$\bar{b}\bar{b}ud$ tetraquarks from lattice QCD

Part 1 – Lattice QCD static potentials and the Born-Oppenheimer approximation

Marc Wagner
Goethe-Universität Frankfurt, Institut für Theoretische Physik
mwagner@itp.uni-frankfurt.de
https://itp.uni-frankfurt.de/∼mwagner/

in collaboration with Pedro Bicudo, Marco Cardoso, Krzystof Cichy, Jakob Hoffmann, Antje Peters, Martin Pflaumer, Jonas Scheunert, Björn Wagenbach, Andre Zimermmane

July 24, 2020

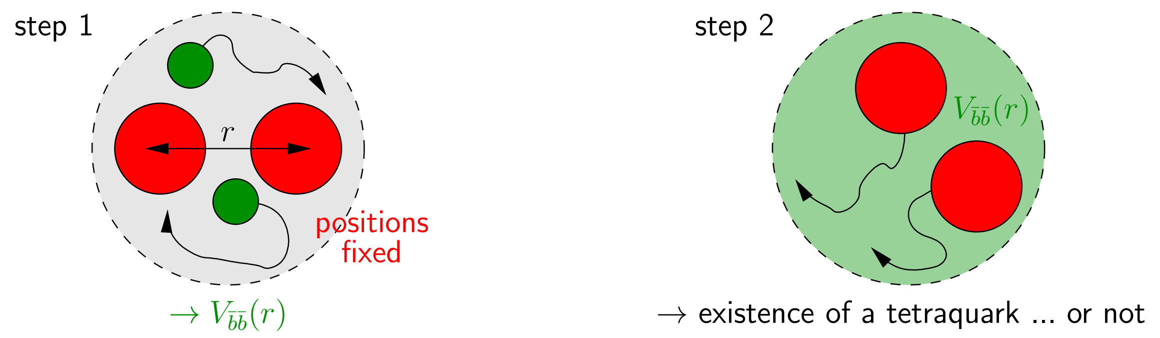
Basic idea: lattice QCD + BO (1)

- Study heavy-heavy-light-light tetraquarks $\bar{b}\bar{b}qq$ in two steps.

  (1) **Compute potentials of two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s, c\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** (→ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

  ((1) + (2) → Born-Oppenheimer approximation).



Basic idea: lattice QCD + BO (2)

- The talk summarizes:
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]

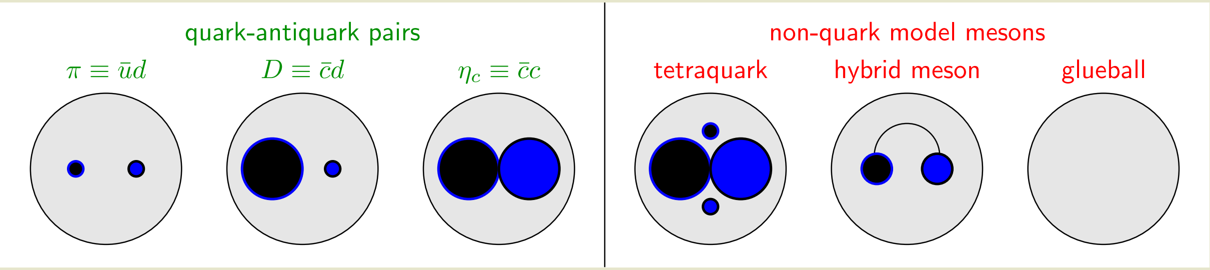
- For recent work from other groups using a similar approach see e.g.:
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571]]
  [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953]]
  [E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]

- Recent related work (quark models, effective field theories, and QCD sum rules):
  [M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666]]
  [E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, 202002 (2017) [arXiv:1707.09575]]
  [Z. G. Wang, Acta Phys. Polon. B **49**, 1781 (2018) [arXiv:1708.04545]]
  [W. Park, S. Noh and S. H. Lee, Acta Phys. Polon. B **50**, 1151-1157 (2019) [arXiv:1809.05257]]
  [B. Wang, Z. W. Liu and X. Liu, Phys. Rev. D **99**, 036007 (2019) [arXiv:1812.04457]]
  [M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie and L. S. Geng, Phys. Rev. D **99**, 094018 (2019) [arXiv:1902.03044]]

- Full lattice QCD work is covered in the second part of this talk by Martin Pflaumer.

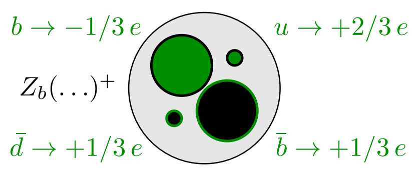
Why are such studies important? (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \ldots$
- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.g. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).
- Certain mesons are poorly understood (e.g. significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
  - **2 quarks and 2 antiquarks (tetraquark)**,
  - **a quark-antiquark pair and gluons (hybrid meson)**,
  - **only gluons (glueball)**.



Why are such studies important? (2)

- Indications for tetraquark structures:
  - Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$:
    - Mass suggests a $b\bar{b}$ pair ...
    - ... but $b\bar{b}$ is electrically neutral ...?
    - **Easy to understand, when assuming a tetraquark structure**: $Z_b(\ldots)^+ \equiv b\bar{b}u\bar{d}$ ($u \rightarrow +2/3\,e$, $\bar{d} \rightarrow -1/3\,e$).



  - Electrically charged Z_c states:
    - Similar to Z_b states.

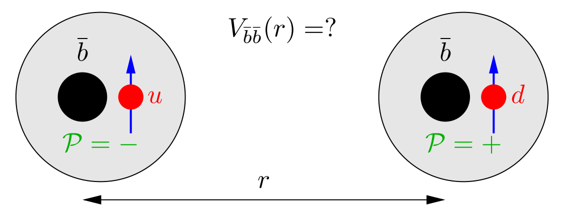
Outline

- $\bar{b}\bar{b}qq$ / BB potentials.
- Stable $\bar{b}\bar{b}qq$ tetraquarks.
- $\bar{b}\bar{b}qq$ tetraquark resonances.
- Inclusion of heavy spin effects.

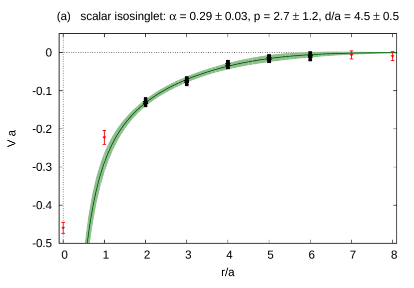
$\bar{b}\bar{b}qq$ / BB potentials (1)

- Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).
- At large $\bar{b}\bar{b}$ separation r, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Consider only pseudoscalar/vector mesons ($j^P = (1/2)^-$, PDG: B, B^*) and scalar/pseudovector mesons ($j^P = (1/2)^+$, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons (j: spin of the light degrees of freedom).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of qq on
  - the "light" quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^* (parity).

$\rightarrow$ Many different channels: attractive as well as repulsive, different asymptotic values ...



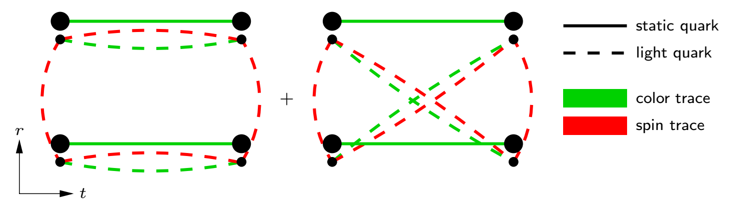
$\bar{b}\bar{b}qq$ / BB potentials (2)



- Rotational symmetry broken by static quarks $\bar{b}\bar{b}$.
- Remaining symmetries and quantum numbers:
  - $j_z \equiv \Lambda$: rotations around the separation axis (e.g. z axis).
  - $P \equiv \eta$: parity.
  - $P_x \equiv \epsilon$: reflection along an axis perpendicular to the separation axis (e.g. x axis).
- To extract the potential(s) of a given sector $(I, I_z, |j_z|, P, P_x)$, compute the temporal correlation function of the trial state(s)

$$\Big(C\Gamma\Big)_{AB}\Big(C\tilde{\Gamma}\Big)_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q^{(1)}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q^{(2)}_B(+\mathbf{r}/2)\Big)|\Omega\rangle.$$

  - $q^{(1)}q^{(2)} \in \{ud - du\,,\, uu\,,\, dd\,,\, ud + du\,,\, ss\,,\, cc\}$ (isospin I, I_z, flavor).
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity P, P_x).
  - $\tilde{\Gamma} \in \{(1 - \gamma_0)\gamma_5\,,\, (1 - \gamma_0)\gamma_j\}$ (irrelevant).



$\bar{b}\bar{b}qq$ / BB potentials (3)

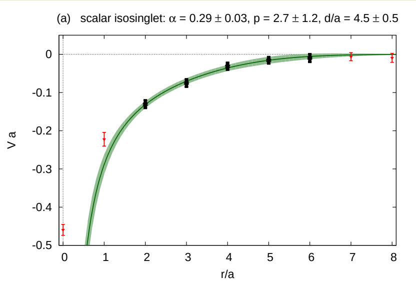
- The most attractive potential has
  - isospin $I = 0$,
  - light quark spin $j_z = 0$,

  and ist asymptotic value at large $\bar{b}\bar{b}$ separations r corresponds to $2 \times m_{B^*}$.

- Parameterize lattice results by

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r} \exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0 .$$

  - $1/r$: 1-gluon exchange at small $\bar{b}\bar{b}$ separations.
  - $\exp(-(r/d)^p)$: color screening at large $\bar{b}\bar{b}$ separations due to meson formation.
  - Fit parameters α, d and V_0 obtained by χ^2 minimizing fits; $p = 2$ from quark models.

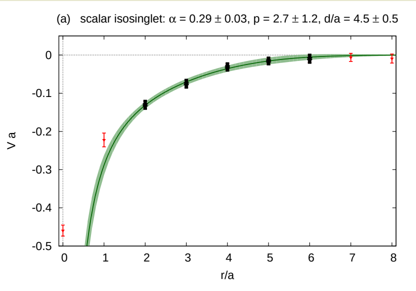


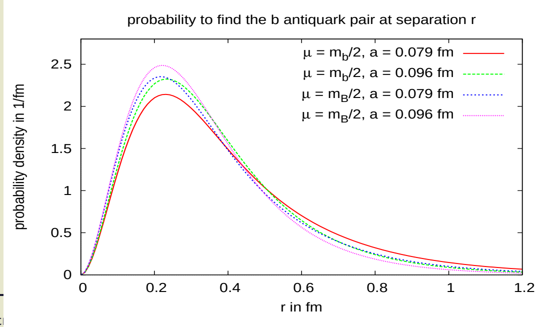
Stable $\bar{b}\bar{b}qq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

$$\Big(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\Big)\psi(\mathbf{r}) \quad = \quad E\psi(\mathbf{r}) \quad , \quad \mu \quad = \quad m_b/2.$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $-E = 90^{+43}_{-36}$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.
- No further bound states ... but there could still be resonances.



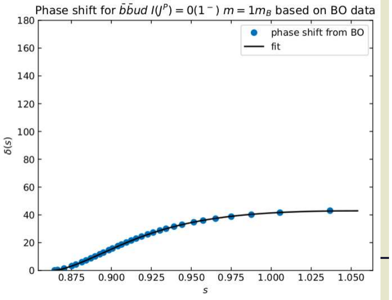


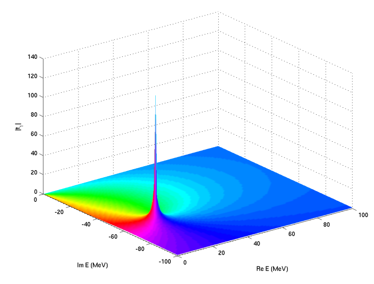
$\bar b\bar b qq$ tetraquark resonances

- Comparatively easy to investigate within our approach (since we have potentials $V_{\bar b\bar b}(r)$, no Lüscher method etc. necessary).

- Use standard methods from scattering theory:
  - Solve Schrödinger equation with potential $V_{\bar b\bar b}(r)$ and appropriate boundary conditions $\rightarrow$ partial wave scattering amplitudes $t_L(E)$.
  - Use partial wave scattering amplitudes $t_L(E)$ to ...
    - ... determine scattering phase shifts via $1 + 2it_L = e^{2i\delta_L}$
      $\rightarrow$ sharp rise (ideally from ≈ 0 to $\approx \pi$) indicates resonance mass.
    - ... determine poles of the scattering amplitudes $t_L(E)$
      $\rightarrow$ real part of a pole $\equiv$ resonance mass ($m = \mathrm{Re}(E_\text{pole})$)
      $\rightarrow$ imaginary part of a pole $\equiv$ resonance width ($\Gamma = -2\mathrm{Im}(E_\text{pole})$).

- There is a resonance for $L = 1$:
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma = 112^{+90}_{-103}$ MeV.
  - Quantum numbers $I(J^P) = 0(1^-)$.





Inclusion of heavy spin effects

- Heavy spin effects have been neglected so far (e.g. mass splitting $m_{B^*} - m_B \approx 46$ MeV).
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $-E = 90^{+43}_{-36}$ MeV.
- Moreover, two competing effects:
  - The attractive $\bar{b}\bar{b}ud$ channel corresponds to a linear combination of BB^* and/or B^*B^*.
  - The BB^* interaction is a superposition of attractive and repulsive $\bar{b}\bar{b}ud$ potentials.
- **Will there still be a bound state, when heavy spin effects are taken into account?**
  - "Yes".
  - We have included heavy spin effects by solving a coupled channel Schrödinger equation.
  - Binding energy $E = -59^{+38}_{-30}$ MeV.
  - Tetraquark is approximately a $50\%/50\%$ superposition of BB^* and B^*B^* (strong attraction more important than light constituents).
- Work in progress: inclusion of heavy spin effects for the $I(J^P) = 0(1^-)$ tetraquark resonance.