The spectrum of and forces between B mesons from lattice QCD

Seminar, University of Nicosia, Cyprus

Marc Wagner
Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik
mwagner@th.physik.uni-frankfurt.de
http://people.physik.hu-berlin.de/∼mcwagner/

March 7, 2012

Outline

- I will discuss two lattice QCD projects (which are related):

  (1) **Computation of the spectrum of radially/orbitally excited B mesons.**

  (2) **Computation of spin/isospin/parity dependent forces between B mesons.**

Part 1

Computation of the spectrum of radially/orbitally excited B mesons

[K. Jansen, C. Michael, A. Shindler and M.W. [ETM Collaboration], JHEP **0812**, 058 (2008)]

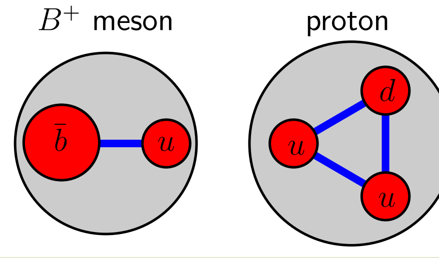
[C. Michael, A. Shindler and M.W. [ETM Collaboration], JHEP **1008**, 009 (2010)]

QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.
- Part of the standard model explaining the formation of hadrons (mesons $= q\bar{q}$, baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.
- Definition of QCD by means of an action simple:

$$
\begin{aligned}
S &= \int d^4x \bigg(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu \Big) + m^{(f)} \Big) \psi^{(f)} + \frac{1}{2g^2} \mathrm{Tr} \Big(F_{\mu\nu} F_{\mu\nu} \Big) \bigg) \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu].
\end{aligned}
$$

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).



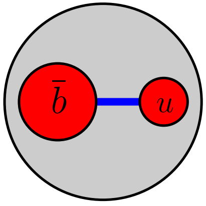
B mesons, static-light mesons

- B meson: a meson made from a heavy b quark ($m_b \approx 4200\,\text{MeV}$) and a light u, d or s quark ($m_l \lesssim 100\,\mathrm{MeV}$), e.g. $B = \{\bar{b}u\,,\,\bar{b}d\}$, $B_s = \bar{b}s$.

- Static limit, i.e. $m_b \to \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $\mathcal{P}$ and half-integer total angular momentum of the light cloud j.

- m_b finite, but heavy:
  - Classify states according to parity $\mathcal{P}$ and total angular momentum J.

$j^\mathcal{P}$ (static-light)	$J^\mathcal{P}$ (finite m_b)
$(1/2)^- \equiv S$	$0^- \equiv B_{(s)}$ $1^- \equiv B^*_{(s)}$
$(1/2)^+ \equiv P_-$ $(3/2)^+ \equiv P_+$	$0^+ \equiv B^*_{(s)0}$ (not in PDG) $1^+ \equiv B^*_{(s)1}$ (not in PDG) $1^+ \equiv B_{(s)1}$ $2^+ \equiv B^*_{(s)2}$
$(3/2)^- \equiv D_-$ $(5/2)^- \equiv D_+$	1^- (no experiment) 2^- (no experiment) 2^- (no experiment) 3^- (no experiment)
$(5/2)^+ \equiv F_-$ ...	2^+ (no experiment) 3^+ (no experiment) ...

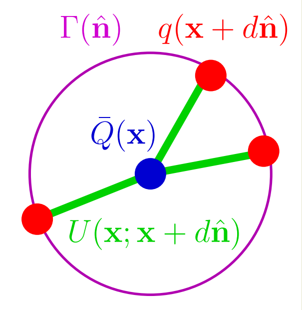


How to compute M(static-light meson)? (1)

- Let $\mathcal{O}(\mathbf{x})$ be a suitable "static-light meson creation operator", i.e. an operator such that $\mathcal{O}(\mathbf{x})|\Omega\rangle$ is a state containing a static-light meson at position $\mathbf{x}$ ($|\Omega\rangle$: vacuum).
- More precisely: ... an operator such that $\mathcal{O}(\mathbf{x})|\Omega\rangle$ has the same quantum numbers ($j^{\mathcal{P}}$, flavor) as the static-light meson of interest.
- Determine the mass of the ground state of the corresponding static-light meson from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times T:

$$\begin{aligned} \mathcal{C}(T) \quad &= \quad \langle\Omega|\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger \mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \quad \langle\Omega|e^{+HT}\Big(\mathcal{O}(\mathbf{x},0)\Big)^\dagger e^{-HT}\mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \sum_n \Big|\langle n|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)T\Big) \quad \approx \quad (\text{for } T \gg 1) \\ &\approx \quad \Big|\langle 0|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{M(\text{static-light meson})} T\Big). \end{aligned}$$

How to compute M(static-light ...



- **General form of a static-light meson creation operator**:

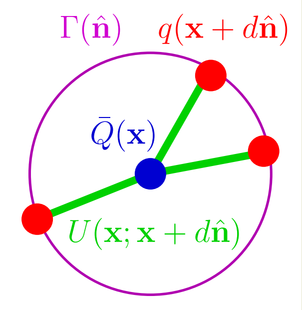
$$\mathcal{O}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

  - $\bar{Q}(\mathbf{x})$ creates an infinitely heavy i.e. static antiquark at position $\mathbf{x}$.
  - $q(\mathbf{x} + d\hat{\mathbf{n}})$ creates a light quark at position $\mathbf{x} + d\hat{\mathbf{n}}$ separated by a distance d from the static antiquark.
  - The spatial parallel transporter

$$U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) = P\left\{\exp\left(+i \int_{\mathbf{x}}^{\mathbf{x}+d\hat{\mathbf{n}}} dz_j\, A_j(\mathbf{z})\right)\right\}$$

    connects the antiquark and the quark in a gauge invariant way via gluons.
  - The integration over the unit sphere $\int d\hat{\mathbf{n}}$ combined with a suitable weight factor $\Gamma(\hat{\mathbf{n}})$ yields well defined total angular momentum J and parity $\mathcal{P}$ ($\Gamma(\hat{\mathbf{n}})$ is a combination of spherical harmonics [$\rightarrow$ angular momentum] and γ-matrices [$\rightarrow$ spin]; Wigner-Eckart theorem).

How to compute M(static-light ...



- **General form of a static-light meson creation operator**:

$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}} \, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **List of operators** (J: total angular momentum; j: total angular momentum of the light cloud; $\mathcal{P}$: parity):

$\Gamma(\hat{\mathbf{n}})$	$J^{\mathcal{P}}$	$j^{\mathcal{P}}$	$\mathrm{O_h}$	lattice $j^{\mathcal{P}}$	notation
$\gamma_5 \ , \ \gamma_5 \gamma_j \hat{n}_j$	$0^- \ [1^-]$	$(1/2)^-$	A_1	$(1/2)^- \ , \ (7/2)^- \ , \ \ldots$	S
$1 \ , \ \gamma_j \hat{n}_j$	$0^+ \ [1^+]$	$(1/2)^+$		$(1/2)^+ \ , \ (7/2)^+ \ , \ \ldots$	P_-
$\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2$ (and cyclic)	$2^+ \ [1^+]$	$(3/2)^+$	E	$(3/2)^+ \ , \ (5/2)^+ \ , \ \ldots$	P_+
$\gamma_5(\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2)$ (and cyclic)	$2^- \ [1^-]$	$(3/2)^-$		$(3/2)^- \ , \ (5/2)^- \ , \ \ldots$	$D_\pm$
$\gamma_1 \hat{n}_2 \hat{n}_3 + \gamma_2 \hat{n}_3 \hat{n}_1 + \gamma_3 \hat{n}_1 \hat{n}_2$	$3^- \ [2^-]$	$(5/2)^-$	A_2	$(5/2)^- \ , \ (7/2)^- \ , \ \ldots$	D_+
$\gamma_5(\gamma_1 \hat{n}_2 \hat{n}_3 + \gamma_2 \hat{n}_3 \hat{n}_1 + \gamma_3 \hat{n}_1 \hat{n}_2)$	$3^+ \ [2^+]$	$(5/2)^+$		$(5/2)^+ \ , \ (7/2)^+ \ , \ \ldots$	$F_\pm$

Lattice QCD (1)

- Goal: compute correlation functions $\mathcal{C}(T)$ of the previously discussed static-light meson creation operators (the corresponding meson masses can directly be read off from their exponential decays).

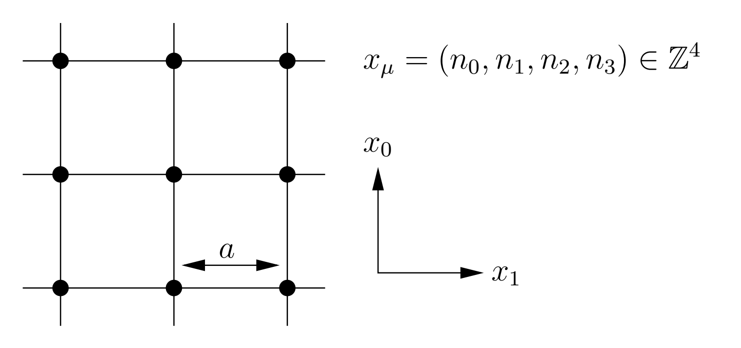
- Use the path integral formulation of QCD,

$$\begin{aligned}\mathcal{C}(T) &= \langle\Omega|\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger\mathcal{O}(\mathbf{x},0)|\Omega\rangle = \\ &= \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger\mathcal{O}(\mathbf{x},0)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.\end{aligned}$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $(\mathcal{O}(\mathbf{x},T))^\dagger\mathcal{O}(\mathbf{x},0)$: function of the quark and gluon fields (cf. previous slides).
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x},t)$ and $A_\mu(\mathbf{x},t)$.
  - $e^{-S[x]}$: weight factor containing the QCD action.

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing
    $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$
    $\longrightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension
    $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus)
    $\longrightarrow$ "no finite size effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \bigg) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    - x_μ: $32^4 \approx 10^6$ lattice sites.
    - $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    - $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    - In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

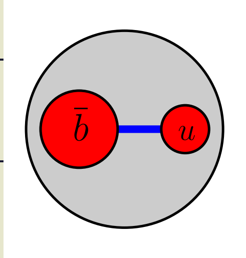
  $\rightarrow$ standard approaches for numerical integration not applicable

  $\rightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

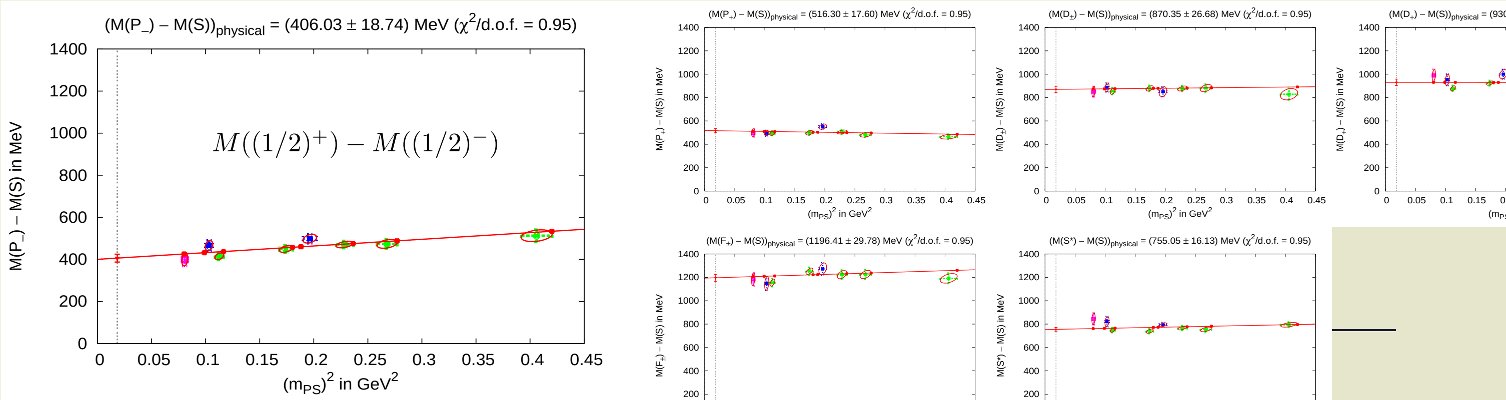
Lattice setup

- Various lattice spacings: $a \approx 0.051\,\text{fm}\,,\ 0.064\,\text{fm}\,,\ 0.080\,\text{fm}$ (corresponding to $48^3 \times 96\,,\ 32^3 \times 64\,,\ 24^3 \times 48$ lattice sites).
- Lattice extensions: $L \approx 2.45\,\text{fm}\,,\ 2.05\,\text{fm}\,,\ 1.92\,\text{fm}$ (periodic boundary conditions).
- Many different pion mass: $m_{\text{PS}} \approx 284 \ldots 637\,\text{MeV}$.

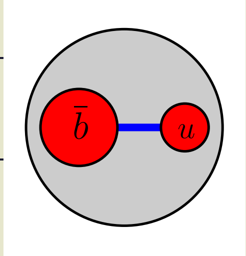
Masses of B and B_s mesons (1)



- Compute static-light meson masses (B/B_s mesons with $m_b \rightarrow \infty$) for different light u/d quark masses and different lattice spacings:
  - Different u/d quark masses to extrapolate to the physical u/d quark mass (due to technical reasons $m_{\mathrm{PS}}^{\text{(lattice)}} \gtrsim 284\,\text{MeV}$, $m_{\mathrm{PS}}^{\text{physical}} \approx 135\,\text{MeV}$).
  - Different lattice spacings to extrapolate to the continuum.
  - Horizontal axis: pion mass $(m_{\mathrm{PS}}^{\text{(lattice)}})^2$.
  - Vertical axis: $M(j^{\mathcal{P}}) - M((1/2)^-)$ mass difference between radially and orbitally excited "B mesons" (B_0^*, B_1^*, B_1, B_2^*, ...) and the "ground state B meson" ($B/B^* \equiv j^{\mathcal{P}} = (1/2)^-$) ... analogous for "$B_s$ mesons".



Masses of B and B_s mesons (2)

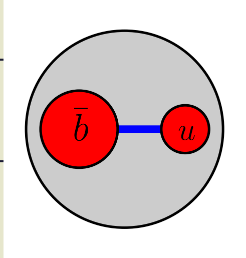


- Summary of the computed static-light meson spectrum:

$j^\mathcal{P}$	alternative notation	B mesons ($\bar{b}u$ or $\bar{b}d$): $M(j^\mathcal{P}) - M((1/2)^-)$ in MeV	B_s mesons ($\bar{b}s$) $M(j^\mathcal{P}) - M((1/2)^-)$ in MeV
$(1/2)^+$	P_-	$406(19)$	$413(12)$
$(3/2)^+$	P_+	$516(18)$	$504(12)$
$(3/2)^-$, $(5/2)^-$	$D_\pm$	$870(27)$	$770(26)$
$(5/2)^-$	D_+	$930(28)$	$960(24)$
$(5/2)^+$, $(7/2)^+$	$F_\pm$	$1196(30)$	$1179(37)$
$(1/2)^-$	S^*	$755(16)$	$751(26)$

- Motivation/achievements:
  - Continuum limit (among the first).
  - Dependence on the light u/d sea quark mass (for the first time).
  - Valuable input for model builders (e.g. no reversal of $M(P_-)$ and $M(P_+)$, ...).

Masses of B and B_s mesons (3)



- Comparison to experimental results:
  - Extrapolation to the physical (finite) b quark mass $m_B \approx 4200$ MeV:
    * Use rather precise experimental results for c quarks, i.e. D mesons.
    * Assume that Heavy Quark Effective Theory (HQET) up to $\mathcal{O}(1/m_Q)$ is "valid" down to the physical charm quark mass.
    * Amounts to "reincluding" hyperfine splitting.

	$M - M(B)$ in MeV			$M - M(B_s)$ in MeV	
name	lattice	experiment	name	lattice	experiment
B_0^*	443(21)		B_{s0}^*	391(8)	
B_1^*	460(22)		B_{s1}^*	440(8)	
B_1	530(12)	444(2)	B_{s1}	526(8)	463(1)
B_2^*	543(12)	464(5)	B_{s2}^*	539(8)	473(1)
B_J^*		418(8)	B_{sJ}^*		487(15)

- Difference between lattice and experimental results: scale setting problem?

Part 2

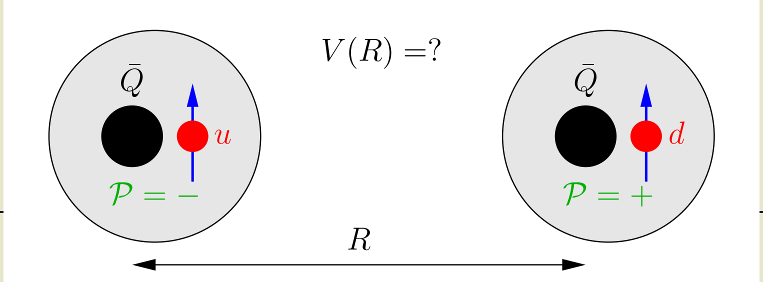
Computation of spin/isospin/parity dependent forces between B mesons.

[M. W. [ETM Collaboration], PoS **LATTICE2010**, 162 (2010)]

[M. W. [ETM Collaboration], Acta Phys. Pol. B Proceedings Supplement, Vol. 4, No. 4, 2011, page 747]

Introduction (1)

- Goal: compute the potential of (or equivalently the force between) two B mesons from first principles by means of lattice QCD:
  - Treat the b quark in the static approximation.
  - Consider only pseudoscalar/vector mesons ($j^{\mathcal{P}} = (1/2)^-$, denoted by S, PDG: B, B^*) and scalar/pseudovector mesons ($j^{\mathcal{P}} = (1/2)^+$, denoted by P_-, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons.
  - Study the dependence of the mesonic potential $V(R)$ on
    * the light quark flavor u and/or d (isospin),
    * the light quark spin (the static quark spin is irrelevant),
    * the type of the meson S and/or P_-.



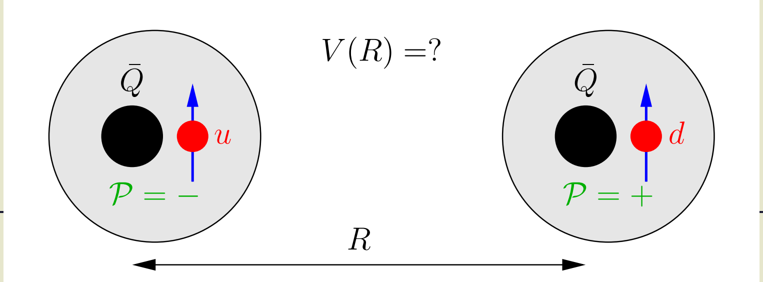
Introduction (2)

- Motivation:
  - First principles computation of a hadronic force.
  - Possible application: determine, whether two B mesons may form bound states (tetraquarks).
  - Until now
    - it has mainly been studied in the quenched approximation,
    - only pseudoscalar (S), but no scalar (P_-) B mesons have been considered.

[C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]

[W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

[G. Bali and M. Hetzenegger, PoS **LATTICE2010**, 142 (2010)]



(Pseudo)scalar B mesons

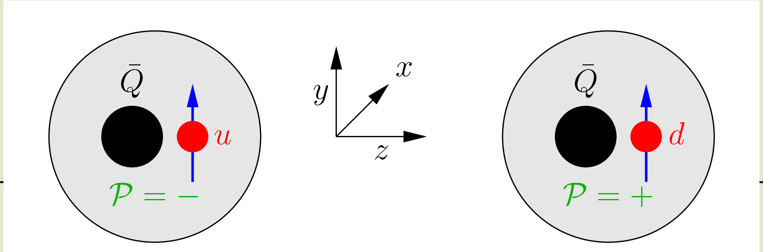
- Symmetries and quantum numbers of static-light mesons:
  - Isospin: $I = 1/2$, $I_z = \pm 1/2$, i.e. $B \equiv \bar{Q}u$ or $B \equiv \bar{Q}d$.
  - Parity: $\mathcal{P} = \pm$,
    - $\mathcal{P} = - \quad \equiv \quad S$ (wave),
    - $\mathcal{P} = + \quad \equiv \quad P_-$ (wave).
  - Rotations:
    - Light cloud angular momentum $j = 1/2$ (for S and P_-), $j_z = \pm 1/2$.
    - Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
- Examples of static-light meson creation operators:
  - $\bar{Q}\gamma_5 q$ (pseudoscalar, i.e. S), $q \in \{u\,,\,d\}$,
  - $\bar{Q}q$ (scalar, i.e. P_-)

(j_z is not well-defined, when using these operators).

BB systems (1)

- Symmetries and quantum numbers of a pair of static-light mesons (separated along the z-axis):
  - Isospin: $I = 0, 1$, $I_z = -1, 0, +1$.
  - Rotations around the z-axis:
    - Angular momentum of the light degrees of freedom $j_z = -1, 0, +1$.
    - Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
  - Parity: $\mathcal{P} = \pm$.
  - If $j_z = 0$, reflection along the x-axis: $\mathcal{P}_x = \pm$.
  - Instead of using $j_z = \pm 1$ one can also label states by $|j_z| = 1$, $\mathcal{P}_x = \pm$.

$\longrightarrow$ Label BB states by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$.



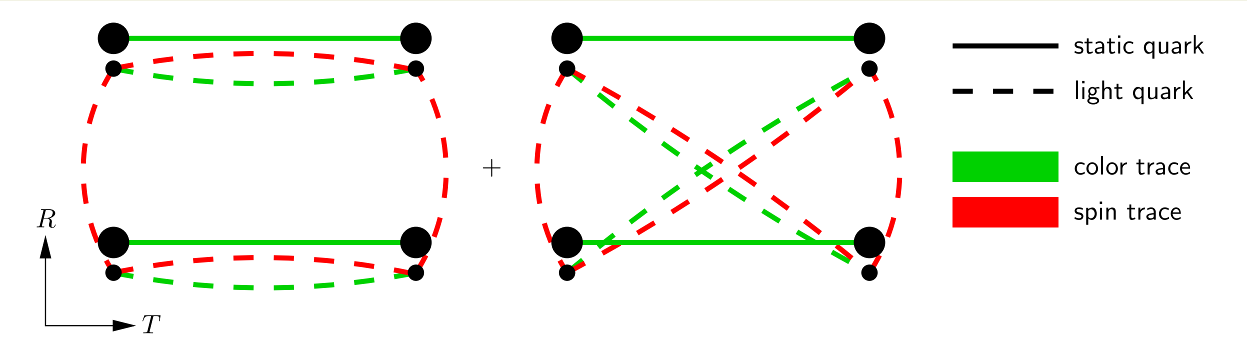
BB systems (2)

- To extract the potential(s) of a given sector (characterized by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$), compute the temporal correlation function of the trial state

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(1)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(2)}(+R/2)\Big)|\Omega\rangle,$$

where

  - $\mathcal{C} = \gamma_0\gamma_2$ (charge conjugation matrix),
  - $q^{(1)}q^{(2)} \in \{ud - du \quad , \quad uu\,,\, dd\,,\, ud + du\}$ (isospin I, I_z) ,
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}$, $\mathcal{P}_x$).



BB systems (3)

- BB creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big).$$

Γ	$\|j_z\|$, $\mathcal{P}$, $\mathcal{P}_x$
1	0, $-$, $-$
$\gamma_0\gamma_5$	0, $+$, $+$
γ_5	0, $+$, $+$
γ_0	0, $+$, $-$
γ_3	0, $-$, $-$
$\gamma_0\gamma_3\gamma_5$	0, $+$, $+$
$\gamma_3\gamma_5$	0, $-$, $+$
$\gamma_0\gamma_3$	0, $-$, $-$
γ_1	1, $-$, $+$
$\gamma_0\gamma_1\gamma_5$	1, $+$, $-$
$\gamma_1\gamma_5$	1, $-$, $-$
$\gamma_0\gamma_1$	1, $-$, $+$
...	...

BB systems (4)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(d)}(+R/2)\Big) \pm (u \leftrightarrow d).$$

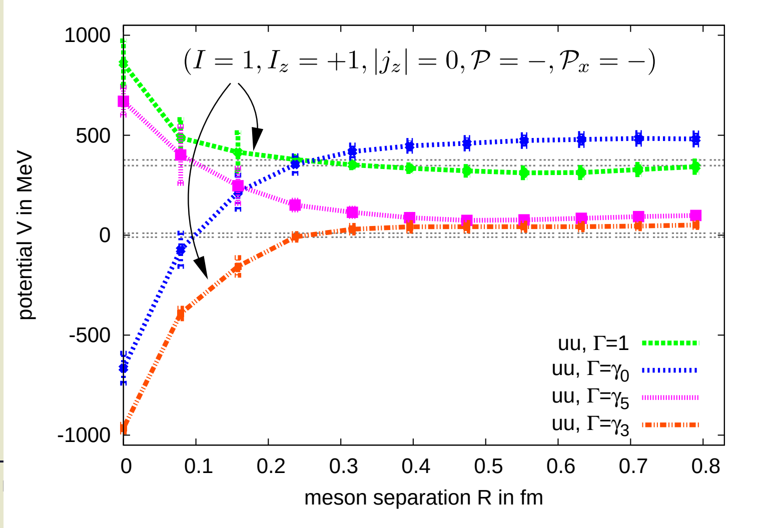
Γ, $\pm$	$\lvert j_z \rvert$, I, $\mathcal{P}$, $\mathcal{P}$
γ_5 , $-$	0, 0, $-$, $+$
γ_0 , $-$	0, 0, $-$, $-$
1 , $-$	0, 0, $+$, $-$
$\gamma_0\gamma_5$, $-$	0, 0, $-$, $+$
$\gamma_3\gamma_5$, $-$	0, 0, $+$, $+$
$\gamma_0\gamma_3$, $-$	0, 0, $+$, $-$
γ_3 , $-$	0, 0, $+$, $-$
$\gamma_0\gamma_3\gamma_5$, $-$	0, 0, $-$, $+$
γ_5 , $+$	0, 1, $+$, $+$
γ_0 , $+$	0, 1, $+$, $-$
1 , $+$	0, 1, $-$, $-$
$\gamma_0\gamma_5$, $+$	0, 1, $+$, $+$
...	...

Lattice setup

- Lattice spacing: $a \approx 0.079\,\mathrm{fm}$.
- Lattice extension: $L \approx 1.90\,\mathrm{fm}$ (periodic boundary conditions).
- Pion mass: $m_{\mathrm{PS}} \approx 340\,\mathrm{MeV}$.

Discussion of results (1)

- Four "types of potentials":
  - Two attractive, two repulsive.
  - Two have asymptotic values, which are larger by $\approx 400\,\mathrm{MeV}$.
- There are cases, where two potentials with identical quantum numbers are completely different (i.e. of different type)
  $\rightarrow$ at least one of the corresponding trial states must have very small ground state overlap
  $\rightarrow$ physical understanding, i.e. interpretation of trial states needed.



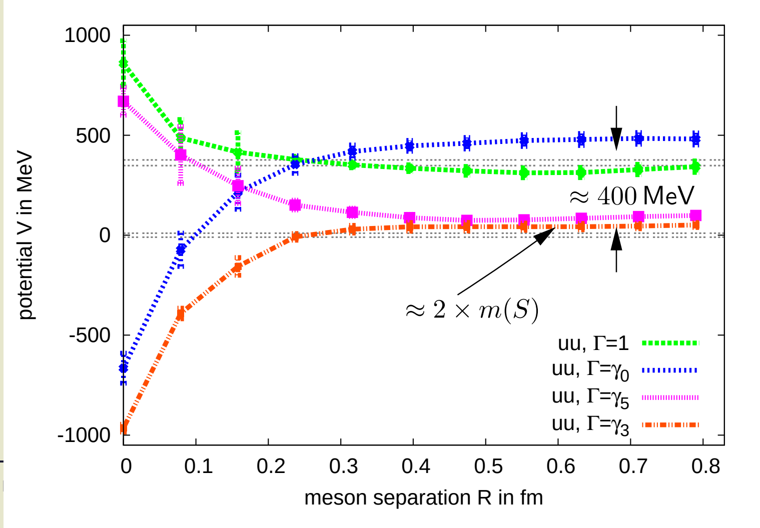
Discussion of results (2)

- Expectation at large meson separation R: $V(R) \approx 2 \times$ meson mass.
  - Potentials with smaller asymptotic value at $\approx 2 \times m(S)$.
  - $m(P_-) - m(S) \approx 400\,\mathrm{MeV}$: approximately the observed difference between different types of potentials.

$\rightarrow$ Two types correspond to $S \leftrightarrow S$ potentials.

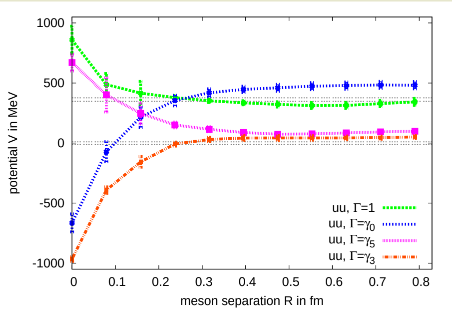
$\rightarrow$ Two types correspond to $S \leftrightarrow P_-$ potentials.

- Can this be understood in detail on the level of the used BB creation operators?

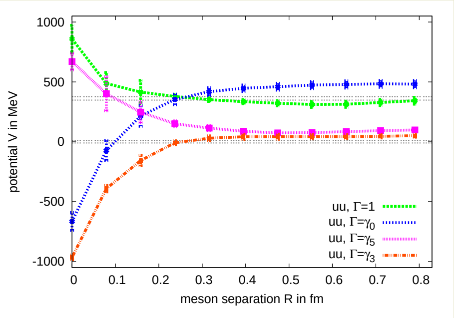


Discussion of results (3)

- Express the BB creation operators in terms of static-light meson creation operators (use suitable spin and parity projectors for the light quarks).
  - Examples:
    - uu, $\Gamma = 1 \quad \rightarrow \quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
      $$(\mathcal{C}1)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \quad \propto$$
      $$\propto \quad S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow.$$
    - uu, $\Gamma = \gamma_3 \quad \rightarrow \quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
      $$(\mathcal{C}\gamma_3)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \quad \propto$$
      $$\propto \quad S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow.$$
  - SS/SP_- content and asymptotic values in agreement for all 64 correlation functions/ potentials
    $\rightarrow$ asymptotic differences understood.

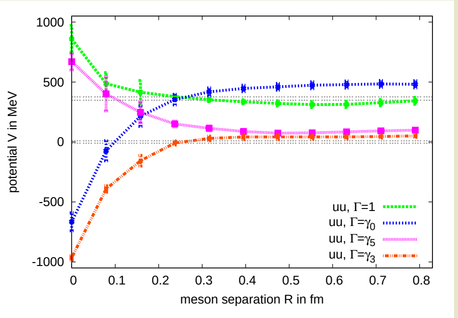


Discussion of results (4)



- Is there a general rule, about when a potential is repulsive and when attractive?
  - $S \leftrightarrow S$ potentials:
    - $(I = 0\ ,\ s = 0)$ or $(I = 1\ ,\ s = 1)$, i.e. $I = s \quad \rightarrow$ attractive
      $(I = 0\ ,\ s = 1)$ or $(I = 1\ ,\ s = 0)$, i.e. $I \neq s \quad \rightarrow$ repulsive
      (s: combined angular momentum of the two mesons).
    - Example: uu, $\Gamma = \gamma_3 \quad \rightarrow \quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
      $$(\mathcal{C}\gamma_3)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \propto$$
      $$\propto \quad S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow .$$
      i.e. $I = 1$, $s = 1$; the numerically obtained potential is attractive, i.e. in agreement with the above stated rule.
    - All 32 $S \leftrightarrow S$ correlation functions/potentials fulfill the rule.
    - Agreement with similar quenched lattice studies.
      [C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]
      [W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

Discussion of results (5)



– $S \leftrightarrow P_-$ potentials:

∗ Do not obey the above stated rule.

∗ It can, however, easily be generalized by including parity, i.e. symmetry or antisymmetry under exchange of S and P_-:
trial state symmetric under meson exchange $\quad\longrightarrow\quad$ attractive
trial state antisymmetric under meson exchange $\quad\longrightarrow\quad$ repulsive
(meson exchange $\equiv$ exchange of flavor, spin and parity).

∗ Example: uu, $\Gamma = \gamma_0 \quad \longrightarrow \quad \mathcal{P} = +$, $\mathcal{P}_x = -$:

$$(\mathcal{C}\gamma_0)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \quad \propto$$

$$\propto \quad S_\uparrow P_\downarrow - S_\downarrow P_\uparrow - P_\uparrow S_\downarrow + P_\downarrow S_\uparrow,$$

i.e. $I = 1$ (symmetric), $s = 0$ (antisymmetric), antisymmetric with respect to $S \leftrightarrow P_-$; the numerically obtained potential is attractive, i.e. in agreement with the above stated general rule.

∗ All 32 $S \leftrightarrow P_-$ correlation functions/potentials (and all 32 $S \leftrightarrow S$ correlation functions/potentials) fulfill the generalized rule.

Discussion of results (6)

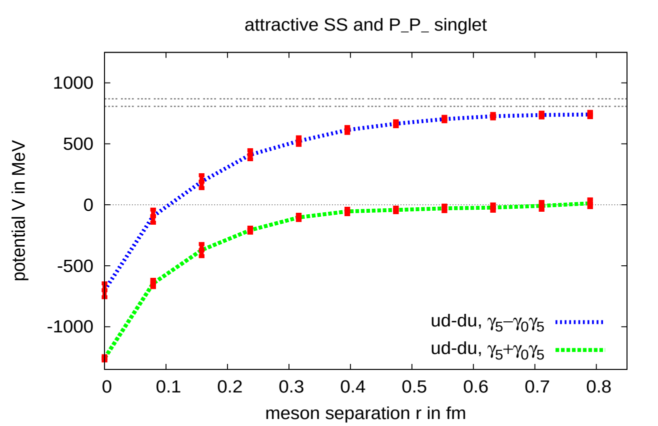
- Improvements after having understood the extraction and interpretation of BB potentials from single correlation functions:
  - Linearly combine BB operators to either eliminate $P_- \leftrightarrow P_-$ or $S \leftrightarrow S$ combinations.
  - Example:

    $ud - du$, $\Gamma = \gamma_5 \quad \rightarrow \quad -S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow + P_\downarrow P_\uparrow$

    $ud - du$, $\Gamma = \gamma_0\gamma_5 \quad \rightarrow \quad -S_\uparrow S_\downarrow + S_\downarrow S_\uparrow + P_\uparrow P_\downarrow - P_\downarrow P_\uparrow$

    $\rightarrow$ use $\gamma_5 + \gamma_0\gamma_5$ to obtain a better signal for the $S \leftrightarrow S$ potential

    $\rightarrow$ use $\gamma_5 - \gamma_0\gamma_5$ to extract the $P_- \leftrightarrow P_-$ potential.



Discussion of results (7)

- Improvements after having understood the extraction and interpretation of BB potentials from single correlation functions:
  - Use correlation matrices instead of single correlation functions to avoid mixing with BB states of lower energy, which is present, because
    - although the product of two specific B meson creation operators closely resembles the corresponding BB state, it will still have a non-vanishing overlap to BB states corresponding to B mesons with different isospin, spin and/or parity,
    - twisted mass lattice QCD explicitly breaks isospin and parity (the breaking is proportional to the lattice spacing a; isospin and parity will be restored in the continuum limit).

Summary of BB states and degeneracies

- Two B mesons, each can have $I_z = \pm 1/2$, $j_z = \pm 1/2$, $\mathcal{P} = \pm$ $\rightarrow 8 \times 8 = 64$ states.

- $S \leftrightarrow S$ potentials:
  - Attractive: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{6}_{I=1,|j_z|=1}$ (10 states).
  - Repulsive: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{2}_{I=0,|j_z|=1}$ (6 states).

- $S \leftrightarrow P_-$ potentials:
  - Attractive: $\underbrace{1 \oplus 1 \oplus 3 \oplus 3}_{|j_z|=0} \oplus \underbrace{2 \oplus 6}_{|j_z|=1}$ (16 states).
  - Repulsive: $\underbrace{1 \oplus 1 \oplus 3 \oplus 3}_{|j_z|=0} \oplus \underbrace{2 \oplus 6}_{|j_z|=1}$ (16 states).

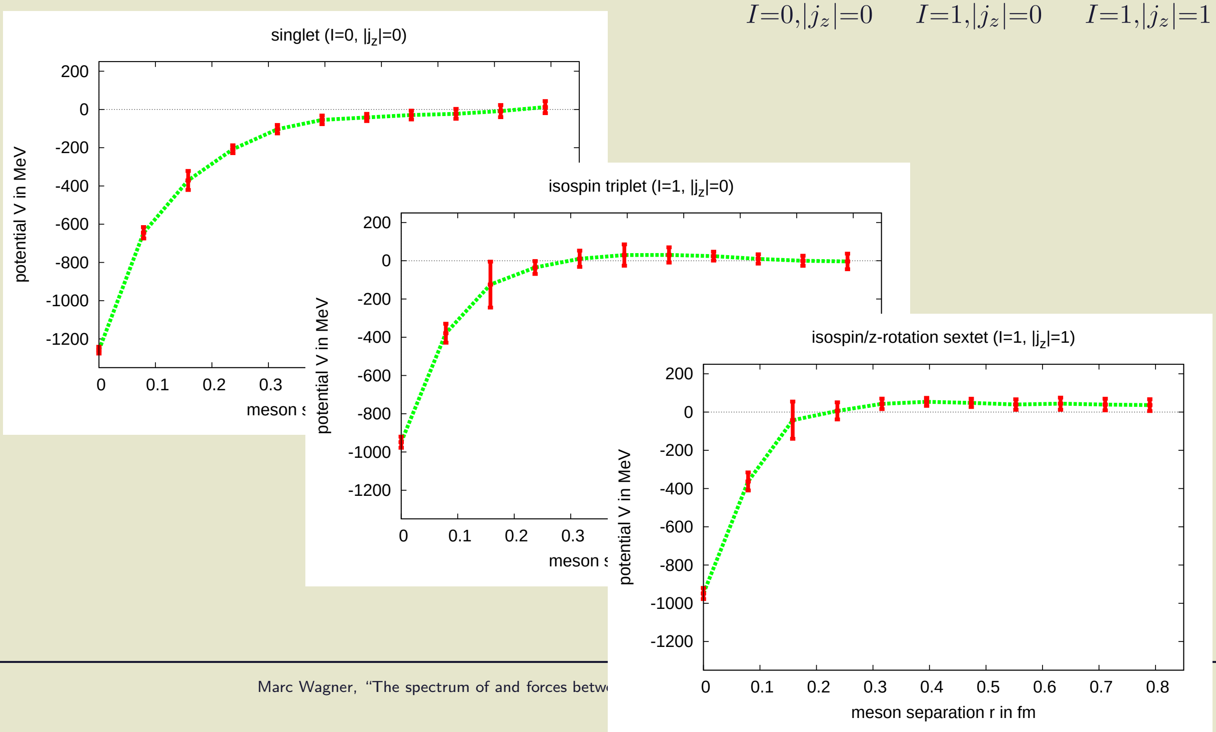
- $P_- \leftrightarrow P_-$ potentials: identical to $S \leftrightarrow S$ potentials.

- In total 24 different potentials.

Attractive $S \leftrightarrow S$ potentials

- Attractive $S \leftrightarrow S$ potentials are relevant, when trying to determine, whether BB may form a bound state.

- Three different attractive $S \leftrightarrow S$ potentials: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{6}_{I=1,|j_z|=1}$.



Summary, conclusions, future plans (1)

- Computation of BB potentials (arbitrary flavor, spin and parity) with "light" dynamical quarks ($m_{\mathrm{PS}} \approx 340\,\mathrm{MeV}$).
  - Qualitative agreement with existing quenched results for $S \leftrightarrow S$ potentials.
  - First lattice computation of $S \leftrightarrow P_-$ and $P_- \leftrightarrow P_-$ potentials.
  - Clear statements about whether a potential of a given channel is attractive or repulsive.

- Statistical accuracy problematic (exponentially decaying correlation functions are quickly lost in statistical noise):
  - Reasonable accuracy for attractive $S \leftrightarrow S$ potentials (interesting, when trying to determine, whether BB may form a bound state).
  - Other (higher) potentials:
    $\rightarrow$ BB potentials are extracted at rather small temporal separations
    $\rightarrow$ slight contamination from excited states cannot be excluded.

Summary, conclusions, future plans (2)

- Further plans and possibilities:
  - Other values of the lattice spacing, the spacetime volume and/or the u/d quark mass.
  - Partially quenched computations, to obtain $B_s B_s$ and/or $B_s B$ potentials.
  - Improve lattice meson potentials at small separations (where the suppression of UV fluctuations due to the lattice cutoff yields wrong results) with corresponding perturbative potentials.
  - Use lattice meson potentials to study, whether BB may form a bound state.