$b\bar{b}ud$ tetraquarks from lattice QCD

Part 1 – Lattice QCD static potentials and the Born-Oppenheimer approximation

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Basic idea: lattice QCD + BO (1)

- Study heavy-heavy-light-light tetraquarks $\bar{b}bqq$ in two steps.
  
  (1) Compute potentials of two static quarks $\bar{b}b$ in the presence of two lighter quarks $qq$ ($q \in \{u, d, s, c\}$) using lattice QCD.

  (2) Check, whether these potentials are sufficiently attractive to host bound states or resonances ($\rightarrow$ tetraquarks) by using techniques from quantum mechanics and scattering theory.

$((1) + (2) \rightarrow$ Born-Oppenheimer approximation).

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Basic idea: lattice QCD + BO (2)

- The talk summarizes:

- For recent work from other groups using a similar approach see e.g.:

- Recent related work (quark models, effective field theories, and QCD sum rules):

- Full lattice QCD work is covered in the second part of this talk by Martin Pflaumer.
Why are such studies important? (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \ldots$

- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.q. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).

- Certain mesons are poorly understood (e.g. significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
  - 2 quarks and 2 antiquarks (tetraquark),
  - a quark-antiquark pair and gluons (hybrid meson),
  - only gluons (glueball).

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Why are such studies important? (2)

- Indications for tetraquark structures:
  - Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$:
    * Mass suggests a $b\bar{b}$ pair ...
    * ... but $b\bar{b}$ is electrically neutral ...?
    * **Easy to understand, when assuming a tetraquark structure:**
      $Z_b(\ldots)^+ \equiv b\bar{b}u\bar{d}$ ($u \rightarrow +2/3 \, e$, $\bar{d} \rightarrow -1/3 \, e$).

    ![diagram]

  - Electrically charged $Z_c$ states:
    * Similar to $Z_b$ states.
Outline

- $\bar{b}bqq / BB$ potentials.
- Stable $\bar{b}bqq$ tetraquarks.
- $\bar{b}bqq$ tetraquark resonances.
- Inclusion of heavy spin effects.
\( \bar{b}\bar{b}qq / BB \) potentials (1)

- Spins of static antiquarks \( \bar{b}\bar{b} \) are irrelevant (they do not appear in the Hamiltonian).
- At large \( \bar{b}\bar{b} \) separation \( r \), the four quarks will form two static-light mesons \( \bar{b}q \) and \( \bar{b}q \).
- Consider only pseudoscalar/vector mesons \( (j^P = (1/2)^-, \text{PDG: } B, B^*) \) and scalar/pseudovector mesons \( (j^P = (1/2)^+, \text{PDG: } B^*_0, B^*_1) \), which are among the lightest static-light mesons \( (j: \text{spin of the light degrees of freedom}) \).
- Compute and study the dependence of \( \bar{b}\bar{b} \) potentials in the presence of \( qq \) on
  - the “light” quark flavors \( q \in \{u, d, s, c\} \) (isospin, flavor),
  - the “light” quark spin (the static quark spin is irrelevant),
  - the type of the meson \( B, B^* \) and/or \( B^*_0, B^*_1 \) (parity).

→ Many different channels: attractive as well as repulsive, different asymptotic values ...
Rotational symmetry broken by static quarks $\bar{b}b$.

Remaining symmetries and quantum numbers:

- $j_z \equiv \Lambda$: rotations around the separation axis (e.g. $z$ axis).
- $P \equiv \eta$: parity.
- $P_x \equiv \epsilon$: reflection along an axis perpendicular to the separation axis (e.g. $x$ axis).

To extract the potential(s) of a given sector $(I, I_z, |j_z|, P, P_x)$, compute the temporal correlation function of the trial state(s)

$$\left(C \Gamma\right)_{AB} \left(C \tilde{\Gamma}\right)_{CD} \left(\bar{Q}_C(-r/2)q_A^{(1)}(-r/2)\right)\left(\bar{Q}_D(+r/2)q_B^{(2)}(+r/2)\right)|\Omega\rangle.$$ 

- $q^{(1)}q^{(2)} \in \{ud - du, uu, dd, ud + du, ss, cc\}$ (isospin $I, I_z$, flavor).
- $\Gamma$ is an arbitrary combination of $\gamma$ matrices (spin $|j_z|$, parity $P, P_x$).
- $\tilde{\Gamma} \in \{(1 - \gamma_0)\gamma_5, (1 - \gamma_0)\gamma_j\}$ (irrelevant).
The most attractive potential has

- isospin \( I = 0 \),
- light quark spin \( j_z = 0 \),

and its asymptotic value at large \( \bar{b}b \) separations \( r \) corresponds to \( 2 \times m_{B^*} \).

Parameterize lattice results by

\[
V_{\bar{b}b}(r) = -\frac{\alpha}{r} \exp \left( -\left(\frac{r}{d}\right)^p \right) + V_0.
\]

- \( 1/r \): 1-gluon exchange at small \( \bar{b}b \) separations.
- \( \exp(-r/d)^p \): color screening at large \( \bar{b}b \) separations due to meson formation.
- Fit parameters \( \alpha, d \) and \( V_0 \) obtained by \( \chi^2 \) minimizing fits; \( p = 2 \) from quark models.
Stable $\overline{b}bqq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\overline{b}b$ using the previously computed $\overline{b}bqq/BB$ potentials,

$$\left(-\frac{1}{2\mu}\nabla^2 + V_{\overline{b}b}(r)\right)\psi(r) = E\psi(r), \quad \mu = m_b/2.$$ 

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\overline{b}bqq$ tetraquarks.

- There is a bound state for orbital angular momentum $L = 0$ of $\overline{b}b$:
  - Binding energy $-E = 90^{+43}_{-36}$ MeV with respect to the $BB^*$ threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.  

- No further bound states ... but there could still be resonances.
**\( \bar{b}bqq \) tetraquark resonances**

- Comparatively easy to investigate within our approach (since we have potentials \( V_{\bar{b}b}(r) \), no Lüsher method etc. necessary).

- Use standard methods from scattering theory:
  - Solve Schrödinger equation with potential \( V_{\bar{b}b}(r) \) and appropriate boundary conditions \( \rightarrow \) partial wave scattering amplitudes \( t_L(E) \).
  - Use partial wave scattering amplitudes \( t_L(E) \) to ...
    * ... determine scattering phase shifts via \( 1 + 2it_L = e^{2i\delta_L} \)
      \( \rightarrow \) sharp rise (ideally from \( \approx 0 \) to \( \approx \pi \)) indicates resonance mass.
    * ... determine poles of the scattering amplitudes \( t_L(E) \)
      \( \rightarrow \) real part of a pole \( \equiv \) resonance mass \( (m = \text{Re}(E_{\text{pole}})) \)
      \( \rightarrow \) imaginary part of a pole \( \equiv \) resonance width \( (\Gamma = -2\text{Im}(E_{\text{pole}})) \).

- There is a resonance for \( L = 1 \):
  - Resonance mass \( E = +17^{+4}_{-4} \) MeV above the \( BB \) threshold.
  - Decay width \( \Gamma = 112^{+90}_{-103} \) MeV.
  - Quantum numbers \( I(J^P) = 0(1^-) \).
Inclusion of heavy spin effects

• Heavy spin effects have been neglected so far (e.g. mass splitting $m_{B^*} - m_B \approx 46$ MeV).

• Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $-E = 90^{+43}_{-36}$ MeV.

• Moreover, two competing effects:
  – The attractive $\bar{b}\bar{b}ud$ channel corresponds to a linear combination of $BB^*$ and/or $B^*B^*$.
  – The $BB^*$ interaction is a superposition of attractive and repulsive $\bar{b}\bar{b}ud$ potentials.

• Will there still be a bound state, when heavy spin effects are taken into account?
  – “Yes”.
  – We have included heavy spin effects by solving a coupled channel Schrödinger equation.
  – Binding energy $E = -59^{+38}_{-30}$ MeV.
  – Tetraquark is approximately a 50%/50% superposition of $BB^*$ and $B^*B^*$ (strong attraction more important than light constituents).

• Work in progress: inclusion of heavy spin effects for the $I(J^P) = 0(1^-)$ tetraquark resonance.

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