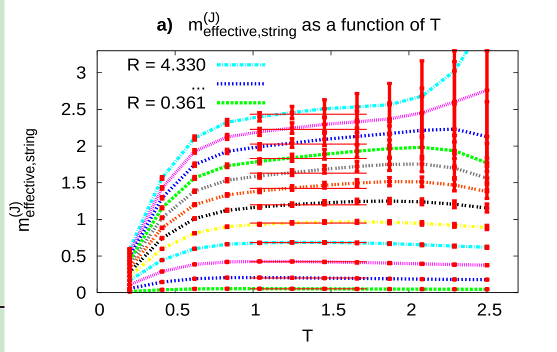
and compute the corresponding potential values from effective mass plateaus,

$$m^{(J)}_{\mathrm{effective,string}}(T) \quad = \quad -\frac{1}{a}\ln\frac{\mathcal{C}^{(J)}_{\mathrm{string}}(T)}{\mathcal{C}^{(J)}_{\mathrm{string}}(T-a)}.$$

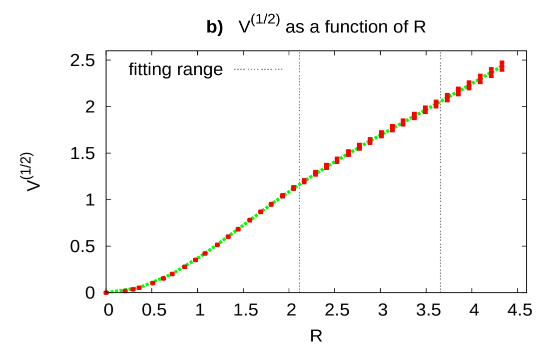
Pure Wilson loop static potentials (2)

- Numerical results for the fundamental representation ($J = 1/2$):
  - The potential is linear for large separations, i.e. confinement.
  - The scale is set by fitting $V^{(1/2)}(R) = V_0 + \sigma R$ and by identifying σ with $\sigma_{\text{physical}} = 4.2/\text{fm}^2$.
  - A coupling constant of $g = 12.5$ (standard choice for results shown in this talk) corresponds to a spacetime region of $L^4 = (1.85\,\text{fm})^4$.
  - Like in lattice gauge theory, the scale can be changed by changing the value of g, e.g. $g = 9.5 \ldots 18.5$ corresponds to $L = 1.55\,\text{fm} \ldots 2.31\,\text{fm}$.

effective masses



fundamental potential

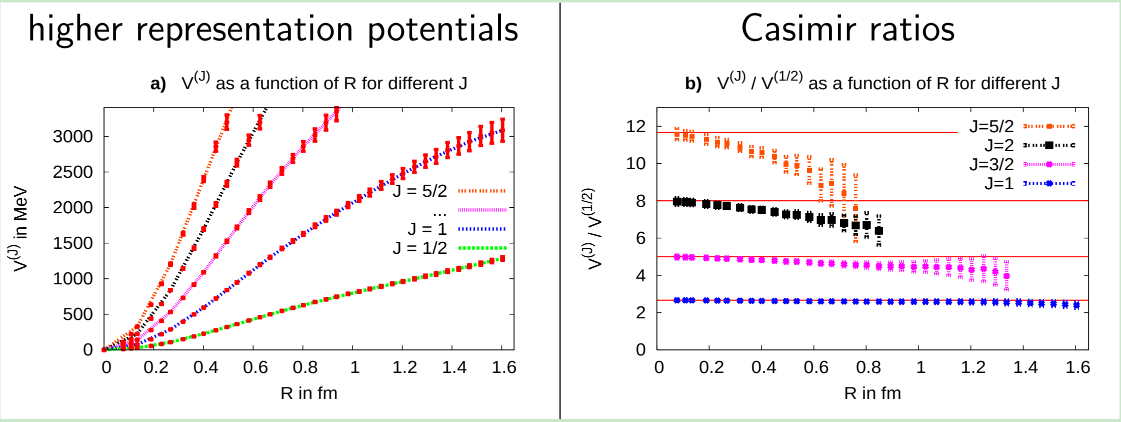


Pure Wilson loop static potentials (3)

- Numerical results for higher representations ($J = 1$, ..., $J = 5/2$):
  - Higher representation potentials exhibit Casimir scaling:

$$V^{(1/2)}(R) \approx \frac{V^{(1)}(R)}{8/3} \approx \frac{V^{(3/2)}(R)}{5} \approx \frac{V^{(2)}(R)}{8} \approx \frac{V^{(5/2)}(R)}{35/3}.$$

  - The adjoint potential ($J = 1$) shows no sign of string breaking even at separations $R \approx 1.6\,\mathrm{fm}$.



Why is string breaking elusive?

- Static potential at small separations R: the string trial state has good overlap to the physical ground state, which is expected to be a string state.
- Static potential at large separations R: the string trial state has poor overlap to the physical ground state, which is expected to be a two gluelump state.
- Solution: Extend the set of trial states by "two-gluelump trial states", which are supposed to have good overlap to the physical ground state at large R,

$$\sum_{j=x,y,z} G_j^{(\ldots)}(\mathbf{x}) G_j^{(\ldots)}(\mathbf{y}) |\Omega\rangle \quad , \quad |\mathbf{x} - \mathbf{y}| \ = \ R$$

with

$$G_j^{(J=1,P=+)}(\mathbf{x}) \ = \ \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) B_j(\mathbf{x})\Big) \quad \text{(magnetic gluelump)}$$

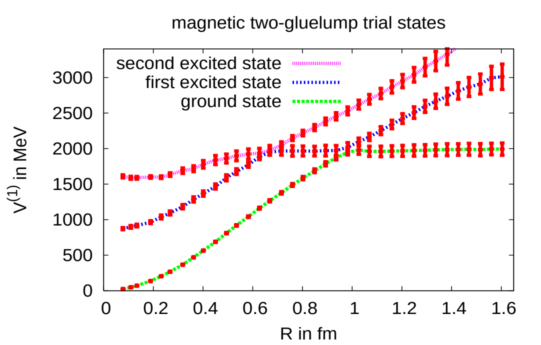
$$G_j^{(J=1,P=-)}(\mathbf{x}) \ = \ \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) \epsilon_{jkl} D_k B_l(\mathbf{x})\Big) \quad \text{(electric gluelump)}$$

$$G_j^{(J=2,P=-)}(\mathbf{x}) \ = \ \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) |\epsilon_{jkl}| D_k B_l(\mathbf{x})\Big).$$

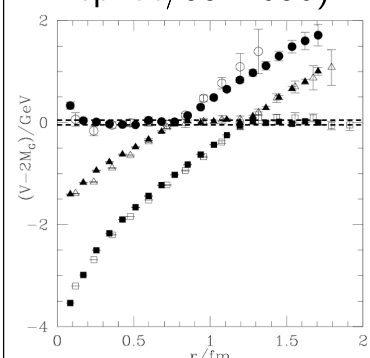
The static adjoint potential (1)

- We extract the adjoint potential from correlation matrices containing both string trial states and magnetic two gluelump trial states via effective masses.
  - The potential saturates at $V^{(1)} \approx 2m^{(J=1,P=+)}$.
  - The string breaking distance $R_{\mathrm{sb}}^{(J=1,P=+)} \approx 1.0\,\mathrm{fm}$ and the level ordering are in qualitative agreement with lattice results ($R_{\mathrm{sb,lattice}}^{(J=1,P=+)} = 1.0\,\mathrm{fm} \ldots 1.25\,\mathrm{fm}$).

magnetic trial states



lattice result (P. de Forcrand, O. Philipsen, hep-lat/9912050)



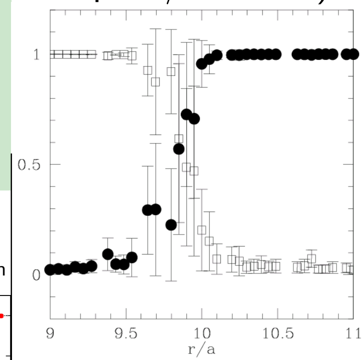
The static adjoint potential (2)

- Mixing analysis to investigate, whether the string really breaks:
  - During the computation of effective masses we obtain approximations

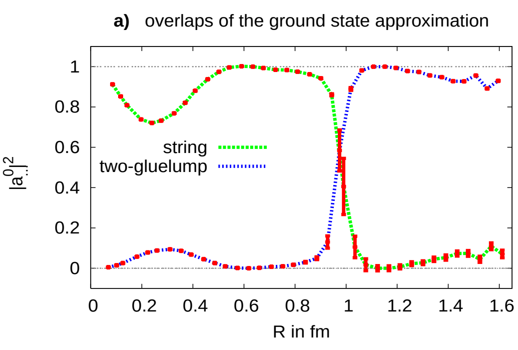
$$|0\rangle \approx a^0_{\text{string}}|\text{string}\rangle + a^0_{\text{two-gluelump}}|\text{two-gluelump}\rangle$$
$$|1\rangle \approx a^1_{\text{string}}|\text{string}\rangle + a^1_{\text{two-gluelump}}|\text{two-gluelump}\rangle,$$

    where $|\text{string}\rangle$ and $|\text{two-gluelump}\rangle$ are normalized trial states.
  - The amplitudes $a^j_{\ldots}$ indicate a smooth but rapid transition between string and two-gluelump states.

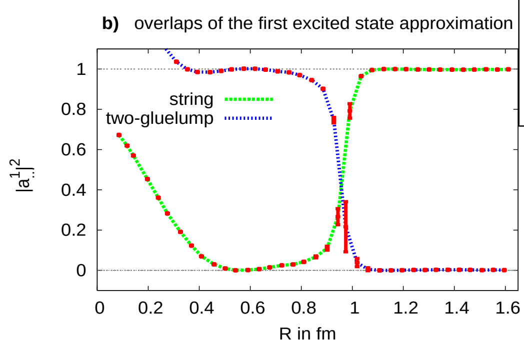
lattice result (P. de Forcrand, O. Philipsen, hep-lat/9912050)



ground state (magnetic)



first excited state (magnetic)



Summary

- The static potential in the fundamental representation is linear for large separations ($\rightarrow$ confinement).
- Static potentials in higher representations exhibit Casimir scaling.
- The static potential in the adjoint representation is in qualitative agreement with lattice results:
  - String breaking at $\approx 1.0\,\mathrm{fm}$.
  - Correct level ordering: the "first excited string state" is below the "two gluelump ground state".
  - A mixing analysis indicates a smooth transition between a string and a two gluelump state, when two static charges are separated adiabatically; the transition region is very narrow.

Outlook

- Apply the PP approach to fermionic theories:
  - First steps regarding the inclusion of fermionic fields in the PP approach have been successful (phase diagram of the GN model, M.W., 2007).
  - Apply the PP approach to QCD:
    * Cheap computation of exact all-to-all propagators.
    * Identification of properties of physically relevant fermionic field configurations.
  - Apply the PP approach to supersymmetric theories:
    * Exact supersymmetry is possible due to exact translational invariance.