Scalar mesons and tetraquarks by means of lattice QCD

Quark Confinement and the Hadron Spectrum X – Munich, Germany

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Introduction, motivation (1)

- The nonet of light scalar mesons ($J^P = 0^+$)
  - $\sigma \equiv f_0(500)$, $I = 0$, $400 \ldots 550\,\text{MeV}$,
  - $\kappa \equiv K_0^*(800)$, $I = 1/2$, $682 \pm 29\,\text{MeV}$,
  - $a_0(980)$, $f_0(980)$, $I = 1$, $980 \pm 20\,\text{MeV}$, $990 \pm 20\,\text{MeV}$

  is poorly understood:
  - All nine states are unexpectedly light (should rather be close to the corresponding $J^P = 1^+, 2^+$ states around $1200 \ldots 1500\,\text{MeV}$).
  - The ordering of states is inverted compared to expectation:
    * E.g. in a $q\bar{q}$ picture the $I = 1$ states $a_0(980)$, $f_0(980)$ must necessarily be formed by two u/d quarks, while the $I = 1/2$ κ states are made from an s and a u/d quark; since $m_s > m_{u/d}$ one would expect $m(\kappa) > m(a_0(980)), m(f_0(980))$.

Introduction, motivation (2)

  * In a tetraquark picture the quark content could be the following: $\kappa \equiv \bar{s}l\bar{l}l$, while $a_0(980)$, $f_0(980) \equiv \bar{s}l\bar{l}s$; this would naturally explain the observed ordering.
- Certain decays also support a tetraquark interpretation: e.g. $a_0(980)$ readily decays to $K + \bar{K}$, which indicates that besides the two light quarks required by $I = 1$ also an $s\bar{s}$ pair is present.

$\rightarrow$ Study these states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.

- Examples of heavy mesons, which are tetraquark candidates:
  - $D_{s0}^*(2317)^\pm$ $(I(J^P) = 0(0^+))$, $D_{s1}(2460)^\pm$ $(I(J^P) = 0(1^+))$,
    [talk by Graham Moir (Tue 14:30), poster by Martin Kalinowski]
  - charmonium states $X(3872)$, $Z(4430)^\pm$, $Z(4050)^\pm$, $Z(4250)^\pm$, ...

Lattice QCD hadron spectroscopy (1)

- Lattice QCD: discretized version of QCD,

$$S = \int d^4x \, \bigg(\sum_{\psi \in \{u,d,s,c,t,b\}} \bar{\psi}\Big(\gamma_\mu\Big(\partial_\mu - iA_\mu\Big) + m^{(\psi)}\Big)\psi + \frac{1}{2g^2}\mathrm{Tr}\Big(F_{\mu\nu}F_{\mu\nu}\Big)\bigg)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- Let $\mathcal{O}$ be a suitable "hadron creation operator", i.e. an operator formed by quark fields ψ and gluonic fields A_μ such that $\mathcal{O}|\Omega\rangle$ is a state containing the hadron of interest ($|\Omega\rangle$: QCD vacuum).

- More precisely: ... an operator such that $\mathcal{O}|\Omega\rangle$ has the same quantum numbers ($J^{\mathcal{PC}}$, flavor) as the hadron of interest.

- Examples:
  - Pion creation operator: $\mathcal{O} = \int d^3x \, \bar{u}(\mathbf{x})\gamma_5 d(\mathbf{x})$.
  - Proton creation operator: $\mathcal{O} = \int d^3x \, \epsilon^{abc} u^a(\mathbf{x})(u^{b,T}(\mathbf{x}) C\gamma_5 d^c(\mathbf{x}))$.

Lattice QCD hadron spectroscopy (2)

- Determine the mass of the ground state of the hadron of interest from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times T:

$$\mathcal{C}(t) \quad = \quad \langle\Omega|\Big(\mathcal{O}(t)\Big)^\dagger\mathcal{O}(0)|\Omega\rangle \quad = \quad \langle\Omega|e^{+Ht}\Big(\mathcal{O}(0)\Big)^\dagger e^{-Ht}\mathcal{O}(0)|\Omega\rangle \quad =$$

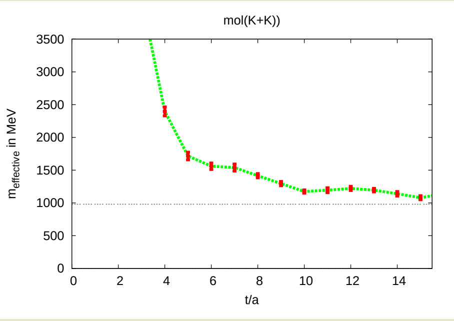
$$= \quad \sum_n \Big|\langle n|\mathcal{O}(0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)t\Big) \quad \approx \quad (\text{for } \text{“}t \gg 1\text{”})$$

$$\approx \quad \Big|\langle 0|\mathcal{O}(0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{m(\text{hadron})}t\Big).$$

- Usually the exponent is determined by identifying the “plateaux-value” of a so-called effective mass:

$$m_{\text{effective}}(t) \quad = \quad \frac{1}{a}\ln\left(\frac{\mathcal{C}(t)}{\mathcal{C}(t+a)}\right) \quad \approx \quad (\text{for } \text{“}t \gg 1\text{”})$$

$$\approx \quad E_0 - E_\Omega \quad = \quad m(\text{hadron}).$$



Tetraquark creation operators

- At the moment we study
  - $a_0(980)$, mass $980 \pm 20\,\text{MeV}$, quantum numbers $I(J^{PC}) = 1(0^{++})$;
  - $\kappa \equiv K_0^*(800)$, mass $682 \pm 29\,\text{MeV}$, quantum numbers $I(J^P) = 1/2(0^+)$.
- Tetraquark operators for $a_0(980)$ (quantum numbers $I(J^{PC}) = 1(0^{++})$):
  - Needs two light quarks due to $I = 1$, e.g. $u\bar{d}$.
  - $a_0(980)$ decays to $K\bar{K}$ ... suggests an $s\bar{s}$ component.
  - Molecule type (models a bound $K\bar{K}$ state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \int d^3x \, \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big).$$

  - Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \int d^3x \, \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

Lattice setup (1)

- Gauge link configurations generated by ETMC.
- 2+1+1 dynamical quark flavors, i.e. u, d, s and c sea quarks.
- Lattice spacing $a = 0.086\,\text{fm}$ (rather fine, computations at even finer lattice spacings planned).
- Various lattice volumes:
  - Small volume $L^3 \times T = 20^3 \times 48$ lattice sites, spatial extension $1.73\,\text{fm}$ $\rightarrow$ rather easy to identify momentum excitations. (Most of the numerical results shown in the following were obtained with this volume.)
  - ...
  - Large volume $L^3 \times T = 32^3 \times 64$ lattice sites, spatial extension $2.75\,\text{fm}$ $\rightarrow$ less finite size effects.
  - Different volumes needed to study resonances in a rigorous way. (Not done yet ... will be one of our next steps.)

Lattice setup (2)

- Various light u/d quark masses, corresponding pion masses $m_{\mathrm{PS}} \approx 280 \ldots 460\,\mathrm{MeV}$ (physical light u/d quark masses $[m_{\mathrm{PS}} = m_\pi \approx 140\,\mathrm{MeV}]$ are technically extremely challenging; because of that in lattice QCD one usually studies several heavier quark masses and extrapolates to the "physical point").

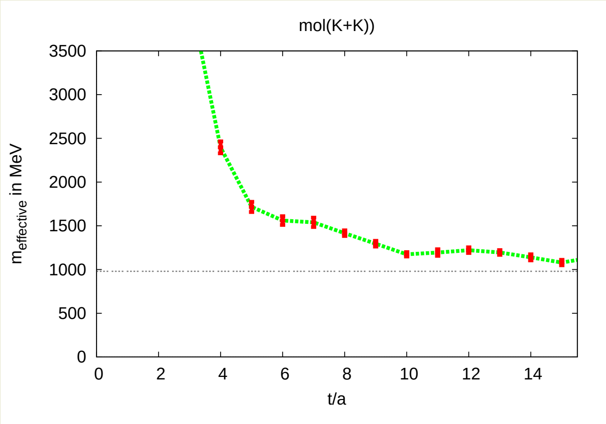
Numerical results $a_0(980)$ (1)

- Effective mass, molecule type operator:

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big).$$

- The effective mass plateaux indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20\,\text{MeV}$.

- Conclusion: $a_0(980)$ is a tetraquark state of $K\bar{K}$ molecule type ...?



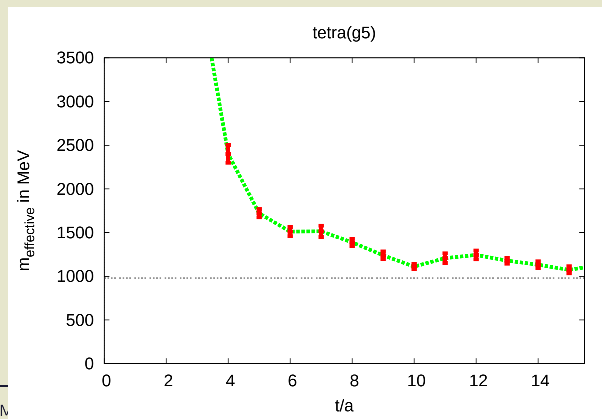
Numerical results $a_0(980)$ (2)

- Effective mass, diquark type operator:

$$\mathcal{O}^{\text{diquark}}_{a_0(980)} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

- The effective mass plateaux indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20\,\text{MeV}$.

- Conclusion: $a_0(980)$ is a tetraquark state of diquark type ...? Or a mixture of $K\bar{K}$ molecule and tetraquark?



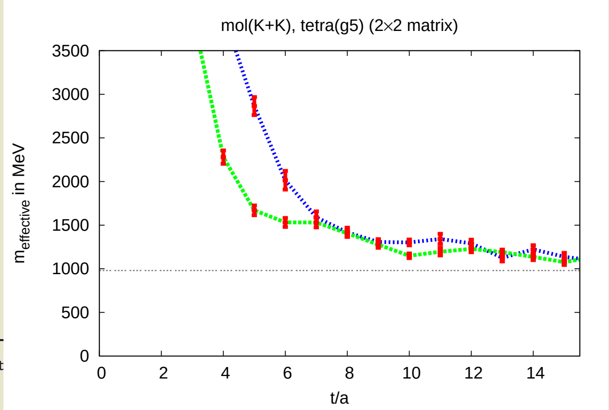
Numerical results $a_0(980)$ (3)

- Study both operators at the same time, extract the two lowest energy eigenstates by diagonalizing a 2×2 correlation matrix ("generalized eigenvalue problem"):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big)$$

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

- Now two orthogonal states roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20\,\text{MeV}$ ...?



Two-particle creation operators (1)

- Explanation: there are two-particle states, which have the same quantum numbers as $a_0(980)$, $I(J^{PC}) = 1(0^{++})$,
  - $K + \bar{K}$ ($m(K) \approx 500\,\text{MeV}$),
  - $\eta_s + \pi$ ($m(\eta_s \equiv \bar{s}\gamma_5 s) \approx 700\,\text{MeV}$, $m(\pi) \approx 300\,\text{MeV}$ in our lattice setup),

  which are both around the expected $a_0(980)$ mass $980 \pm 20\,\text{MeV}$.

- What we have seen in the previous plots might actually be two-particle states (our operators are of tetraquark type, but they nevertheless generate overlap [possibly small, but certainly not vanishing] to two-particle states).

- To determine, whether there is a bound $a_0(980)$ tetraquark state, we need to resolve the above listed two-particle states $K + \bar{K}$ and $\eta_s + \pi$ and check, whether there is an additional 3rd state in the mass region around $980 \pm 20\,\text{MeV}$; to this end we need operators of two-particle type.

Two-particle creation operators (2)

- Two-particle operators with quantum numbers $I(J^{PC}) = 1(0^{++})$:
  - Two-particle $K + \bar{K}$ type:

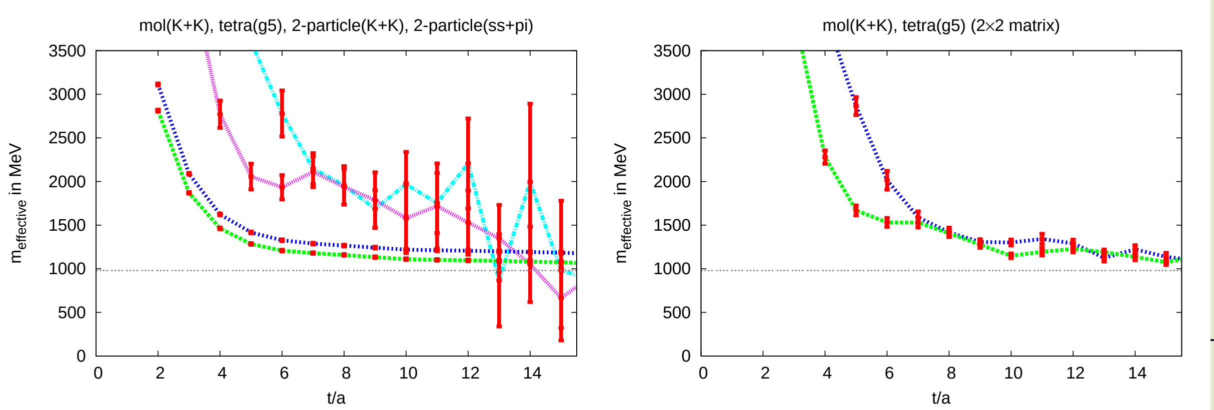
$$\mathcal{O}_{a_0(980)}^{K+\bar{K}\text{ two-particle}} = \Big(\sum_{\mathbf{x}} \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\sum_{\mathbf{y}} \bar{d}(\mathbf{y})\gamma_5 s(\mathbf{y})\Big).$$

  - Two-particle $\eta_s + \pi$ type:

$$\mathcal{O}_{a_0(980)}^{\eta_s+\pi\text{ two-particle}} = \Big(\sum_{\mathbf{x}} \bar{s}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big)\Big(\sum_{\mathbf{y}} \bar{d}(\mathbf{y})\gamma_5 u(\mathbf{y})\Big).$$

Numerical results $a_0(980)$ (4)

- Study all four operators ($K\bar{K}$ molecule, diquark, $K + \bar{K}$ two-particle, $\eta_s + \pi$ two-particle) at the same time, extract the four lowest energy eigenstates by diagonalizing a 4×4 correlation matrix (left plot).
  - Still only two low-lying states around 980 ± 20 MeV, the 2nd and 3rd excitation are ≈ 750 MeV heavier.
  - The signal of the low-lying states is of much better quality than before (when we only considered tetraquark operators)
    $\longrightarrow$ suggests that the observed low-lying states have much better overlap to the two-particle operators and are most likely of two-particle type.

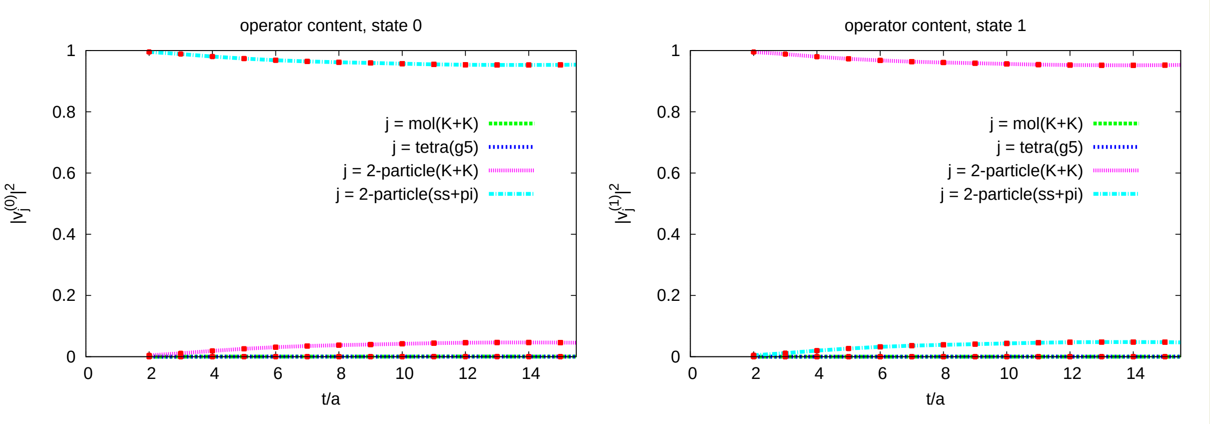


Numerical results $a_0(980)$ (5)

- When determining low-lying eigenstates from a correlation matrix, one does not only obtain their mass, but also information about their operator content, i.e. which percentage of which operator is present in an extracted state:

  $\rightarrow$ The ground state is a $\eta_s + \pi$ state ($\gtrsim 95\%$ two-particle $\eta_s + \pi$ content).

  $\rightarrow$ The first excitation is a $K + \bar{K}$ state ($\gtrsim 95\%$ two-particle $K + \bar{K}$ content).

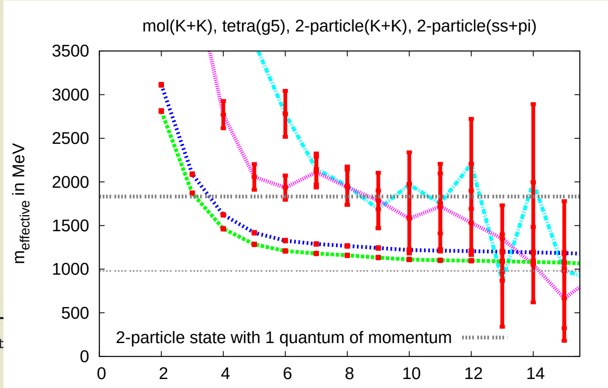


Numerical results $a_0(980)$ (6)

- What about the 2nd and 3rd excitation? ... Are these tetraquark states? ... What is their nature?

- Two-particle states with one relative quantum of momentum (one particle has momentum $+p_{\textrm{min}} = +2\pi/L$ the other $-p_{\textrm{min}}$) also have quantum numbers $I(J^{PC}) = 1(0^{++})$; their masses can easily be estimated:
  - $p_{\textrm{min}} = 2\pi/L \approx 715\,\textrm{MeV}$ (the results presented correspond to the small lattice with spatial extension $L = 1.73\,\textrm{fm}$);
  - $m(K(+p_{\textrm{min}}) + \bar{K}(-p_{\textrm{min}})) \approx 2\sqrt{m(K)^2 + p_{\textrm{min}}^2} \approx 1750\,\textrm{MeV}$;
  - $m(\eta(+p_{\textrm{min}}) + \pi(-p_{\textrm{min}})) \approx \sqrt{m(\eta)^2 + p_{\textrm{min}}^2} + \sqrt{m(\pi)^2 + p_{\textrm{min}}^2} \approx$
    $\approx 1780\,\textrm{MeV}$;

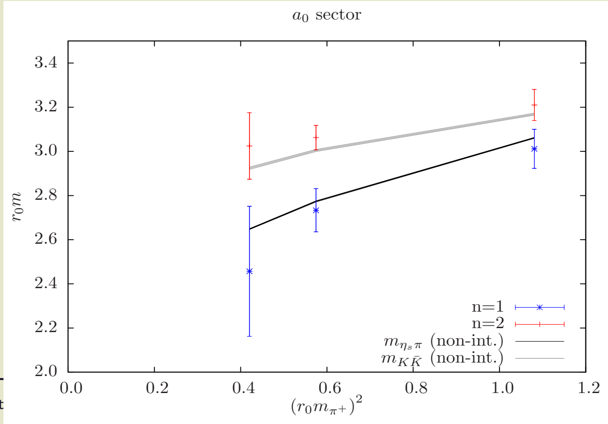
  these estimated mass values are consistent with the observed mass values of the 2nd and 3rd excitation
  $\rightarrow$ suggests to interpret these states as
  two-particle states.



Numerical results $a_0(980)$ (7)

- Summary regarding the presented "$a_0(980)$ results":
  - In the $a_0(980)$ sector (quantum numbers $I(J^{PC}) = 1(0^{++})$) we do not observe any low-lying (mass $\lesssim 1750\,\text{MeV}$) tetraquark state, even though we employed operators of tetraquark structure ($K\bar{K}$ molecule, diquark).
  - The experimentally measured mass for $a_0(980)$ is $980 \pm 20\,\text{MeV}$.
  - Conclusion: $a_0(980)$ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state (unlikely, lattice results indicate the opposite) or a rather unstable resonance.

- Similar results for the range of light quark masses investigated ($m_{\text{PS}} \approx 280 \ldots 460\,\text{MeV}$).



Numerical results κ

- Tetraquark operators for κ (quantum numbers $I(J^P) = 1/2(0^+)$):
  - Molecule type (models a bound $K\pi$ state):

$$\mathcal{O}_\kappa^{K\pi \text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 q(\mathbf{x})\Big)\Big(\bar{q}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big) \quad , \quad q\bar{q} = u\bar{u} + d\bar{d}$$

  - Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_\kappa^{\text{diquark}} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{q}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}q^{d,T}(\mathbf{x})C\gamma_5 u^e(\mathbf{x})\Big).$$

- An analysis yields only the expected low-lying two-particle $K + \pi$ energy levels.
- Conclusions: κ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state (unlikely, lattice results indicate the opposite) or a rather unstable resonance.

Conflict with existing lattice results

- In a similar recent lattice study of $\sigma \equiv f_0(500)$ and $\kappa \equiv K_0^*(800)$ bound tetraquark states have been observed in both sectors.

  [S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. -F. Liu, N. Mathur and D. Mohler, Phys. Rev. D **82**, 094507 (2010) [arXiv:1005.0948 [hep-lat]]]

- In particular for κ this conflict has to be resolved.

$a_0(980)$ and κ as resonances

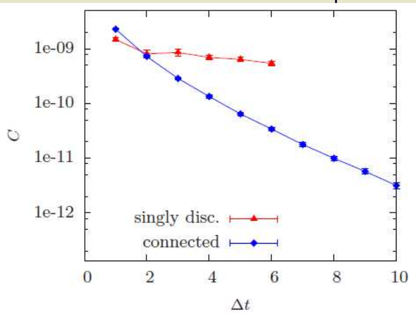
- A lattice study of $a_0(980)$ and κ as resonances requires rather precise computations of the masses of the two-particle states $K + \bar{K}$, $\eta + \pi$ and $K + \pi$ for various spatial volumes.
- Technically very challenging.
- No results yet.

Sources of systematic error, outlook (1)

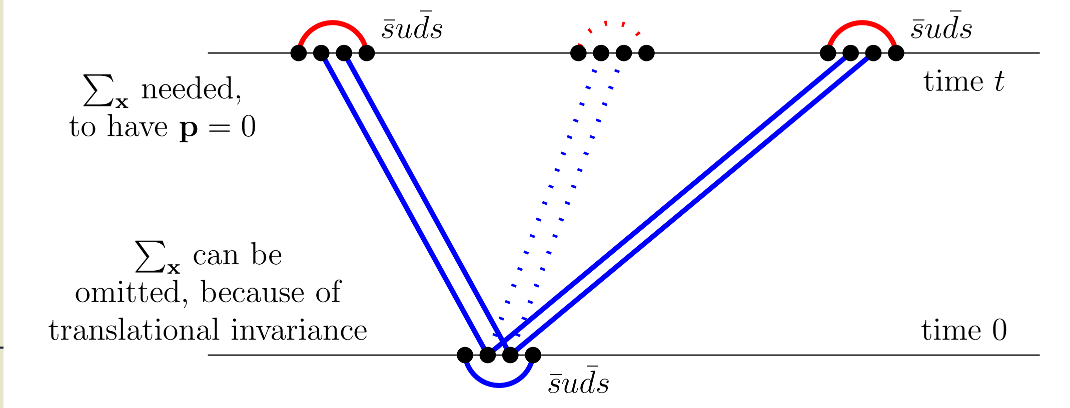
- The computations presented are technically rather challenging; there are several possible sources of systematic error, which have not yet been studied, but which need to be addressed in the future:
  - Inclusion of (singly) disconnected disconnected diagrams.
  - Include also $q\bar{q}$ creation operators (implies singly disconnected diagrams), e.g. for $a_0(980)$

$$\mathcal{O}^{q\bar{q}}_{a_0(980)} = \sum_{\mathbf{x}} \bar{d}(\mathbf{x}) u(\mathbf{x}).$$

Singly disconnected diagrams



- Missing diagrams for e.g. $a_0(980)$, κ, $D^*_{s0}(2317)^\pm$, ...
- Blue and red lines represent quark propagators:
  - Blue: point-to-all propagators applicable.
  - Red: due to $\sum_{\mathbf{x}}$, all-to-all propagators needed.
  - All-to-all propagators can only be estimated stochastically; using several stochastic all-to-all propagators results in a poor signal-to-noise ratio.
    $\longrightarrow$ combine three point-to-all (blue) and one stochastic all-to-all (red) propagator.



Sources of systematic error, outlook (2)

- Continuum limit (at the moment only a single value of the lattice spacing, $a = 0.086\,\mathrm{fm}$, has been considered).
- Finite volume studies (extrapolate the here presented results to infinite spatial volume, determine resonance properties).
- The techniques and codes developed can be used with only minor modifications to study other tetraquark candidates, e.g.
  - $\sigma \equiv f_0(500)$, $f_0(980)$,
  - $D_{s0}^*(2317)^\pm$, $D_{s1}(2460)^\pm$,
  - charmonium states $X(3872)$, $Z(4430)^\pm$, $Z(4050)^\pm$, $Z(4250)^\pm$, ...