Fermionic fields in the pseudoparticle approach

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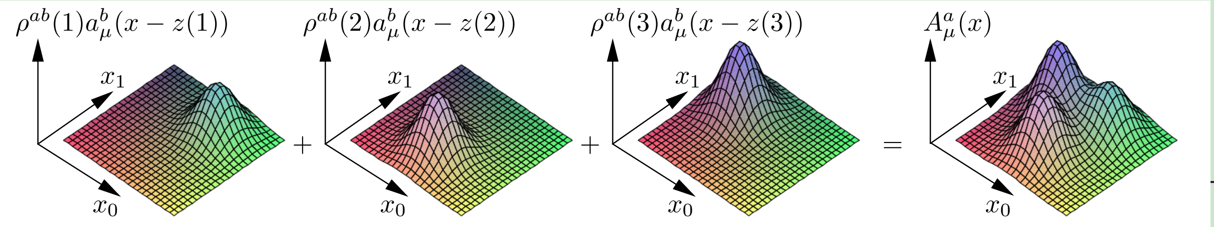
Introduction (1)

- Models for SU(2) Yang-Mills theory with a small number of degrees of freedom:
  - Regular gauge instanton and meron models (F. Lenz, J. W. Negele, M. Thies, 2003).
  - Pseudoparticle approach (F. Lenz, M.W., 2005).
  - Calorons with non-trivial holonomy (P. Gerhold, E.-M. Ilgenfritz, M. Müller-Preussker, 2006).
  - Ensemble of dyons (D. Diakonov, V. Petrov, 2007).
- Basic principle: restrict the path integral to those gauge field configurations, which can be represented by a linear superposition of a small number of localized building blocks (instantons, merons, akyrons, calorons, dyons, ...).



Introduction (2)

- Successes of these models:
  - Linear potential between two static charges at large separations (confinement).
  - Confinement-deconfinement phase transition.
  - String tension, topological susceptibility, critical temperature, ... in qualitative agreement with lattice results.
- Intention: get a better understanding of confining gauge field configurations and the mechanism of confinement.
- **How can fermions be included in such models?**
- In this talk: first steps in this direction.
  - Basic principle of the PP approach in fermionic theories.
  - Testing ground: the $1+1$-dimensional Gross-Neveu model in the large-N-limit (phase diagram, chiral condensate).

Basic principle (1)

- Starting point: action and partition function of **any theory with quadratic fermion interaction**:

$$S[\psi,\bar{\psi},\phi] = \int d^{d+1}x \left(\bar{\psi}Q(\phi)\psi + \mathcal{L}(\phi)\right)$$

$$Z = \int D\psi \, D\bar{\psi} \, D\phi \, e^{-S[\psi,\bar{\psi},\phi]}$$

(Q: Dirac operator; ϕ: any type and number of bosonic fields, e.g. the non-Abelian gauge field in QCD).

Basic principle (2)

- Consider only those fermionic field configurations, which can be represented by a linear superposition a fixed number of localized building blocks:

$$\psi(x) \quad = \quad \sum_j \underbrace{\eta_j G_j(x)}_{j\text{-th PP}}$$

(η_j: Grassmann valued spinors; G_j: functions, which are localized in space as well as in time, i.e. PPs).

- Define the functional integration over all fermionic field configurations as an integration over the Grassmann valued spinors:

$$\int D\psi \, D\bar{\psi} \ldots \quad = \quad \int \left(\prod_j d\eta_j \, d\bar{\eta}_j \right) \ldots$$

Basic principle (3)

- Integrate out the fermions:

$$S_{\text{effective}}[\phi] \quad = \quad \int d^{d+1}x\, \mathcal{L}(\phi) - \ln\Big(\det\Big(\langle G_j|Q|G_{j'}\rangle\Big)\Big)$$

$$Z \quad \propto \quad \int D\phi\, e^{-S_{\text{effective}}[\phi]}$$

  ($\langle G_j|Q|G_{j'}\rangle$ is a finite matrix; **"Q-regularization"**).

- If $\det(Q)$ is real and positive, $\det(Q) = \sqrt{\det(Q^\dagger Q)}$. This suggests another PP regularization:

$$S_{\text{effective}}[\phi] \quad = \quad \int d^{d+1}x\, \mathcal{L}(\phi) - \frac{1}{2}\ln\Big(\det\Big(\langle G_j|Q^\dagger Q|G_{j'}\rangle\Big)\Big)$$

  (**"$Q^\dagger Q$-regularization"**).

- **The "$Q^\dagger Q$-regularization" has significant advantages over the naive "Q-regularization".**

Q versus $Q^\dagger Q$ (1)

- For the sake of simplicity: consider all PPs G_j to be orthonormal, i.e. $\langle G_j | G_{j'} \rangle = \delta_{jj'}$ (this is not a restriction!).
- The problem of the Q-regularization:
  - **Applying the Dirac operator Q to one of the PPs $G_{j'}$ in general yields a function, which is (partially) outside the PP function space $\mathrm{span}\{G_n\}$, i.e.**

    $$QG_{j'}(x) \quad = \quad \sum_k a_{j'k} G_k(x) + h_{j'} H_{j'}(x)$$

    ($H_{j'}$ normalized, $H_{j'} \perp \mathrm{span}\{G_n\}$).
  - If $|\sum_k a_{j'k} G_k| \gg |h_{j'}| \quad \rightarrow \quad$ no problem.
  - If $|\sum_k a_{j'k} G_k| \lesssim |h_{j'}| \quad \rightarrow \quad$ when computing the fermionic matrix $\langle G_j | Q | G_{j'} \rangle$, a significant part of $QG_{j'}$ is simply ignored, just because $H_{j'}$ is perpendicular to the PP function space $\mathrm{span}\{G_n\}$.

Q versus $Q^\dagger Q$ (2)

- The advantage of the $Q^\dagger Q$-regularization:
  - **Both the left hand sides $\langle G_j | Q^\dagger$ and the right hand sides $Q | G_{j'} \rangle$ of the matrix elements $\langle G_j | Q^\dagger Q | G_{j'} \rangle$ might be outside the PP function space $\mathrm{span}\{G_n\}$, but they form the same function space, $\mathrm{span}\{Q G_n\}$, in which their overlap is computed.**
- For "better arguments" cf. M.W., arXiv:0704.3023 [hep-lat].

Testing ground: Gross-Neveu model (1)

- Action and partition function of the $1+1$-dimensional Gross-Neveu model:

$$S = \int d^2x \left(\sum_{n=1}^{N} \bar{\psi}^{(n)} \Big(\gamma_0(\partial_0 + \mu) + \gamma_1 \partial_1\Big) \psi^{(n)} - \frac{g^2}{2} \left(\sum_{n=1}^{N} \bar{\psi}^{(n)} \psi^{(n)}\right)^2\right)$$

$$Z = \int \left(\prod_{n=1}^{N} D\psi^{(n)} \, D\bar{\psi}^{(n)}\right) e^{-S}$$

(N: number of flavors; μ: chemical potential; g: coupling constant).

Testing ground: Gross-Neveu model (2)

- Introduce a real scalar field σ and integrate out the fermions:

$$S_{\text{effective}} = N\left(\frac{1}{2\lambda}\int d^2x\,\sigma^2 - \ln\left(\det\left(\gamma_0(\partial_0+\mu)+\gamma_1\partial_1+\sigma\right)\right)\right)$$

$$Z \propto \int D\sigma\, e^{-S_{\text{effective}}}$$

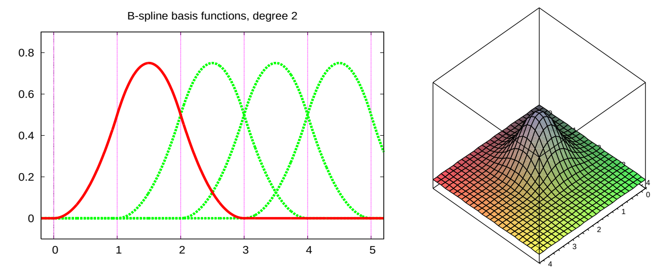
($\lambda = Ng^2$).

- Large-N limit:
  - $N \to \infty$, $\lambda = Ng^2 = \text{constant}$.
  - There is no need to compute the σ-path integral anymore.
  - It is sufficient to minimize $S_{\text{effective}}$ with respect to σ.
  - $\sigma = -g^2 \sum_{n=1}^{N} \bar{\psi}^{(n)}\psi^{(n)}$ (chiral condensate).

Fermionic PPs

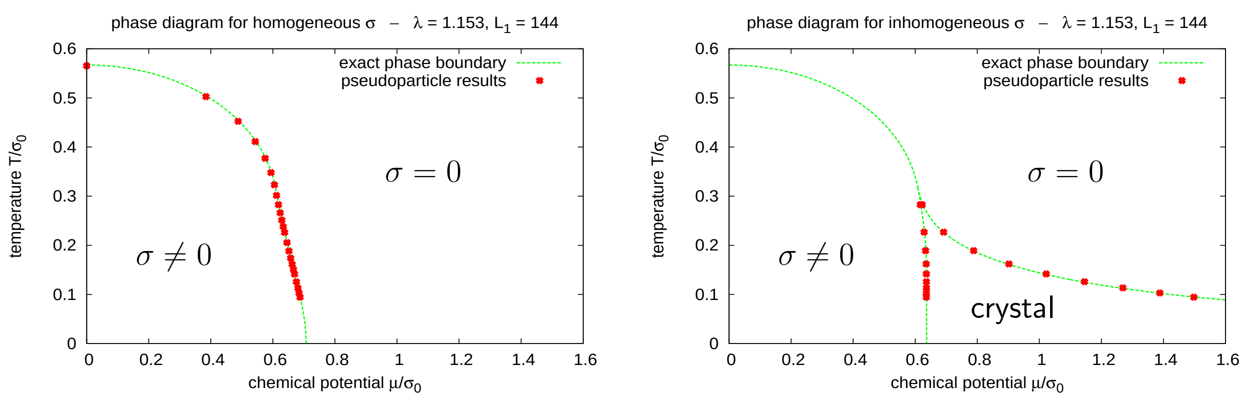
- Fermionic PPs in this talk: a large number of uniformly distributed "hat functions".
  - B-spline basis functions of degree 2.
  - 8 ... 56 PPs in time direction, 144 PPs in space direction.
  - Antiperiodic boundary conditions in time direction, periodic boundary conditions in space direction.
- "Sensible set of field configurations" (any not too heavily oscillating field configuration can be approximated)
  $\longrightarrow$ we can expect to reproduce correct Gross-Neveu results.
- Piecewise polynomial functions
  $\longrightarrow$ certain integrals can be calculated analytically.

PPs G_j as functions of x



Phase diagram

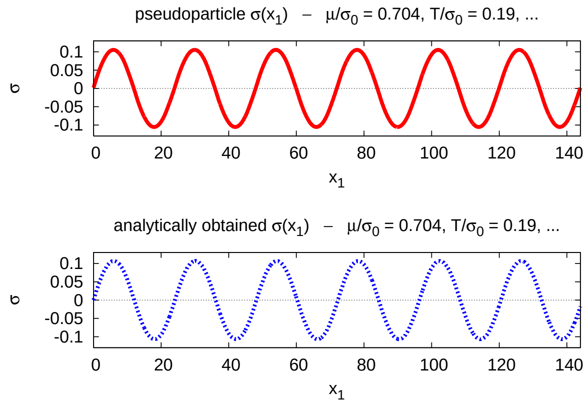
- Q**-regularization: completely wrong and useless results.**
  - No improvement, when using a larger number of PPs.
  - No improvement, when using a different type of PPs.
- $Q^\dagger Q$**-regularization: excellent agreement with analytical results.**
  - Homogeneous chiral condensate: analytical results by U. Wolff, 1985.
  - Inhomogeneous chiral condensate: analytical results by O. Schnetz, M. Thies, K. Urlichs, 2004.



Chiral condensate

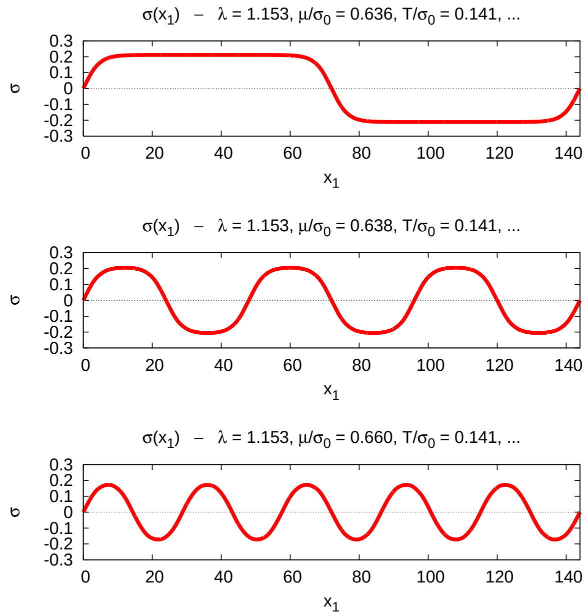
- $Q^\dagger Q$**-regularization: excellent agreement with analytical results.**
  - The $\sin$-like behavior "inside the crystal phase" changes to a kink-antikink structure, when approaching the left phase boundary.

PP result



analytical result

close to the left phase boundary



inside the crystal phase

Summary

- Always apply the $Q^\dagger Q$-regularization and not the naive Q-regularization, when including fermionic fields in the pseudoparticle approach.
- The application of the PP approach to compute the phase diagram of the $1{+}1$-dimensional Gross-Neveu model in the large-N-limit has been a successful test.

Outlook

- Next steps:
  - Apply the PP approach to QCD.
  - Try to identify a small number of physically relevant fermionic PPs (PPs, which are able to approximate typical low lying eigenmodes of the Dirac operator?).
- Current research:
  - Chiral symmetry breaking by computing the low lying eigenmodes of the Dirac operator in the quenched approximation (Banks-Casher relation).
- Goals:
  - **Obtain a model with a small number of degrees of freedom, which exhibits chiral symmetry breaking and a confinement deconfinement phase transition at the same time.**
  - Compute further fermionic observables: pion masses, ...