Adjoint string breaking in the pseudoparticle approach

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May 30, 2008

Outline

Part I: introduction to the pseudoparticle approach

- Basic principle, pseudoparticle ensembles and their building blocks.

Part II: adjoint string breaking in the pseudoparticle approach

- Pure Wilson loop static potentials, Casimir scaling.
- The gluelump spectrum.
- The static adjoint potential, string breaking.

Summary, further results, ongoing projects

Part I: introduction to the pseudoparticle approach

Basic principle (1)

- Pseudoparticle approach (PP approach; F. Lenz, M.W., 2005):
  - A numerical technique to approximate Euclidean path integrals.
  - In this talk: application to SU(2) Yang-Mills theory,

$$\left\langle \mathcal{O} \right\rangle = \frac{1}{Z} \int DA\, \mathcal{O}[A] e^{-S[A]}$$

$$S[A] = \frac{1}{4g^2} \int d^4x\, F_{\mu\nu}^a F_{\mu\nu}^a \quad , \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c .$$

  - Goals:
    - ∗ Build a model for SU(2) Yang-Mills theory with a small number of physically relevant degrees of freedom.
    - ∗ Analyze the importance of certain classes of gauge field configurations with respect to confinement and other essential properties of SU(2) Yang-Mills theory.

Basic principle (2)

- Related work:
  - Ensembles of regular gauge instantons and merons (F. Lenz, J. W. Negele, M. Thies, 2003).
  - Ensembles of calorons with non-trivial holonomy (P. Gerhold, E.-M. Ilgenfritz, M. Müller-Preussker, 2006).
  - Ensembles of dyons (D. Diakonov, V. Petrov, 2007).

Basic principle (3)

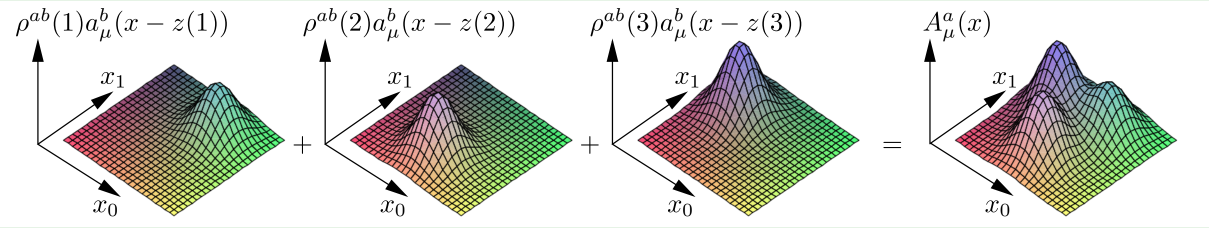
- PP: any gauge field configuration a^a_μ, which is localized in space and in time.
- Consider only those gauge field configurations, which can be written as a sum of a fixed number (≈ 500) of PPs:

$$A^a_\mu(x) \;=\; \sum_j \rho^{ab}(j) a^b_\mu(x - z(j))$$

(j: PP index; $\rho^{ab}(j)$: degrees of freedom of the j-th PP, i.e. amplitude and color orientation; $z(j)$: position of the j-th PP).

- Define the functional integration as an integration over the PP degrees of freedom:

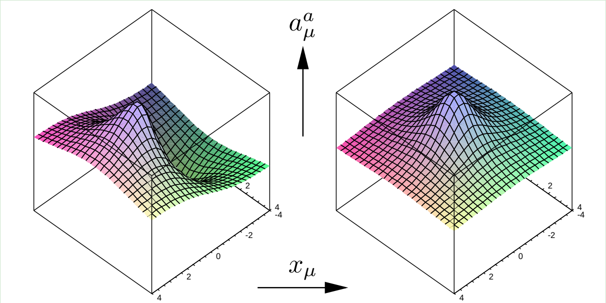
$$\int DA \ldots \quad \rightarrow \quad \int \left(\prod_j d\rho^{ab}(j) \right) \ldots$$



Building blocks of PP ensembles

- Building blocks of PP ensembles: "instantons", "antiinstantons", akyrons (λ: PP size).

$$
\begin{aligned}
a^a_{\mu,\text{instanton}}(x) &= \eta^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2} \\
a^a_{\mu,\text{antiinstanton}}(x) &= \bar{\eta}^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2} \\
a^a_{\mu,\text{akyron}}(x) &= \delta^{a1} \frac{x_\mu}{x^2 + \lambda^2} .
\end{aligned}
$$



- Instantons, antiinstantons and akyrons form a basis of all gauge field configurations in the "continuum limit".

- Degrees of freedom: amplitudes $\mathcal{A}(i)$, color orientations $\mathcal{C}^{ab}(i)$, positions $z(i)$.

$$
\begin{aligned}
A^a_\mu(x) &= \mathcal{A}(i)\mathcal{C}^{ab}(i) a^a_{\mu,\text{instanton}}(x - z(i)) \\
A^a_\mu(x) &= \mathcal{A}(i)\mathcal{C}^{ab}(i) a^a_{\mu,\text{antiinstanton}}(x - z(i)) \\
A^a_\mu(x) &= \mathcal{A}(i)\mathcal{C}^{ab}(i) a^a_{\mu,\text{akyron}}(x - z(i)) .
\end{aligned}
$$

PP ensembles (1)

- PP ensemble: a fixed number of PPs (in this talk: 625) inside a periodic spacetime hypercube (in this talk: extension 5.0^4).
- Gauge field:

$$A_\mu^a(x) = \sum_i \mathcal{A}(i)\mathcal{C}^{ab}(i) a^b_{\mu,\mathrm{instanton}}(x - z(i)) + \sum_j \mathcal{A}(j)\mathcal{C}^{ab}(j) a^b_{\mu,\mathrm{antiinstanton}}(x - z(j)) + \sum_k \mathcal{A}(k)\mathcal{C}^{ab}(k) a^b_{\mu,\mathrm{akyron}}(x - z(k)).$$

- Choose color orientations $\mathcal{C}^{ab}(i)$ and positions $z(i)$ randomly.
- **A_μ^a is not a classical solution (not even close to a classical solution), i.e. the PP approach is not a semiclassical method.**
  - Long range interactions between PPs.
  - Variable amplitudes $\mathcal{A}(i)$.

PP ensembles (2)

- Approximation of the path integral:

$$\left\langle \mathcal{O} \right\rangle \quad = \quad \frac{1}{Z} \int \left(\prod_i d\mathcal{A}(i) \right) \mathcal{O}(\mathcal{A}(i)) e^{-S(\mathcal{A}(i))}$$

(integration over PP amplitudes).

- Solve this multidimensional integral via standard Monte-Carlo methods.

Part II: adjoint string breaking in the pseudoparticle approach

String breaking

- **Static potential $V(R)$:** the energy of the lowest state containing two static sources ϕ and $\phi^\dagger$ ("two infinitely heavy quarks") at separation R ($+$ light particles [gluons, dynamical quarks, ...]).
- **String breaking in QCD:** when two static sources are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by dynamical quarks; a pair of (essentially) non-interacting static-light mesons is formed.
- **String breaking in pure SU(2) Yang-Mills theory:**
  - Fundamental representation ($\phi^{(1/2)} = (\phi_1, \phi_2)$): no string breaking, since gluons are not able to screen sources in the fundamental representation.
  - Adjoint representation ($\phi^{(1)} = (\phi_1, \phi_2, \phi_3)$ or $\phi^{(1)} = \phi_a \sigma_a / 2$): when two static sources are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by gluons; a pair of (essentially) non-interacting gluelumps is formed.

Pure Wilson loop static potentials (1)

- The starting point to extract the energy of the lowest state containing two static sources $\phi^{(J)}$ and $(\phi^{(J)})^\dagger$ in spin-J-representation are "string trial states"

$$S^{(J)}(\mathbf{x},\mathbf{y})|\Omega\rangle \;\; = \;\; (\phi^{(J)}(\mathbf{x}))^\dagger U^{(J)}(\mathbf{x};\mathbf{y})\phi^{(J)}(\mathbf{y})|\Omega\rangle \quad , \quad |\mathbf{x}-\mathbf{y}| \;\; = \;\; R.$$

- We consider temporal correlations

$$\mathcal{C}^{(J)}_{\textrm{string}}(T) \;\; = \;\; \langle\Omega|\Big(S^{(J)}(\mathbf{x},\mathbf{y},T)\Big)^\dagger S^{(J)}(\mathbf{x},\mathbf{y},0)|\Omega\rangle$$

and compute the corresponding potential values from effective mass plateaus,

$$m^{(\ldots)}_{\textrm{effective,string}}(T) \;\; = \;\; -\frac{1}{a}\ln\frac{\mathcal{C}^{(\ldots)}_{\textrm{string}}(T)}{\mathcal{C}^{(\ldots)}_{\textrm{string}}(T-a)}.$$

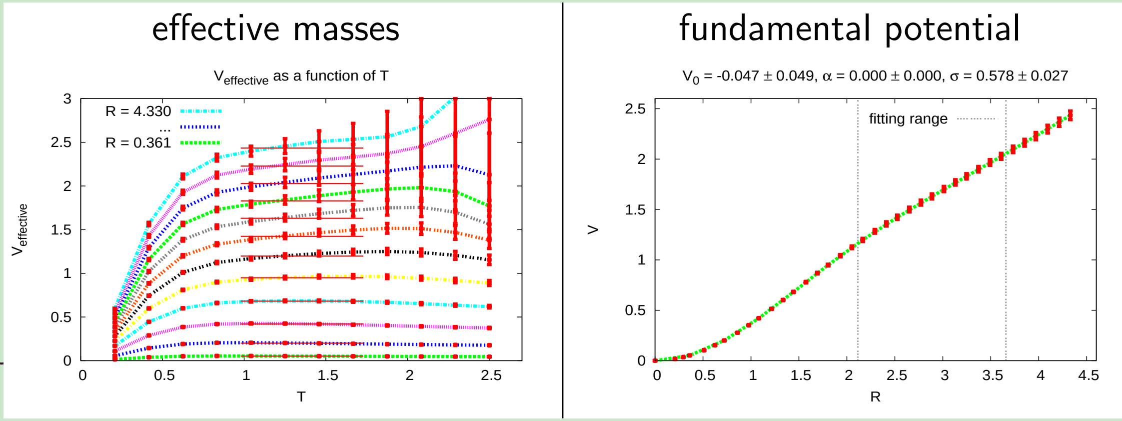
Pure Wilson loop static potentials (2)

- Integrating out the static sources yields expectation values of polynomials of fundamental representation Wilson loops $W_{(R,T)}$:

$$\begin{aligned}
\mathcal{C}^{(1/2)}_{\mathrm{string}}(T) &\propto \Big\langle W_{(R,T)} \Big\rangle \\
\mathcal{C}^{(1)}_{\mathrm{string}}(T) &\propto \Big\langle W^{(1)}_{(R,T)} \Big\rangle = \Big\langle \frac{4}{3}(W_{(R,T)})^2 - \frac{1}{3} \Big\rangle \\
\mathcal{C}^{(3/2)}_{\mathrm{string}}(T) &\propto \Big\langle W^{(3/2)}_{(R,T)} \Big\rangle = \Big\langle 2(W_{(R,T)})^3 - W_{(R,T)} \Big\rangle \\
\mathcal{C}^{(2)}_{\mathrm{string}}(T) &\propto \Big\langle W^{(2)}_{(R,T)} \Big\rangle = \Big\langle \frac{16}{5}(W_{(R,T)})^4 - \frac{12}{5}(W_{(R,T)})^2 + \frac{1}{5} \Big\rangle \\
\mathcal{C}^{(5/2)}_{\mathrm{string}}(T) &\propto \Big\langle W^{(5/2)}_{(R,T)} \Big\rangle = \Big\langle \frac{16}{3}(W_{(R,T)})^5 - \frac{16}{3}(W_{(R,T)})^3 + W_{(R,T)} \Big\rangle \\
&\ldots
\end{aligned}$$

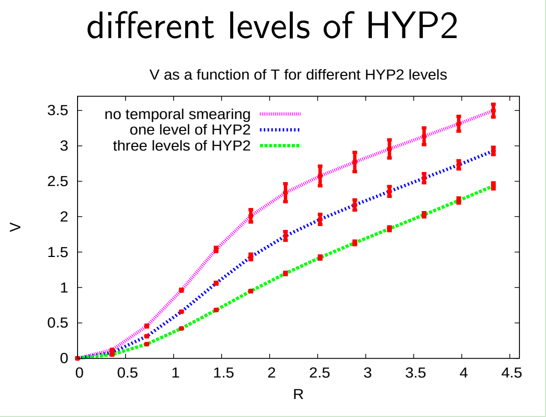
Pure Wilson loop static potentials (3)

- Numerical results for the fundamental representation ($J = 1/2$):
  - The potential is linear for large separations, i.e. confinement.
  - The scale is set by fitting $V(R) = V_0 + \sigma R$ and by identifying σ with $\sigma_{\mathrm{physical}} = 4.2/\mathrm{fm}^2$.
  - A coupling constant of $g = 12.5$ (standard choice for results shown in this talk) corresponds to a spacetime region of $L^4 = (1.85\,\mathrm{fm})^4$.
  - Like in lattice gauge theory, the scale can be changed by changing the value of g, e.g. $g = 9.5 \ldots 18.5$ corresponds to $L = 1.55\,\mathrm{fm} \ldots 2.31\,\mathrm{fm}$.



Numerical techniques (1)

- **Latticization of pseudoparticle gauge field configurations:** allows the use of efficient lattice techniques, while the underlying model is still a continuum model.
- **Smearing techniques:**
  - **HYP2 smearing of temporal links (3 iterations):** removes UV fluctuations and reduces the self energy of the static sources; the signal-to-noise ratio is significantly improved (e.g. for the adjoint potential computation time is reduced by a factor of ≈ 200).



  - **APE smearing of spatial links (5, 15 and 35 iterations):** increases ground state overlaps of trial states; effective mass plateaus are reached at smaller temporal separations.

Numerical techniques (2)

- **Variational technique:**
  - Instead of a single trial state $|\Phi\rangle$ consider a set of trial states $\{|\Phi^{(1)}\rangle\, ,\, \ldots\, ,\, |\Phi^{(N)}\rangle\}$ differing e.g. by their APE smearing levels.
  - Diagonalize the corresponding correlation matrices

    $$\mathcal{C}_{AB}(T) \quad = \quad \langle\Phi^{(A)}(T)|\Phi^{(B)}(0)\rangle$$

    according to

    $$\mathcal{C}_{AB}(T_0)v_B^{(n)} \quad = \quad \mathcal{C}_{AB}(T_0-a)v_B^{(n)}\lambda^{(n)}.$$

  - Approximations of low lying states are then given by

    $$|n\rangle \quad \approx \quad v_A^{(n)}|\Phi^{(A)}\rangle,$$

    the corresponding effective masses by

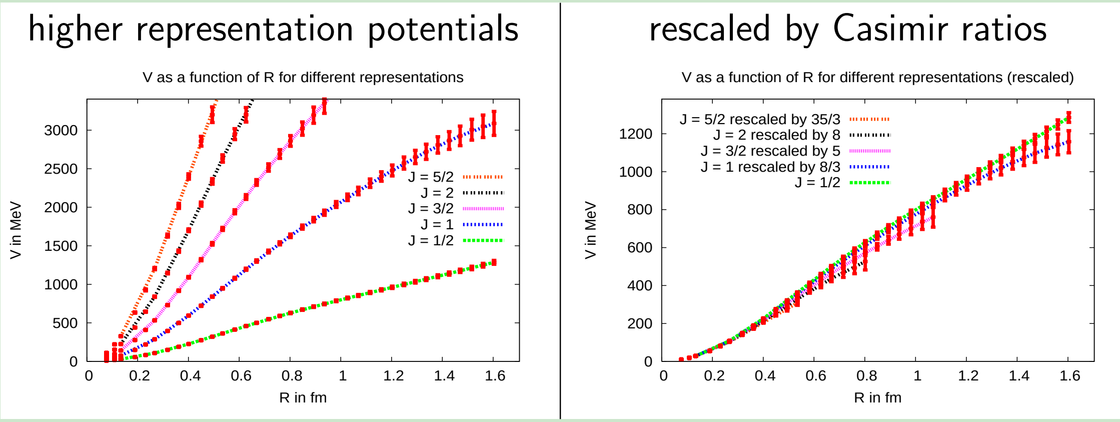
    $$m_{\text{effective}}^{(n)}(T) \quad = \quad -\frac{1}{a}\ln\frac{(v_A^{(n)})^\dagger\mathcal{C}_{AB}(T)v_B^{(n)}}{(v_A^{(n)})^\dagger\mathcal{C}_{AB}(T-a)v_B^{(n)}}.$$

Pure Wilson loop static potentials (4)

- Numerical results for higher representations ($J = 1$, ..., $J = 5/2$):
  - Higher representation potentials exhibit Casimir scaling:

$$V^{(1/2)}(R) \approx \frac{V^{(1)}(R)}{8/3} \approx \frac{V^{(3/2)}(R)}{5} \approx \frac{V^{(2)}(R)}{8} \approx \frac{V^{(5/2)}(R)}{35/3}.$$

  - The adjoint potential ($J = 1$) shows no sign of string breaking even at separations $R \approx 1.6\,\mathrm{fm}$.



Why is string breaking elusive?

- Static potential at small separations R: the string trial state has good overlap to the physical ground state, which is expected to be a string state.
- Static potential at large separations R: the string trial state has poor overlap to the physical ground state, which is expected to be a two gluelump state.
- Solution: Extend the set of trial states by "two-gluelump trial states", which are supposed to have good overlap to the physical ground state at large R,

$$G_j^{(\ldots)}(\mathbf{x}) G_j^{(\ldots)}(\mathbf{y}) |\Omega\rangle \quad , \quad |\mathbf{x} - \mathbf{y}| \;=\; R$$

with

$$\begin{aligned} G_j^{(\mathrm{magnetic}, J=1)}(\mathbf{x}) &= \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) B_j(\mathbf{x})\Big) \quad , \quad j = x, y, z \\ G_z^{(\mathrm{electric}, J=1)}(\mathbf{x}) &= \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) \Big(D_x B_y(\mathbf{x}) - D_y B_x(\mathbf{x})\Big)\Big) \quad \text{(and cyclic)} \\ G_z^{(\mathrm{electric}, J=2)}(\mathbf{x}) &= \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) \Big(D_x B_y(\mathbf{x}) + D_y B_x(\mathbf{x})\Big)\Big) \quad \text{(and cyclic).} \end{aligned}$$

The gluelump spectrum (1)

- Symmetry group of states constrained by a single static source $\phi^{(1)}(\mathbf{x})$: $\mathrm{SO}(3) \otimes P$.
- "Gluelump trial states" with well defined quantum numbers:
  - Magnetic gluelump: $J = 1$, $P = +$,

    $$G_j^{(\mathrm{magnetic},J=1)}(\mathbf{x})|\Omega\rangle \quad = \quad \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x}) B_j(\mathbf{x})\Big)|\Omega\rangle \quad , \quad j = x, y, z.$$

  - Electric gluelump: $J = 1$, $P = -$,

    $$G_z^{(\mathrm{electric},J=1)}(\mathbf{x})|\Omega\rangle \quad = \quad \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x})\Big(D_x B_y(\mathbf{x}) - D_y B_x(\mathbf{x})\Big)\Big)|\Omega\rangle$$

    (and cyclic).
  - Electric gluelump: $J = 2$, $P = -$,

    $$G_z^{(\mathrm{electric},J=2)}(\mathbf{x})|\Omega\rangle \quad = \quad \mathrm{Tr}\Big(\phi^{(1)}(\mathbf{x})\Big(D_x B_y(\mathbf{x}) + D_y B_x(\mathbf{x})\Big)\Big)|\Omega\rangle$$

    (and cyclic).

The gluelump spectrum (2)

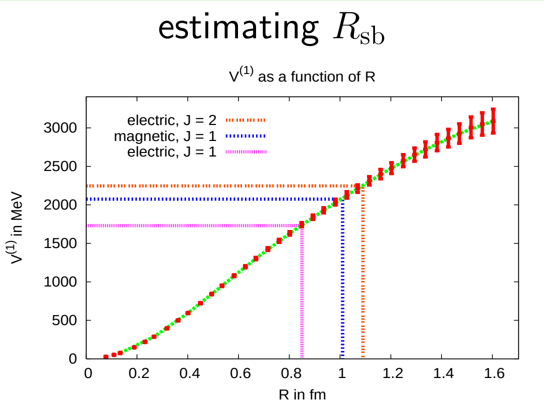
- We consider temporal correlations of gluelump trial states,

$$\mathcal{C}^{(\ldots)}_{\mathrm{gluelump}}(T) \quad = \quad \langle\Omega|\Big(G^{(\ldots)}_j(\mathbf{x},T)\Big)^\dagger G^{(\ldots)}_j(\mathbf{x},T)|\Omega\rangle,$$

  and compute the corresponding gluelump masses from effective mass plateaus,

$$m^{(\ldots)}_{\mathrm{effective,gluelump}}(T) \quad = \quad -\frac{1}{a}\ln\frac{\mathcal{C}^{(\ldots)}_{\mathrm{gluelump}}(T)}{\mathcal{C}^{(\ldots)}_{\mathrm{gluelump}}(T-a)}.$$

The gluelump ...



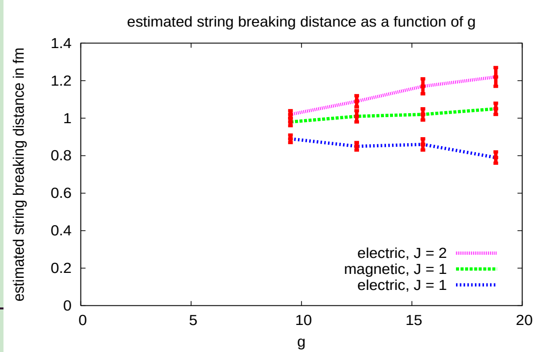
- Numerical results related to gluelump masses:
  - Gluelump masses depend on the regularization and tend to infinity in the continuum limit, i.e. gluelump masses are not physically meaningful.
  - Differences between gluelump masses are physical observables.
  - The string breaking distance R_{sb} can be estimated by solving $V^{(1)}(R_{\mathrm{sb}}) = 2m^{(\ldots)}$.

gluelump	$m^{(\ldots)}$	$m - m^{(\mathrm{electric},J=1)}$	estimated R_{sb}
magnetic, $J = 1$	$1037(21)\,\mathrm{MeV}$	$172(38)\,\mathrm{MeV}$	$1.01(3)\,\mathrm{fm}$
electric, $J = 1$	$865(17)\,\mathrm{MeV}$	–	$0.85(2)\,\mathrm{fm}$
electric, $J = 2$	$1123(23)\,\mathrm{MeV}$	$257(40)\,\mathrm{MeV}$	$1.09(3)\,\mathrm{fm}$

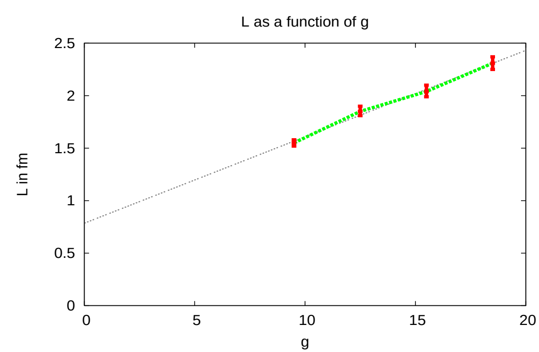
Scaling

- Like in lattice gauge theory the physical scale can be changed by changing the value of the coupling constant g (the scale is set by identifying σ with $\sigma_{\mathrm{physical}} = 4.2/\mathrm{fm}^2$).
- Previous "pseudoparticle papers": the dimensionless ratios $\chi^{1/4}/\sigma^{1/2}$ and $T_{\mathrm{critical}}/\sigma^{1/2}$ are essentially independent of g.
- The estimated string breaking distances $R_{\mathrm{sb}}(\mathrm{magnetic}, J = 1)$, $R_{\mathrm{sb}}(\mathrm{electric}, J = 1)$ and $R_{\mathrm{sb}}(\mathrm{electric}, J = 2)$ vary by $\approx 5\%$, $\approx 10\%$ and $\approx 20\%$, while the extension of the spacetime region is increased by $\approx 50\%$.

estimated R_{sb}



extension of spacetime



The static adjoint potential (1)

- Symmetry group of states constrained by a pair of static sources $\phi^{(1)}(\mathbf{x})$ and $\phi^{(1)}(\mathbf{y})$:
  - $J_z = 0 \quad \rightarrow \quad \mathrm{SO}(2) \otimes P_z \otimes P_x$.
  - $J_z \neq 0 \quad \rightarrow \quad \mathrm{SO}(2) \otimes P_z$.
- The string trial states

$$(\phi^{(1)}(\mathbf{x}))^\dagger U^{(1)}(\mathbf{x};\mathbf{y})\phi^{(1)}(\mathbf{y})|\Omega\rangle \quad , \quad |\mathbf{x}-\mathbf{y}| \quad = \quad R$$

have quantum numbers $J_z = 0$, $P_z = +$ and $P_x = +$.

- Since we want to observe the decay of a string state into a two-gluelump state, we need two-gluelump trial states with the same quantum numbers:

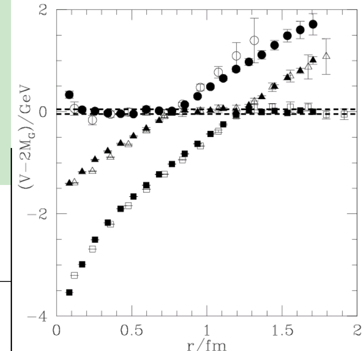
$$\Big(G_x^{(\ldots)}(\mathbf{x})G_x^{(\ldots)}(\mathbf{y}) + G_y^{(\ldots)}(\mathbf{x})G_y^{(\ldots)}(\mathbf{y}) + G_z^{(\ldots)}(\mathbf{x})G_z^{(\ldots)}(\mathbf{y})\Big)|\Omega\rangle$$
$$\Big(G_x^{(\ldots)}(\mathbf{x})G_x^{(\ldots)}(\mathbf{y}) + G_y^{(\ldots)}(\mathbf{x})G_y^{(\ldots)}(\mathbf{y}) - 2G_z^{(\ldots)}(\mathbf{x})G_z^{(\ldots)}(\mathbf{y})\Big)|\Omega\rangle.$$

The static adjoint potential (2)

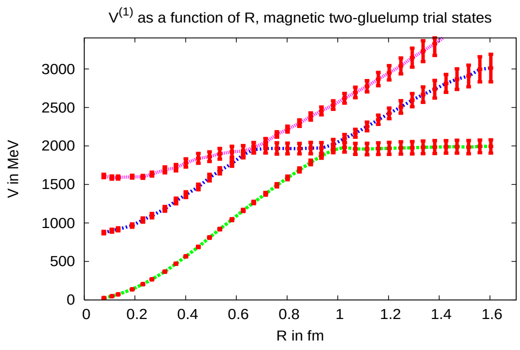
- We extract the adjoint potential from correlation matrices containing both string trial states and two gluelump trial states via effective masses.
  - The potential saturates at $V \approx 2m^{(\dots)}$.
  - The string breaking distances
    - $R_{\mathrm{sb}}^{(\mathrm{magnetic},J=1)} \approx 1.0\,\mathrm{fm}$
    - $R_{\mathrm{sb}}^{(\mathrm{electric},J=1)} \approx 0.85\,\mathrm{fm}$

  and the level ordering in qualitative agreement with lattice results ($R_{\mathrm{sb,lattice}}^{(\mathrm{magnetic},J=1)} = 1.0\,\mathrm{fm} \dots 1.25\,\mathrm{fm}$).

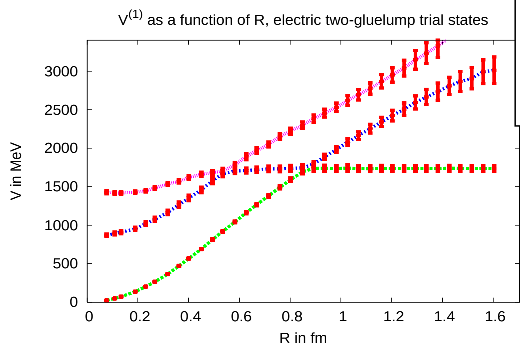
lattice result (P. de Forcrand, O. Philipsen, hep-lat/9912050)



magnetic trial states



electric trial states



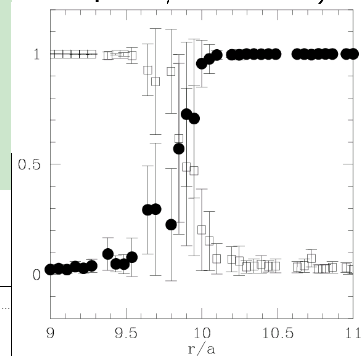
The static adjoint potential (3)

- Mixing analysis to investigate, whether the string really breaks:
  - During the computation of effective masses we obtain approximations

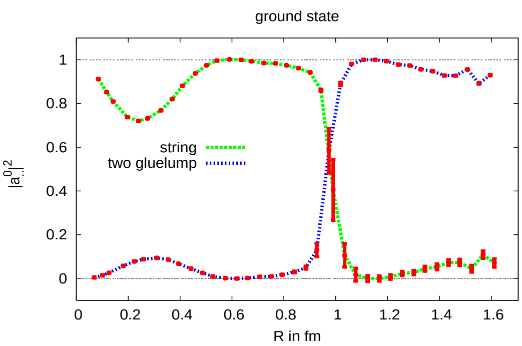
$$|0\rangle \approx a^0_{\text{string}}|\text{string}\rangle + a^0_{\text{two-gluelump}}|\text{two-gluelump}\rangle$$
$$|1\rangle \approx a^1_{\text{string}}|\text{string}\rangle + a^1_{\text{two-gluelump}}|\text{two-gluelump}\rangle,$$

    where $|\text{string}\rangle$ and $|\text{two-gluelump}\rangle$ are normalized trial states.
  - The amplitudes $a^j_{\ldots}$ indicate a smooth but rapid transition between string and two-gluelump states.

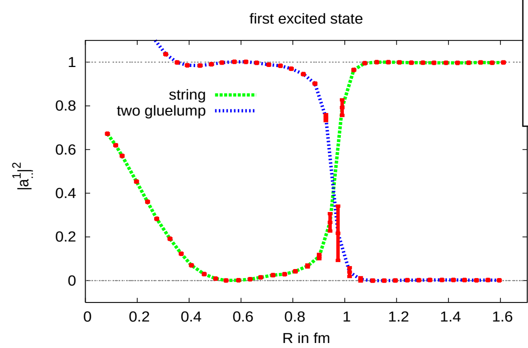
lattice result (P. de Forcrand, O. Philipsen, hep-lat/9912050)



ground state (magnetic)



first excited state (magnetic)



Summary

- The static potential in the fundamental representation is linear for large separations ($\rightarrow$ confinement).
- Static potentials in higher representations exhibit Casimir scaling.
- The static potential in the adjoint representation is in qualitative agreement with lattice results:
  - String breaking at $R_{\mathrm{sb}} \approx 1.0\,\mathrm{fm}$.
  - Level ordering correct: the "first excited string state" is below the "two gluelump ground state".
  - Mixing analysis indicates a smooth transition between a string and a two gluelump state, when two static charges are separated adiabatically; the transition region is very narrow.

Further results and ongoing projects (1)

- SU(2) Yang-Mills theory:
  - Dimensionless ratios $\chi^{1/4}/\sigma^{1/2}$ (χ: topological susceptibility) and $T_\mathrm{critical}/\sigma^{1/2}$ (T_critical: critical temperature of the confinement deconfinement phase transition) exhibit excellent scaling behaviors and are in qualitative agreement with lattice results (F. Lenz, M.W., 2005).
  - Identification of typical properties of physically relevant gauge field configurations: extended structures, transverse degrees of freedom (M.W., 2006).
  - Glueball spectrum in qualitative agreement with lattice results (F. Lenz, J. W. Negele, M. Thies, 2007).
  - Cluster formation of topological charge seems to be in agreement with lattice results (E.-M. Ilgenfritz, S. Solbrig, 2008).

Further results and ongoing projects (2)

- $1+1$ dimensional Gross-Neveu model:
  - Fermionic fields successfully treated within the pseudoparticle approach (M.W., 2007).
- Schwinger model (ongoing):
  - Goal: identification of physically relevant fermionic field configurations (E. Radatz, M.W.).
- $1+1$ dimensional Wess-Zumino model (ongoing):
  - Goal: pseudoparticle regularization of supersymmetric theories, which are difficult to handle on the lattice, due to lack of translational invariance.