Challenges in multi-quark studies

Workshop "Spectroscopy and Hadron structure from lattice QCD" at "Electromagnetic Interactions with Nucleons and Nuclei", Paphos

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Outline

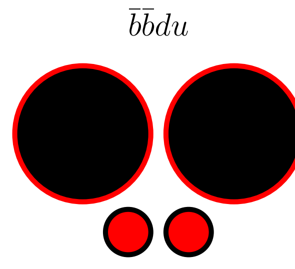
- **A brief discussion of challenges and problems in multi-quark studies.**
- Focus on 4-quark systems, i.e. tetraquarks.
- Challenges and problems when studying unstable particles/scattering in [Talk by M. Petschlies, "Approaches for unstable particles"]
- Three challenges/problems will be discussed in the following:

  (1) "**Is the multi-quark system I am interested in easy or hard to study?**"
  (Why are most multi-quark systems so challenging?)

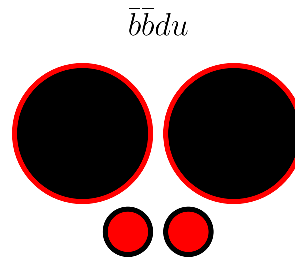
  (2) "**Which techniques do I use to compute correlators?**"
  (Certainly not easy to decide ... in any case, the computation will be technically difficult.)

  (3) After the computation ... "**Why are the errors so large?**"
  (What can one do to mitigate this problem?)

Highly excited states (1)



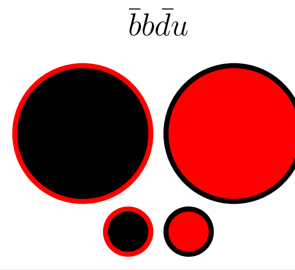
- **"Is the multi-quark system I am interested in easy or hard to study?"**
- In many cases interesting multi-quark states are highly excited states
  $\rightarrow$ difficult multi-quark systems.
- In few cases this is not the case
  $\rightarrow$ comparatively simple multi-quark systems.
- Start with a very simple four-quark system, which can most-likely form a tetraquark: $\bar{b}\bar{b}ud$.
- $\bar{q}q$ annihilation not possible, i.e. system will always have at least four quarks, no mixing with or decay to a quark-antiquark system.
- Decay: $\bar{b}\bar{b}ud \rightarrow B^{(*)} + B^{(*)}$ (B or B^* depends on quantum numbers of $\bar{b}\bar{b}ud$).
- If forces between the four quarks $\bar{b}\bar{b}ud$ are sufficiently attractive,
  $m(\bar{b}\bar{b}ud) < m(B^{(*)}) + m(B^{(*)})$, i.e. $\bar{b}\bar{b}ud$ is a stable tetraquark.
  - One just has to compute the mass of the lowest state in the sector (comparatively simple) ...
  - ... then check, whether $m(\bar{b}\bar{b}ud) < m(B^{(*)}) + m(B^{(*)})$.
- Recent lattice QCD studies provide strong evidence that this is the case, i.e. that there is a stable $\bar{b}\bar{b}ud$ tetraquark.



Highly excited states (2)

- Static $\bar{b}$ quarks, Born-Oppenheimer approximation:
  - Step 1: Compute the $\bar{b}\bar{b}$ potential in the presence of two lighter quarks qq.
  - Step 2: Solve the Schrödinger equation for the relative coordinate of the two $\bar{b}$ quarks to check, whether $m(\bar{b}\bar{b}ud) < m(B^{(*)}) + m(B^{(*)})$.
  - Prediction of a stable $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$, $m(\bar{b}\bar{b}ud) - (m(B) + m(B^*)) \approx -50\,\text{MeV}$.
    [P. Bicudo, M. Wagner, Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274 [hep-ph]]]
    [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953 [hep-lat]].
  - No stable tetraquark for other quantum numbers or flavors $ud \rightarrow \{ss, cc\}$.
    [P. Bicudo *et al.*, Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613 [hep-lat]]]
  - One can also search for resonances, using techniques from scattering theory
    $\rightarrow$ prediction of a $\bar{b}\bar{b}ud$ tetraquark resonance with quantum numbers $I(J^P) = 0(1^-)$,
    $m(\bar{b}\bar{b}ud) - (m(B) + m(B)) \approx +20\,\text{MeV}$, $\Gamma(\bar{b}\bar{b}ud) \approx 100\,\text{MeV}$.
    [P. Bicudo *et al.*, Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383 [hep-lat]]]
- $\bar{b}$ quarks with NRQCD:
  - $m(\bar{b}\bar{b}ud) < (m(B) + m(B^*))$ confirms the stable $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$.
    [A. Francis *et al.*, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214 [hep-lat]]]

Highly excited states (3)



- Now consider $\bar{b}b\bar{q}q$ instead of $\bar{b}\bar{b}qq$, where $q \in \{u, d\}$.
- Even though similar at first glance, the $\bar{b}b\bar{q}q$ is drastically more complicated to study.
- $\bar{b}b$ annihilation possible, but might be negligible.
- $\bar{q}q$ annihilation possible, but can be excluded by studying $\bar{q}q = \bar{u}d$, i.e. $I = 1$.
- Decay: $\bar{b}b\bar{u}d \rightarrow \bar{B}^{(*)} + B^{(*)}$ (as for $\bar{b}\bar{b}qq$).
- Decays: $\bar{b}b\bar{u}d \rightarrow \eta_b + \pi$ and/or $\bar{b}b\bar{u}d \rightarrow \Upsilon(1S) + \pi$ (not possible for $\bar{b}\bar{b}qq$).
- Expectation from experimental measurements of $Z_b^\pm$: $m(\bar{b}b\bar{u}d) \approx 2m(B) \approx 10560\,\text{MeV}$ $\rightarrow$ many states with the same quantum numbers below.
  - $m(\eta_b) + m(\pi) = 9540\,\text{MeV}$.
  - $m(\Upsilon(1S)) + m(\pi) = 9600\,\text{MeV}$.
  - Several momentum excitations of $\eta_b + \pi$ and $\Upsilon(1S) + \pi$ below $\bar{B}B$.
  - Also states with η_b or $\Upsilon(1S)$ and more than one π ... and ...
- All these states have to be resolved in a computation ... or one needs solid arguments that decays into these states can be neglected (e.g. because trial states have tiny overlaps etc.).

Highly excited states (4)

- $\bar{b}b\bar{d}u$, i.e. Z_b states, just one example, many multi-quark systems exhibit similar problems.
- Z_c states very similar to Z_b states.
  - E.g. $Z_c(3900)^+$, quantum numbers $J^P = 1^+$:
    $m_{Z_c(3900)^+} = 3889\,\text{MeV}$.
  - 2-meson states with the same quantum numbers:
    $m_{J/\psi} + m_\pi = (3097 + 139)\,\text{MeV} = 3236\,\text{MeV}$
    $m_{\eta_c} + m_\rho = (2984 + 775)\,\text{MeV} = 3759\,\text{MeV}$
    $m_D + m_{D^*} = (1870 + 2007)\,\text{MeV} = 3877\,\text{MeV}$
    ... further 2-meson states ... additionally states with relative momentum ...

$\rightarrow$ All these states should be determined with a single computation at the same time.

- $a_0(980)$ meson:
  - Quark content: $\bar{d}u\bar{s}s$ (at least a significant 4-quark component seems to be present).
    [Talk by T. Leontiou, "Lattice QCD investigation of the structure of the $a_0(980)$ meson"]
  - $m(a_0(980)) \approx 2m(K) \approx 1000\,\text{MeV}$.
  - Can decay to $\eta + \pi$ and momentum excitations ($m(\eta) + m(\pi) \approx 700\,\text{MeV}$).
  - ...

Highly excited states (5)

- A very nice recent lattice QCD paper about charged Z_c states:
  [S. Prelovsek, C. B. Lang, L. Leskovec and D. Mohler, Phys. Rev. D **91**, 014504 (2015)
  [arXiv:1405.7623 [hep-lat]]]

- **For most multi-quark systems it will be necessary to determine a number of lower states with the same quantum numbers precisely ...**

- ... and to interpret the corresponding data appropriately ("extension of Lüscher's finite volume method", not covered in my talk, also very difficult).
  [Talk by J. Dudek, "Coupled-channel meson resonances from lattice QCD"]
  [Talk by M. Petschlies, "Approaches for unstable particles" ...?]

Study of the Z_c^+ channel using lattice QCD

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(Dated: January 21, 2015)

Recently experimentalists have discovered several charged charmonium-like hadrons Z_c^+ with unconventional quark content $\bar{c}c\bar{d}u$. We perform a search for Z_c^+ with mass below 4.2 GeV in the channel $I^G(J^{PC}) = 1^+(1^{+-})$ using lattice QCD. The major challenge is presented by the two-meson states $J/\psi\,\pi$, $\psi_{2S}\pi$, $\psi_{1D}\pi$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\eta_c\rho$ that are inevitably present in this channel. The spectrum of eigenstates is extracted using a number of meson-meson and diquark-antidiquark interpolating fields. For our pion mass of 266 MeV we find all the expected two-meson states but no additional candidate for Z_c^+ below 4.2 GeV. Possible reasons for not seeing an additional eigenstate related to Z_c^+ are discussed. We also illustrate how a simulation incorporating interpolators with a

Computation of multi-quark diagrams (1)

- "**Which techniques do I use to compute correlators?**"
- Multi-quark studies typically require correlation matrices with many different operators.
- E.g. to study the $a_0(980)$, which might have quark content $\bar{d}u$ and/or $\bar{d}u\bar{s}s$,

$$\mathcal{O}^{q\bar{q}} = \frac{1}{\sqrt{V_s}} \sum_{\mathbf{x}} \Big(\bar{d}(\mathbf{x})u(\mathbf{x})\Big) \quad \text{quark-antiquark}$$

$$\mathcal{O}^{K\bar{K},\text{point}} = \frac{1}{\sqrt{V_s}} \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big) \quad \text{mesonic molecule}$$

$$\mathcal{O}^{\eta_s\pi,\text{point}} = \frac{1}{\sqrt{V_s}} \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big) \quad \text{mesonic molecule}$$

$$\mathcal{O}^{Q\bar{Q}} = \frac{1}{\sqrt{V_s}} \sum_{\mathbf{x}} \epsilon_{abc}\Big(\bar{s}_b(\mathbf{x})(C\gamma_5)\bar{d}_c^T(\mathbf{x})\Big)\epsilon_{ade}\Big(u_d^T(\mathbf{x})(C\gamma_5)s_e(\mathbf{x})\Big) \quad \text{diquark-antidiquark}$$

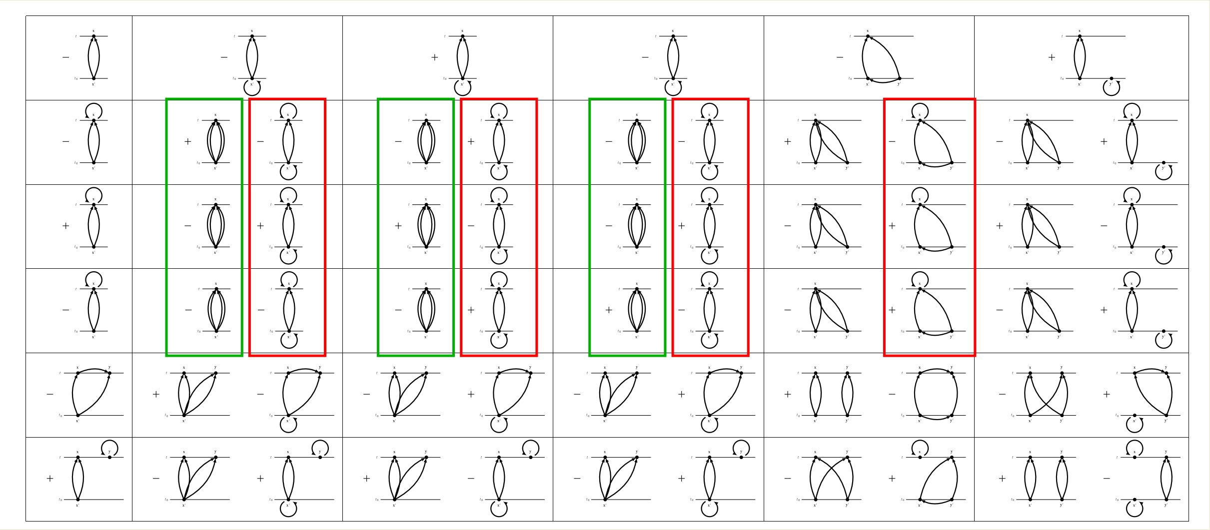
$$\mathcal{O}^{K\bar{K},\text{2part}} = \frac{1}{V_s} \sum_{\mathbf{x},\mathbf{y}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{y})\gamma_5 s(\mathbf{y})\Big) \quad \text{2-meson scattering}$$

$$\mathcal{O}^{\eta_s\pi,2part} = \frac{1}{V_s} \sum_{\mathbf{x},\mathbf{y}} \Big(\bar{s}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{y})\gamma_5 u(\mathbf{y})\Big) \quad \text{2-meson scattering.}$$

[Talk by T. Leontiou, "Lattice QCD investigation of the structure of the $a_0(980)$ meson"]

Computation of multi-quark diagrams (2)

- Many diagrams have to be computed, some are **easy**, others are more **difficult**.



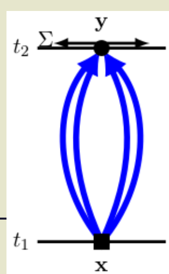
Computation of multi-quark diagrams (3)

(A) Easy: due to translational invariance one of the two spatial sums can be omitted (marked by black boxes) and one can use point-to-all propagators (blue).

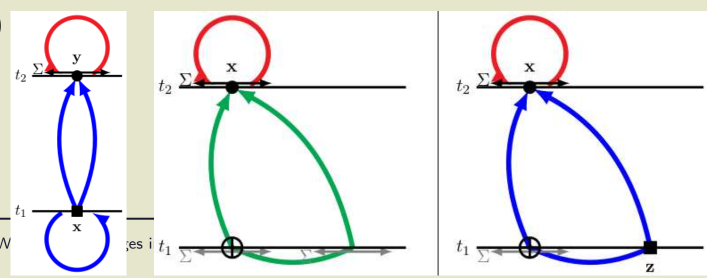
(B) More difficult: one spatial sum can be omitted, but at least one all-to-all propagator needed.

- All-to-all propagators are often estimated stochastically:
  - Quite good, when combined with the one-end trick (green): number of noise terms is reduced by $\sqrt{V_s}$.
  - Not so good, if one-end trick not possible (red): strong statistical fluctuations.
  - $M > 1$ stochastic propagators: typically a desaster, because $\#$ noise terms $\propto V_s^M$.
- Diagrams shown on previous slide can be computed with reasonable accuracy, if techniques (point-to-all, stohastic, one-end trick, sequential propagators) are properly combined.
  [A. Abdel-Rehim et al, Comput. Phys. Commun. **220**, 97 (2017) [arXiv:1701.07228 [hep-lat]]]

(A)



(B)



Computation of multi-quark diagrams (4)

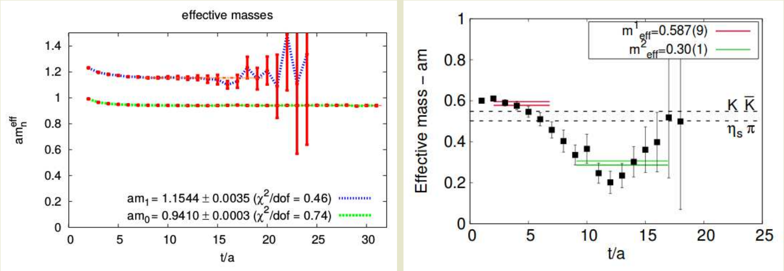
- A more recent technique, to compute all-to-all propagators for LapH-smeared quark fields without introducing additional stochastic noise, is **distillation**.
  [M. Peardon *et al.* [HS Collaboration], Phys. Rev. D **80**, 054506 (2009) [arXiv:0905.2160 [hep-lat]]]
  - Difficult to set up, i.e. one first has to invest a significant amount of resources.
  - Once you have the propagators, they can be used in a very flexible way for many projects (all-to-all propagators).
- Meanwhile, distillation very popular in multi-quark studies
  $\rightarrow$ might indicate that this is the most powerful method for many multi-quark systems.
- In current literature three different approaches for propagator computation:
  (1) "Standard methods" (i.e. no distillation).
  (2) Distillation.
  (3) Stochastic distillation:
    * Essentially a mixture of (1) and (2), a cheaper version of distillation, which introduces additional stochastic noise.
    * When increasing the number of quarks, one should have to face problems similar to those caused by ordinary stochastic propagators ...? (# noise terms $\propto N^M$ [$N \approx \mathcal{O}(100)$: # eigenvectors; M: # propagators])

Computation of multi-quark diagrams (5)

- Question, when starting a new multi-quark project:
  "Which approach is the most efficient/the most suited for my project?"
  - For me each time very difficult to decide.
  - Probably no universal answer, will depend on physical observables, on diagrams, on scale of the project, available resources, etc.
  - As far as I know, no comprehensive comparisons of "standard methods", distillation and stochastic distillation available in the literature.
  - **Such studies might be very helpful for practitioners in our field.**
  - Request/suggestion: If you have done/will do comparative studies, please publish them.

Exponential noise-to-signal ratio (1)

- "**Why are the errors so large?**"
- Two examples of effective masses $m_{\text{eff}} = \ln(C(t)/C(t+1))$ corresponding to correlators $C(t)$ with errors $\Delta C(t)$:



(A) π meson (e.g. $\mathcal{O} = \bar{d}\gamma_5 u$) and other pseudoscalar mesons,

$$\frac{\text{noise}}{\text{signal}} = \frac{\Delta C(t)}{C(t)} \approx \text{const.}$$

(B) a_0 meson (e.g. $\mathcal{O} = \bar{d}u$) and most multi-quark correlators,

$$\frac{\text{noise}}{\text{signal}} = \frac{\Delta C(t)}{C(t)} \approx \# \exp(+\alpha t).$$

Exponential noise-to-signal ratio (2)

Explanation of constant versus exponential behavior (1):

- Correlator $C(t) = \langle \mathcal{O}^\dagger(t) \mathcal{O}(0) \rangle$.
- Estimate this correlator from N samples $C_j(t)$ (stochastically independent), e.g. 1 sample per gauge link configuration (as it is often the case, when using point-to-all propagators):

$$\overline{C}(t) = \frac{1}{N} \sum_j C_j(t) \sim \# \exp(-mt)$$

$$\Delta C(t) = \left(\frac{1}{N^2} \sum_j \Big(C_j(t) - \overline{C}(t)\Big)^2\right)^{1/2} = \left(\frac{1}{N^2} \sum_j C_j^2(t) - \frac{1}{N}\overline{C}^2(t)\right)^{1/2} =$$

$$= \frac{1}{\sqrt{N}}\Big(\overline{C}_{\mathcal{O}\tilde{\mathcal{O}}}(t) - \overline{C}^2(t)\Big)^{1/2} \sim \frac{1}{\sqrt{N}}\Big(\# \exp(-m_{\mathcal{O}\mathcal{O}'} t) - \# \exp(-2mt)\Big)^{1/2}.$$

  - m is the mass of the lightest state probed by $\mathcal{O}$, i.e. in the investigated sector.
  - $\overline{C}_{\mathcal{O}\tilde{\mathcal{O}}}(t)$ is the estimate for $\langle \mathcal{O}^\dagger(t) \mathcal{O}'^\dagger(t) \mathcal{O}(0) \mathcal{O}'(0) \rangle$, where $\mathcal{O}'$ is $\mathcal{O}$ with $q \rightarrow q'$ (i.e. a QCD-like world with twice as many quark flavors).
  - $m_{\mathcal{O}\mathcal{O}'}$ is the mass of the lightest state in the sector probed by $\mathcal{O}\mathcal{O}'$.

Exponential noise-to-signal ratio (3)

Explanation of constant versus exponential behavior (2):

(A) π meson, $\mathcal{O} = \bar{d}\gamma_5 u$ (spatial sum not written)
$\rightarrow \mathcal{O}\mathcal{O}' = \bar{d}\gamma_5 u \bar{d}'\gamma_5 u'$
$\rightarrow$ lightest state in the sector probed by $\mathcal{O}\mathcal{O}'$ is $\pi + \pi$, i.e.

$$\Delta C(t) = \frac{1}{\sqrt{N}}\Big(\#\exp(-2m_\pi t) - \#\exp(-2m_\pi t)\Big)^{1/2} \sim \frac{\#}{\sqrt{N}}\exp(-m_\pi t)$$

$$\frac{\text{noise}}{\text{signal}} = \frac{\Delta C(t)}{\overline{C}(t)} = \frac{\Delta C(t)}{\#\exp(-m_\pi t)} \sim \frac{\#}{\sqrt{N}}.$$

(B) $a_0(980)$ meson, $\mathcal{O} = \bar{d}u$ or $\mathcal{O} = (\bar{d}s)(\bar{s}u)$ (molecule) or diquark-antidiquark or ...
$\rightarrow \mathcal{O}\mathcal{O}' = \bar{d}u\bar{d}'u'$
$\rightarrow$ lightest state in the sector probed by $\mathcal{O}\mathcal{O}'$ is also $\pi + \pi$, i.e.

$$\Delta C(t) = \frac{1}{\sqrt{N}}\Big(\#\exp(-2m_\pi t) - \#\exp(-2m_{a_0(980)} t)\Big)^{1/2} \sim \frac{\#}{\sqrt{N}}\exp(-m_\pi t)$$

$$\frac{\text{noise}}{\text{signal}} = \frac{\Delta C(t)}{\overline{C}(t)} = \frac{\Delta C(t)}{\#\exp(-m_{a_0(980)} t)} \sim \frac{\#}{\sqrt{N}}\exp(+(m_{a_0(980)} - m_\pi)t)$$

(can be verified numerically).
[A. Abdel-Rehim *et al.*, Comput. Phys. Commun. **220**, 97 (2017) [arXiv:1701.07228 [hep-lat]]]

Exponential noise-to-signal ratio (4)

Explanation of constant versus exponential behavior (3):

- Summary of the calculation:
  - Operator $\mathcal{O}$ to extract mass m.
  - "**If the square of the operator $\mathcal{O}$ probes a sector, where the lightest state is lighter than $2m$, then the signal-to-noise ratio increases exponentially.**"
  - Quite often the case in multi-quark studies.

Exponential noise-to-signal ratio (5)

- Attempts to mitigate or solve the problem:
  - To mitigate the problem, try to extract information at small t, e.g.
    - optimize operators, such that they almost exclusively excite states of interest,
    - use smaller lattice spacing in temporal direction,
      [Talk by M. Peardon, "Spectroscopy of charmed mesons and baryons"]
    - use analysis methods, which exploit correlators at small t, e.g. AMIAS.
      [Talk by T. Leontiou, "Lattice QCD investigation of the structure of the $a_0(980)$ meson"]
  - Methods for error reduction, to solve the problem ...?
  - Recent proposals are:
    - [L. Giusti, M. Ce, S. Schaefer, arXiv:1710.09212 [hep-lat]]
      ... "Multi-boson block factorization of fermions", leads to a local action in gauge fields (and auxiliary boson fields), allows multi-level Monte Carlo integration
      $\rightarrow$ exponential error reduction.
    - [M. L. Wagman and M. J. Savage, arXiv:1704.07356 [hep-lat]]
      ... "Taming the signal-to-noise problem in lattice QCD by phase reweighting".
    - Not specifically designed for multi-quark studies.
    - Tested only for rather elementary observables.
- **To make a significant step regarding precision, this problem needs to be solved.**

Outline

- Three challenges/problems have been discussed:

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  (Why are most multi-quark systems so challenging?)

  (2) "**Which techniques do I use to compute correlators?**"
  (Certainly not easy to decide ... in any case, the computation will be technically difficult.)

  (3) After the computation ... "**Why are the errors so large?**"
  (What can one do to mitigate this problem?)