Properties of confining gauge field configurations in the pseudoparticle approach

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Outline

PP = pseudoparticle

- Basic principle.
- Numerical results (static quark antiquark potential, topological susceptibility, critical temperature).
- Properties of confining gauge field configurations:
  - PPs of different size.
  - PPs with Gaussian profile.
  - Instantons, antiinstantons and akyrons.
- Summary.
- Outlook.

Basic principle (1)

- Pseudoparticle approach (PP approach):
  - A numerical technique to approximate Euclidean path integrals (in this talk: SU(2) Yang-Mills theory ≈ "QCD with infinitely heavy quarks"):

$$\Big\langle \mathcal{O} \Big\rangle = \frac{1}{Z} \int DA\, \mathcal{O}[A] e^{-S[A]}$$

$$S[A] = \frac{1}{4g^2} \int d^4x\, F_{\mu\nu}^a F_{\mu\nu}^a \quad , \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c .$$

  - A tool to analyze the importance of certain classes of gauge field configurations with respect to confinement.

Basic principle (2)

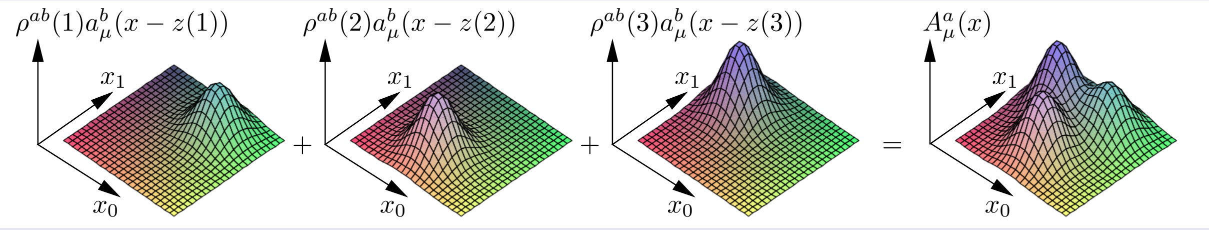
- PP: any gauge field configuration a^a_μ, which is localized in space and in time.
- Consider only those gauge field configurations, which can be written as a sum of a fixed number (≈ 400) of PPs:

$$A^a_\mu(x) \;=\; \sum_i \rho^{ab}(i) a^b_\mu(x - z(i)).$$

(i: PP index; $\rho^{ab}(i)$: degrees of freedom of the i-th PP, i.e. amplitude and color orientation; $z(i)$: position of the i-th PP).

- Approximate the path integral by an integration over PP degrees of freedom:

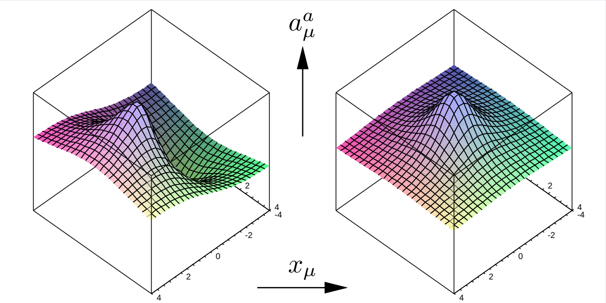
$$\int DA \ldots \quad \rightarrow \quad \int \left(\prod_i d\rho^{ab}(i) \right) \ldots$$



Building blocks of PP ensembles

- Building blocks of PP ensembles: "instantons", "antiinstantons", akyrons (λ: PP size).

$$
\begin{aligned}
a^a_{\mu,\mathrm{instanton}}(x) &= \eta^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2} \\
a^a_{\mu,\mathrm{antiinstanton}}(x) &= \bar{\eta}^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2} \\
a^a_{\mu,\mathrm{akyron}}(x) &= \delta^{a1} \frac{x_\mu}{x^2 + \lambda^2}.
\end{aligned}
$$



- Instantons, antiinstantons and akyrons form a basis of all gauge field configurations in the "continuum limit".

- Degrees of freedom: amplitudes $\mathcal{A}(i)$, color orientations $\mathcal{C}^{ab}(i)$, positions $z(i)$.

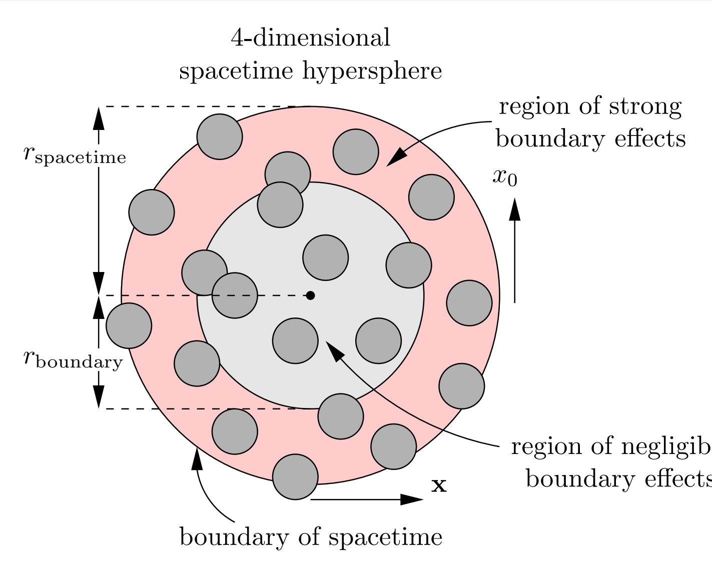
$$
\begin{aligned}
A^a_\mu(x) &= \mathcal{A}(i)\mathcal{C}^{ab}(i) a^a_{\mu,\mathrm{instanton}}(x - z(i)) \\
A^a_\mu(x) &= \mathcal{A}(i)\mathcal{C}^{ab}(i) a^a_{\mu,\mathrm{antiinstanton}}(x - z(i)) \\
A^a_\mu(x) &= \mathcal{A}(i)\mathcal{C}^{ab}(i) a^a_{\mu,\mathrm{akyron}}(x - z(i)).
\end{aligned}
$$

PP ensembles (1)

- PP ensemble: a fixed number of PPs inside a "spacetime hypersphere".
- Gauge field:

$$
\begin{aligned}
A_\mu^a(x) \;&=\; \sum_i \mathcal{A}(i)\mathcal{C}^{ab}(i) a^b_{\mu,\mathrm{instanton}}(x - z(i)) + \\
&\sum_j \mathcal{A}(j)\mathcal{C}^{ab}(j) a^b_{\mu,\mathrm{antiinstanton}}(x - z(j)) + \\
&\sum_k \mathcal{A}(k)\mathcal{C}^{ab}(k) a^b_{\mu,\mathrm{akyron}}(x - z(k)).
\end{aligned}
$$

- Choose color orientations $\mathcal{C}^{ab}(i)$ and positions $z(i)$ randomly.
- A_μ^a is no classical solution (not even close to a classical solution)!
- Long range interactions between PPs.



PP ensembles (2)

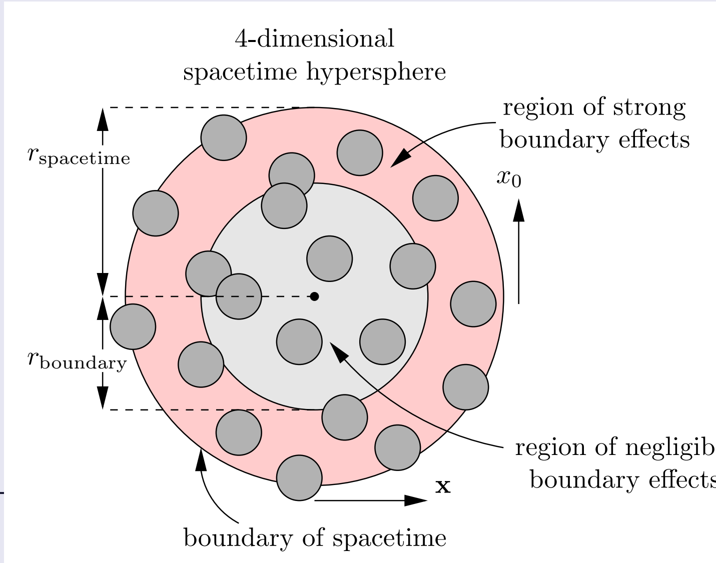
- Approximation of the path integral:

$$\left\langle \mathcal{O} \right\rangle \;=\; \frac{1}{Z} \int \left(\prod_i d\mathcal{A}(i) \right) \mathcal{O}(\mathcal{A}(i)) e^{-S(\mathcal{A}(i))}$$

  (integration over PP amplitudes).

- Solve this multidimensional integral via Monte-Carlo simulations.

- Exclude boundary effects: observables have to be "measured" sufficiently far away from the boundary.



Quark antiquark potential (1)

- Common tool to determine the potential of a static quark antiquark pair: Wilson loops (z: closed spacetime curve),

$$W_z[A] = \frac{1}{2}\mathrm{Tr}\left(P\left\{\exp\left(i\oint dz_\mu\, A_\mu(z)\right)\right\}\right).$$

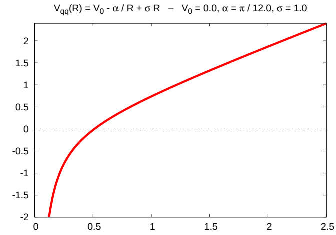
- Rectangular Wilson loop (R, T: spatial and temporal extension): $W_{(R,T)}$.
- Wilson loops $\leftrightarrow$ quark antiquark potential (R: quark antiquark separation):

$$V_{\mathrm{q}\bar{\mathrm{q}}}(R) = -\lim_{T\rightarrow\infty}\frac{1}{T}\ln\left\langle W_{(R,T)}\right\rangle.$$

- Assumption: the potential for large quark antiquark separations can be parameterized according to

$$V_{\mathrm{q}\bar{\mathrm{q}}}(R) = V_0 - \frac{\alpha}{R} + \sigma R.$$

$V_{\mathrm{q}\bar{\mathrm{q}}}$ plotted against R



Quark antiquark potential (2)

Method 1: Determine the string tension σ and the Coulomb coefficient α

- "Guess" the functional dependence of ensemble averages of Wilson loops:

$$-\ln\Big\langle W_{(R,T)}\Big\rangle \quad = \quad V_0\Big(R+T\Big) - \alpha\left(\frac{R}{T}+\frac{T}{R}\right) + \beta + \sigma RT.$$

- Determine the string tension σ and the Coulomb coefficient α by fitting the "Wilson loop ansatz" to Monte-Carlo data for $-\ln\langle W_{(R,T)}\rangle$.

- Several approaches:
  - Area perimeter fits.
  - Creutz ratios.
  - Generalized Creutz ratios.
  - ...

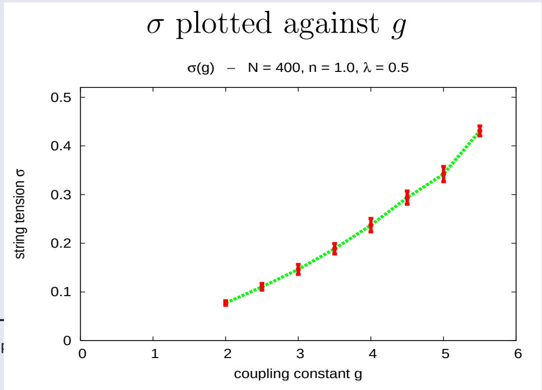
Quark antiquark potential (3)

Method 1: Determine the string tension σ and the Coulomb coefficient α

- Parameterization of the quark antiquark potential:

$$V_{q\bar{q}}(R) \quad = \quad V_0 - \frac{\alpha}{R} + \sigma R.$$

- Results for PP ensembles containing ≈ 400 PPs:
  - String tension $\sigma > 0$
    $\longrightarrow$ linear potential for large quark antiquark separations, confinement.
  - σ is an increasing function of the coupling constant g
    $\longrightarrow$ adjust the physical scale by choosing appropriate values for g.
  - Coulomb coefficient $\alpha > 0$
    $\longrightarrow$ attractive "Coulomb-like" correction (as predicted by the bosonic string picture and by lattice calculations).



Quark antiquark potential (4)

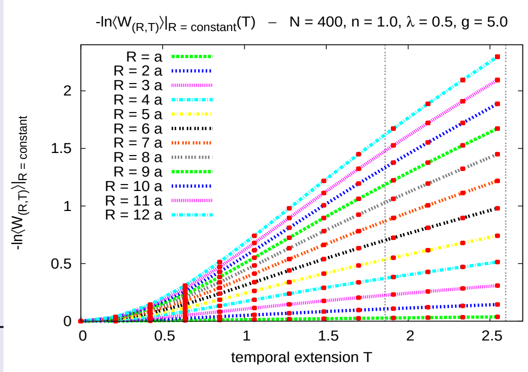
Method 2: Calculate the quark antiquark potential directly

- For large T:

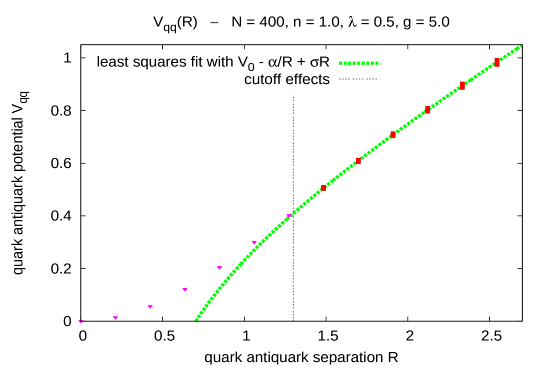
$$V_{q\bar{q}}(R)T \quad \approx \quad -\ln\left\langle W_{(R,T)}\right\rangle.$$

- From the slope of $-\ln\langle W_{(R,T)}\rangle|_{R=\text{constant}}$ we can read off $V_{q\bar{q}}(R)$.
- Results are in agreement with our previous results.

$-\ln\langle W_{(R,T)}\rangle|_{R=\text{constant}}$ plotted against T



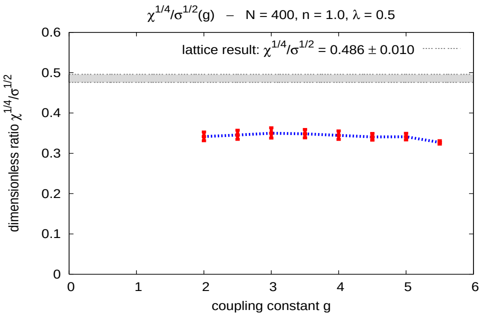
$V_{q\bar{q}}$ plotted against R



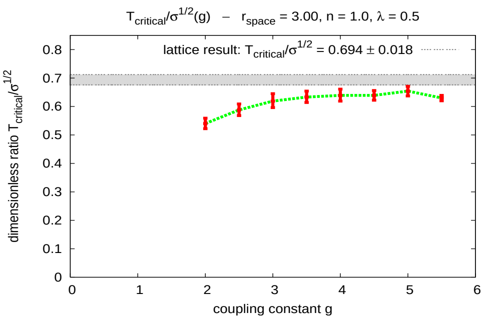
Quantitative results

- For quantitative results, including the string tension, we need other dimensionful quantities:
  - Topological susceptibility $\chi = \langle Q_V^2 \rangle / V$.
  - Critical temperature of the confinement deconfinement phase transition T_{critical}.
- Dimensionless quantities (physically meaningful): $\chi^{1/4}/\sigma^{1/2}$, $T_{\mathrm{critical}}/\sigma^{1/2}$.
- Consider different $g = 2.0 \ldots 5.5$ (diameter of the spacetime hypersphere $0.8\,\mathrm{fm} \ldots 1.9\,\mathrm{fm}$).
- Results are in qualitative agreement with results from lattice calculations.
- Consistent scaling behavior of σ, χ and T_{critical}.

$\chi^{1/4}/\sigma^{1/2}$ plotted against g



$T_{\mathrm{critical}}/\sigma^{1/2}$ plotted against g



Properties of confining gauge field ...

- What are essential properties of confining gauge field configurations?
- Which gauge field configurations are responsible for confinement?
- Apply the PP approach with different types of PPs to study the effect of different classes of gauge field configurations on confinement:
  - PPs of different size.
  - PPs with a limited range of interaction (PPs with Gaussian profile).
  - PPs without topological charge (akyrons).

PPs of different size (1)

- Consider ensembles with PPs of different size λ:

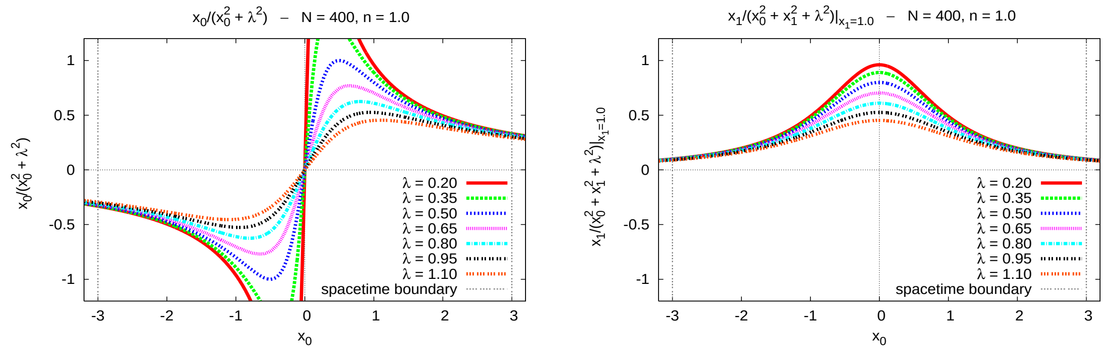
$$a^a_{\mu,\text{instanton}}(x) = \eta^a_{\mu\nu}\frac{x_\nu}{x^2+\lambda^2} \quad , \quad a^a_{\mu,\text{antiinstanton}}(x) = \bar{\eta}^a_{\mu\nu}\frac{x_\nu}{x^2+\lambda^2} \quad ,$$

$$a^a_{\mu,\text{akyron}}(x) = \delta^{a1}\frac{x_\mu}{x^2+\lambda^2}.$$

- λ strongly affects the shape of a PP near its center but has essentially no effect on the $1/|x|$ long range behavior.

- Typical PP profiles:

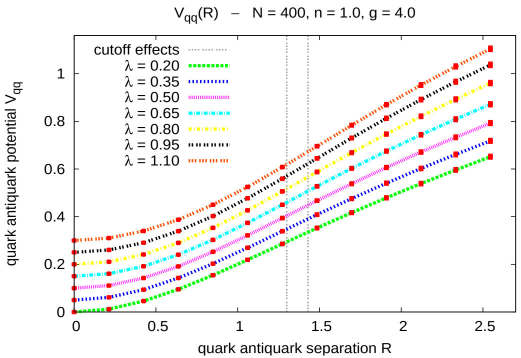
$x_0/(x_0^2+\lambda^2)$ plotted against x_0 $\quad$ $x_1/(x_0^2+x_1^2+\lambda^2)|_{x_1=1.0}$ plotted against x_0



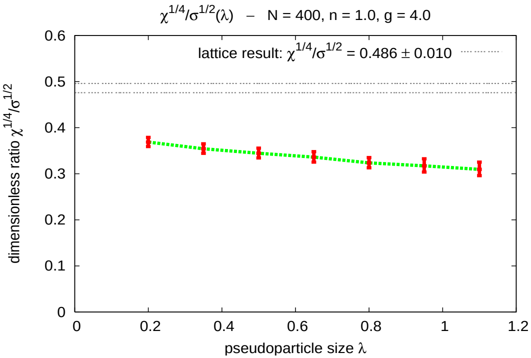
PPs of different size (2)

- $\lambda = 0.2, \ldots, 1.1$.
- The $q\bar{q}$ potential and the dimensionless ratio $\chi^{1/4}/\sigma^{1/2}$ are essentially unaffected by the PP size λ.

$V_{q\bar{q}}$ plotted against R



$\chi^{1/4}/\sigma^{1/2}$ plotted against λ

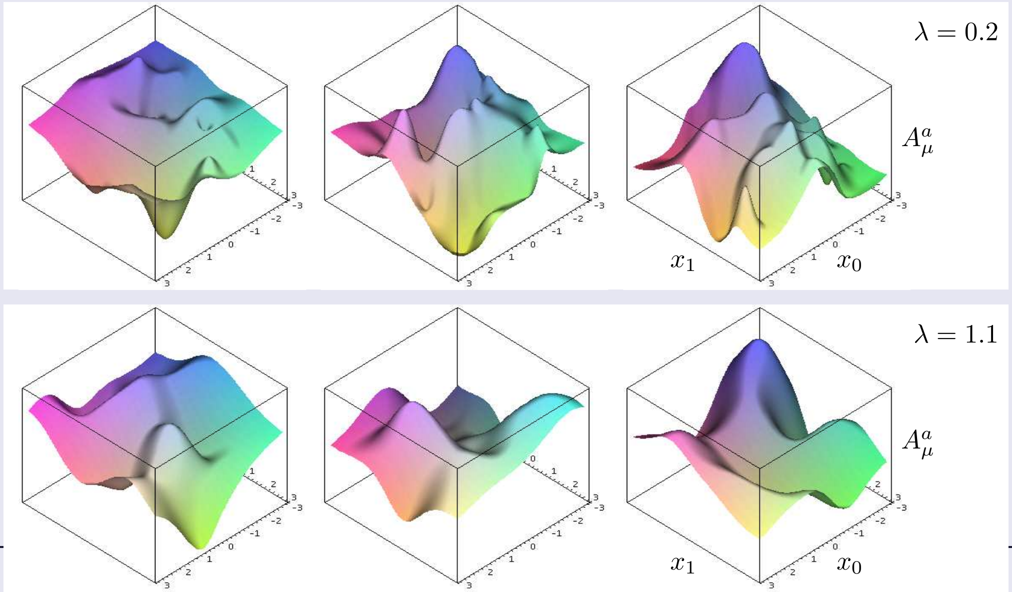


$\rightarrow$ Confinement is a consequence of the $1/|x|$ long range behavior of the PPs, which is unaffected by the size parameter λ.

PPs of different size (3)

Typical gauge field configurations (PP size $\lambda = 0.2 \leftrightarrow \lambda = 1.1$)

- The global structure of typical gauge field configurations is the same.
- For $\lambda = 0.2$ there are additional local UV fluctuations. These UV fluctuations have no effect on confinement and the string tension.



PPs with Gaussian profile (1)

- Consider ensembles with Gaussian localized PPs of different size λ:

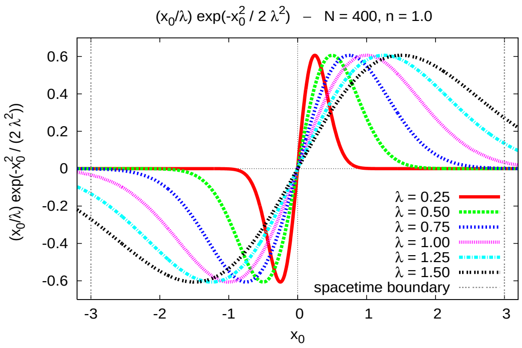
$$a^a_{\mu,\text{instanton}}(x) = \eta^a_{\mu\nu} x_\nu e^{-x^2/2\lambda^2} \quad , \quad a^a_{\mu,\text{antiinstanton}}(x) = \bar{\eta}^a_{\mu\nu} x_\nu e^{-x^2/2\lambda^2} \quad ,$$

$$a^a_{\mu,\text{akyron}}(x) = \delta^{a1} x_\mu e^{-x^2/2\lambda^2}.$$

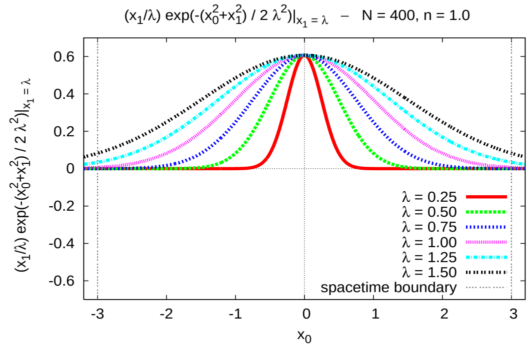
- Gaussian localized PPs have a limited range of interaction, which is proportional to their size λ.

- Typical PP profiles:

$(x_0/\lambda)\exp(-x_0^2/2\lambda^2)$ plotted against x_0

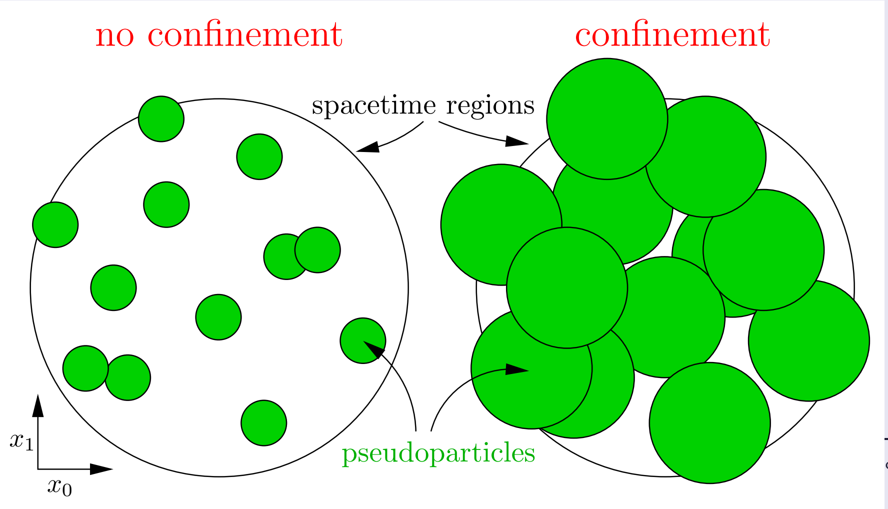


$(x_1/\lambda)\exp(-(x_0^2+x_1^2)/2\lambda^2)|_{x1=\lambda}$ plotted against x_0

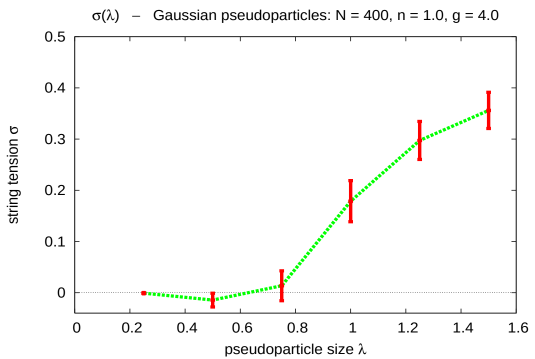


PPs with Gaussian profile (2)

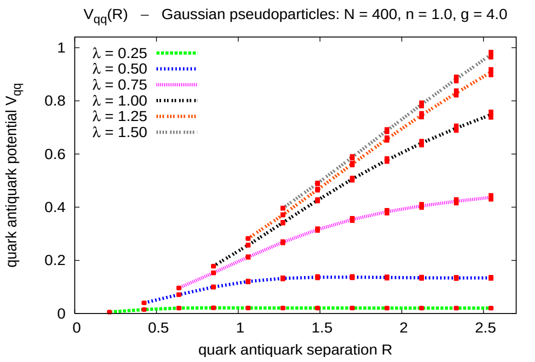
- Short range PPs ($\lambda \leq 0.50$)
  $\longrightarrow$ little overlap between neighboring PPs.
  $\longrightarrow$ no confinement.
- Long range PPs ($\lambda \geq 1.00$)
  $\longrightarrow$ significant overlap between neighboring PPs.
  $\longrightarrow$ confinement.
- PP percolation $\longleftrightarrow$ confinement.



σ plotted against λ



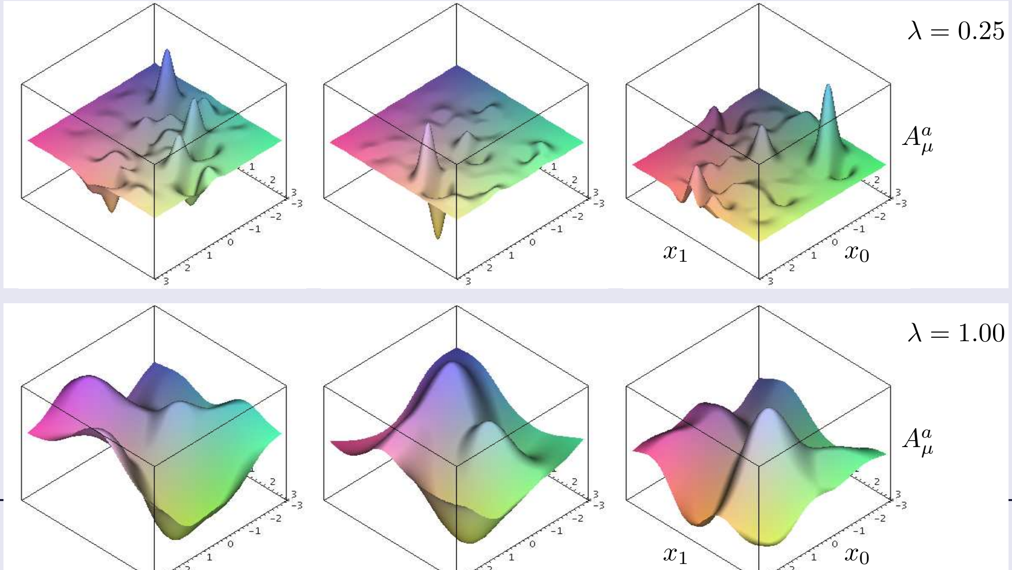
$V_{q\bar{q}}$ plotted against R



PPs with Gaussian profile (3)

Typical gauge field configurations ($\lambda = 0.25 \leftrightarrow \lambda = 1.00$)

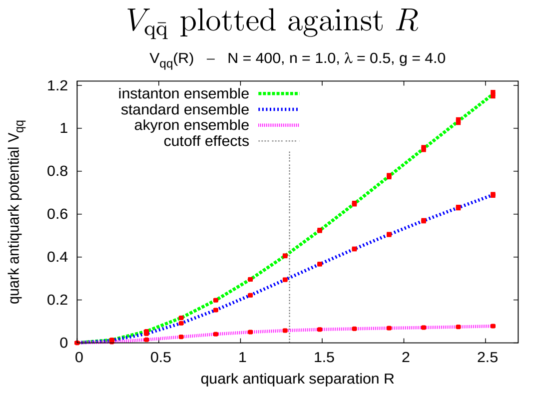
- $\lambda = 0.25$: local UV fluctuations $\rightarrow$ no confinement.
- $\lambda = 1.00$: global excitations $\rightarrow$ confinement.
- Gauge field configurations responsible for confinement contain extended structures and large area excitations.



Instantons, antiinstantons and akyrons

- Consider the following ensembles:
  - Akyron ensemble: 400 akyrons (topological charge density $q = 0$).
  - Standard ensemble: 150 instantons, 150 antiinstantons, 100 akyrons.
  - Instanton ensemble: 200 instantons, 200 antiinstantons.
- No confinement in the akyron ensemble
  $\longrightarrow$ akyrons alone are not suited to reproduce Yang-Mills physics.
  $\longrightarrow$ supports the common expectation that confinement and topological charge are closely related.
- Standard ensemble $\leftrightarrow$ instanton ensemble
  $\longrightarrow (\chi^{1/4}/\sigma^{1/2})_{\text{standard}} = 0.35$
  $(\chi^{1/4}/\sigma^{1/2})_{\text{instanton}} = 0.26$

  $(\chi^{1/4}/\sigma^{1/2})_{\text{lattice}} = 0.49.$
  $\longrightarrow$ using akyrons is beneficial with respect to quantitative results.



Summary

- The PP approach with ≈ 400 instantons, antiinstantons and akyrons is able to reproduce many essential features of SU(2) Yang-Mills theory:
  - Linear quark antiquark potential for large separations $\rightarrow$ confinement.
  - Consistent scaling behavior of σ, χ and T_{critical}.
  - Dimensionless quantities $\chi^{1/4}/\sigma^{1/2}$, $T_{\mathrm{critical}}/\sigma^{1/2}$ are in qualitative agreement with results from lattice calculations.
- Essential properties of confining gauge field configurations:
  - Long range PPs necessary for confinement (PP percolation) $\rightarrow$ confinement $\leftrightarrow$ extended structures and large area excitations.
  - Instantons and antiinstantons (PPs with non-vanishing topological charge density) necessary for confinement $\rightarrow$ confinement $\leftrightarrow$ topological charge.
  - Short range behavior of PPs irrelevant for confinement $\rightarrow$ confinement not affected by UV fluctuations.

Outlook

- Calculate other observables:
  - Glueball masses.
  - Energy density and pressure at finite temperature.
  - Free energy of a static quark antiquark pair at finite temperature.
- Include fermions in the pseudoparticle approach (a model, which exhibits both chiral symmetry breaking and a confinement deconfinement phase transition).