Structure of a $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$

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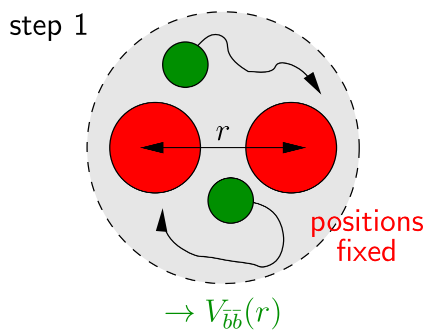
Basic idea: lattice QCD + BO

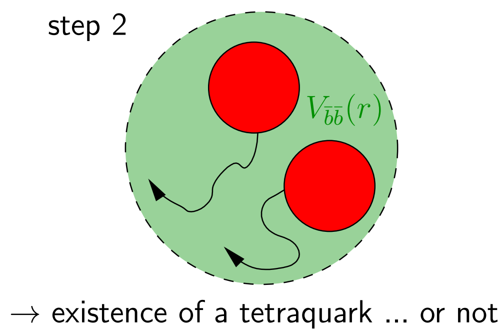
- Study heavy-heavy-light-light tetraquarks $\bar{b}\bar{b}qq$ in two steps.

  (1) **Compute potentials of two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s, c\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

((1) + (2) $\rightarrow$ Born-Oppenheimer approximation).





Previous work on $\bar{b}\bar{b}qq$ tetraquarks

- Lattice QCD static potentials and Born-Oppenheimer approximation.
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [M.W., PoS **LATTICE2010**, 162 (2010) [arXiv:1008.1538]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571]]
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [Z. S. Brown, K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]
  [P. Bicudo, A. Peters, S. Velten, M.W., arXiv:2101.00723]

- Full lattice QCD (b quarks with Non Relativistic QCD) [list not complete]:
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214 [hep-lat]]]
  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285 [hep-lat]]]
  [L. Leskovec, S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197] [hep-lat]]]

- Other approches: quark models, effective field theories, QCD sum rules ... [list not complete]:
  [M. Karliner, J. L. Rosner, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666]]
  [E. J. Eichten, C. Quigg, Phys. Rev. Lett. **119**, 202002 (2017) [arXiv:1707.09575]]
  [Z. G. Wang, Acta Phys. Polon. B **49**, 1781 (2018) [arXiv:1708.04545]]
  [W. Park, S. Noh, S. H. Lee, Acta Phys. Polon. B **50**, 1151-1157 (2019) [arXiv:1809.05257]]
  [B. Wang, Z. W. Liu, X. Liu, Phys. Rev. D **99**, 036007 (2019) [arXiv:1812.04457]]
  [M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie, L. S. Geng, Phys. Rev. D **99**, 094018 (2019) [arXiv:1902.03044]]

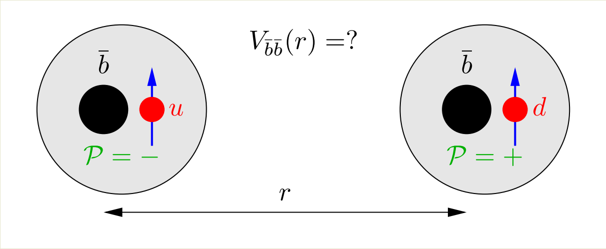
Outline

- $\bar{b}\bar{b}qq$ / BB potentials.
- Stable $\bar{b}\bar{b}qq$ tetraquarks.
- Structure of a $\bar{b}\bar{b}qq$ tetraquark with quantum numbers $I(J^P) = 0(^-)$ (meson-meson versus diquark-antidiquark structure).

$\bar{b}\bar{b}qq$ / BB potentials (1)

- At large $\bar{b}\bar{b}$ separation r, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of qq on
  - the "light" quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^* (parity).

$\rightarrow$ Many different channels: attractive as well as repulsive, different asymptotic values ...

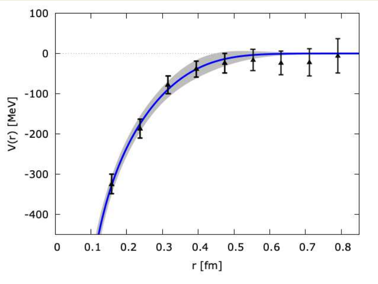


$\bar{b}\bar{b}qq$ / BB potentials (2)

- To determine potentials, compute temporal correlation functions of operators

$$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}_C^a(-\mathbf{r}/2)\psi_A^{(f)a}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D^b(+\mathbf{r}/2)\psi_B^{(f')b}(+\mathbf{r}/2)\Big).$$

- The most attractive potential of a $B^{(*)}B^*$ meson pair has $(I,|j_z|,P,P_x) = (0,0,+,-)$:
  - $C = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $\psi^{(f)}\psi^{(f')} = ud - du$, $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,(1-\gamma_0)\gamma_5\}$.
  - $\bar{Q}\bar{Q} = \bar{b}\bar{b}$, $\tilde{\Gamma} \in \{(1+\gamma_0)\gamma_5\,,\,(1+\gamma_0)\gamma_j\}$ (irrelevant).



- Parameterize lattice results by

$$V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r) \quad = \quad -\frac{\alpha}{r}\exp\bigg(-\Big(\frac{r}{d}\Big)^p\bigg) + V_0$$

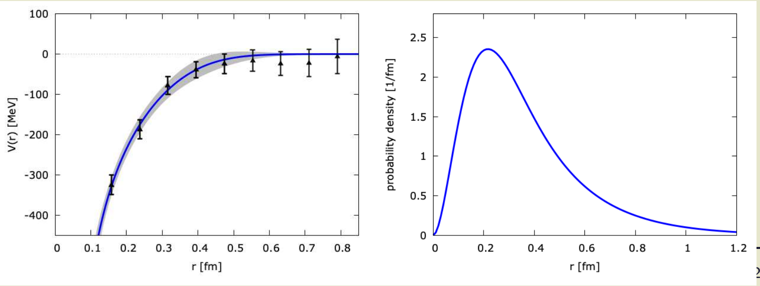
(1-gluon exchange at small r; color screening at large r).
[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]

Stable $\bar{b}\bar{b}qq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

$$\left(\frac{1}{m_b}\left(-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right)+V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)-2m_{\mathrm{sl}}\right)R(r) \quad = \quad ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $-E = 38(18)$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.
- No further bound states.
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]



Structure of the $\bar{b}\bar{b}qq$ tetraquark (1)

- Two types of operators, which probe the same sector:
  [P. Bicudo, A. Peters, S. Velten, M.W., arXiv:2101.00723]
  - **Meson-meson operator** (BB):

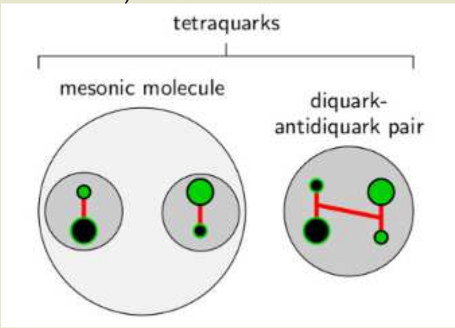
    $$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)\psi^{(f)a}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)\psi^{(f')b}_B(+\mathbf{r}/2)\Big)$$

    with $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,\gamma_5\}$ $(\rightarrow (j_z,\mathcal{P},\mathcal{P}_x) = (0,-,+))$.
  - **Diquark-antidiquark operator** (Dd):

    $$\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\Big(\psi^{(f)b}_A(\mathbf{z})(\mathcal{C}\Gamma)_{AB}\psi^{(f')c}_B(\mathbf{z})\Big)$$
    $$\epsilon^{ade}\Big(\bar{Q}^f_C(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2;\mathbf{z})(\mathcal{C}\tilde{\Gamma})_{CD}\bar{Q}^g_D(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2;\mathbf{z})\Big)$$

    with $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,\gamma_5\}$ $(\rightarrow (j_z,\mathcal{P},\mathcal{P}_x) = (0,-,+))$.
- $\psi^{(f)}\psi^{(f')} = ud - du$ $(\rightarrow I = 0)$.
- $\tilde{\Gamma} = (1+\gamma_0)\gamma_3$ (essentially irrelevant).
- Compute the 4×4 correlation matrix

  $$C_{jk}(t) = \langle\Omega|\mathcal{O}^\dagger_j(t)\mathcal{O}_k(0)|\Omega\rangle.$$



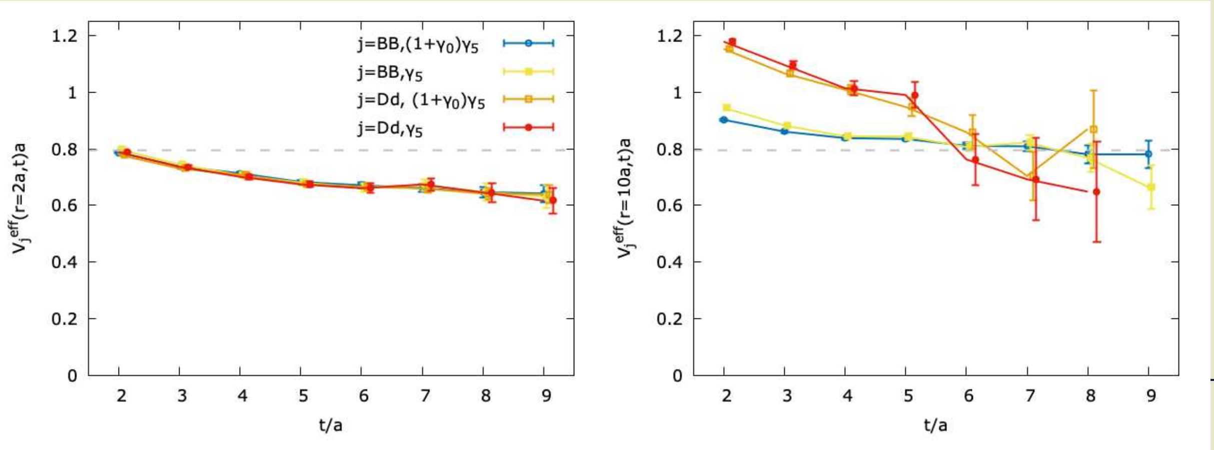
Structure of the $\bar{b}\bar{b}qq$ tetraquark (2)

- Effective energies corresponding to diagonal elements of the correlation matrix,

$$V_j^{\text{eff}}(r,t) \quad = \quad -\frac{1}{a}\log\left(\frac{C_{jj}(t)}{C_{jj}(t-a)}\right) \quad \text{(no sum over } j\text{)}.$$

- For large $\bar{b}\bar{b}$ separations (right plot $r \approx 0.79\,\text{fm}$), BB effective energies reach plateaus at smaller t separations than Dd effective energies.
  $\rightarrow$ BB dominates at large r, Dd not important (energetically disfavored due to flux tube).

- For small $\bar{b}\bar{b}$ separations (left plot $r \approx 0.16\,\text{fm}$), BB and Dd effective energies similar.
  $\rightarrow$ More detailed investigation at small r necessary.



Structure of the $\bar{b}\bar{b}qq$ tetraquark (3)

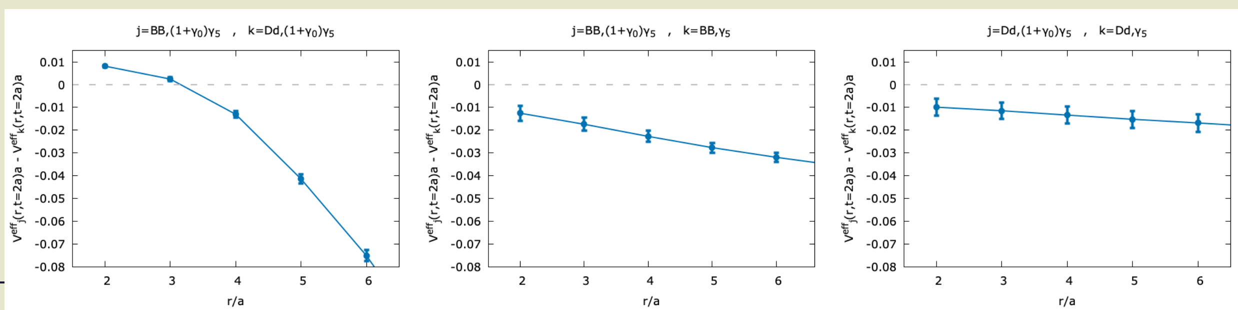
- Differences of effective energies corresponding to diagonal elements of the correlation matrix at small temporal separation $t = 2a$ as functions of the $\bar{b}\bar{b}$ separation r,

  $$V_j^{\text{eff}}(r, t = 2a) - V_k^{\text{eff}}(r, t = 2a).$$

- BB versus Dd (left): Dd dominates for $r \lesssim 3.15\,a \approx 0.25\,\text{fm}$, while BB dominates for $r \gtrsim 3.15\,a \approx 0.25\,\text{fm}$.

- BB operators (center): $\Gamma = (1 + \gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Expected. Via a Fierz transformation one can show that $\Gamma = (1 + \gamma_0)\gamma_5$ generates exclusively ground state mesons, while γ_5 also generates parity excitations.)

- Dd operators (right): $\Gamma = (1 + \gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Interesting. In the literature mostly γ_5 is discussed.)



Structure of the $\bar{b}\bar{b}qq$ tetraquark (4)

- Optimize trial states

$$|\Phi_{b,d}\rangle \quad = \quad b|\Phi_{BB,(1+\gamma_0)\gamma_5}\rangle + d|\Phi_{Dd,(1+\gamma_0)\gamma_5}\rangle$$

  ($|\Phi_j\rangle = \mathcal{O}_j|\Omega\rangle$) by minimizing effective energies

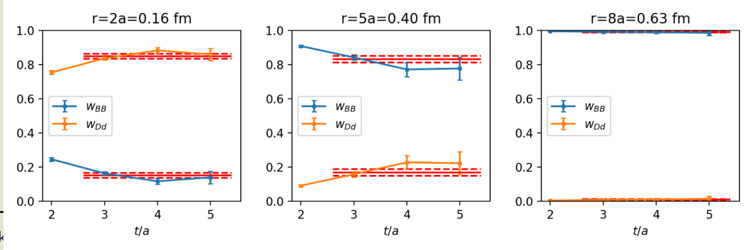
$$V^{\text{eff}}_{b,d}(r,t) \quad = \quad -\frac{1}{a}\log\left(\frac{C_{[b,d][b,d]}(t)}{C_{[b,d][b,d]}(t-a)}\right) \quad , \quad C_{[b,d][b,d]}(t) = \left(\begin{array}{c} b \\ d \end{array}\right)^\dagger_j C_{jk}(t) \left(\begin{array}{c} b \\ d \end{array}\right)_k .$$

  with respect to $b, d \in \mathbb{C}$.

- Since norm and phase of b and d are irrelevant, consider relative weights of BB and Dd,

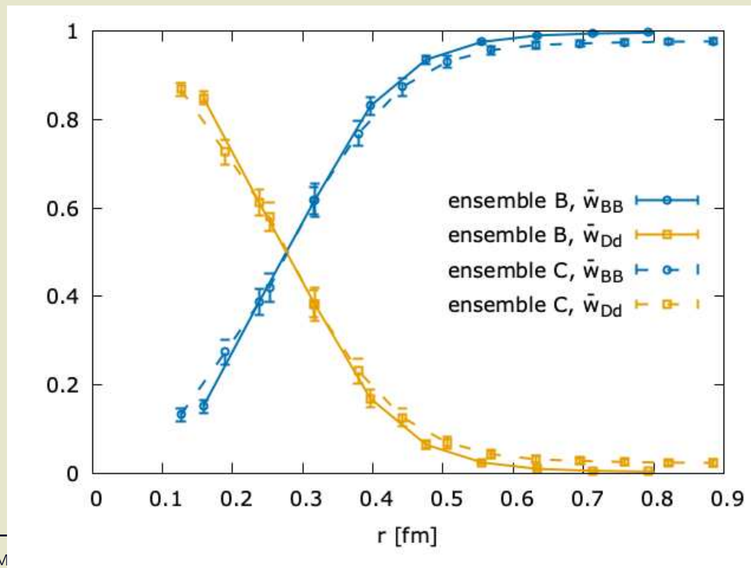
$$w_{BB} \quad = \quad \frac{|b|^2}{|b|^2 + |d|^2} \quad , \quad w_{Dd} \quad = \quad \frac{|d|^2}{|b|^2 + |d|^2} \quad = \quad 1 - w_{BB}.$$

- For fixed $\bar{b}\bar{b}$ separation r, w_{BB} and w_{Dd} depend only weakly on t.
  $\rightarrow$ w_{BB} and w_{Dd} estimate the
  percentage of BB and of Dd.



Structure of the $\bar{b}\bar{b}qq$ tetraquark (5)

- w_{BB} and w_{Dd} as functions of the $\bar{b}\bar{b}$ separation r (for two ensembles, $a \approx 0.079$ fm and $a \approx 0.063$ fm).
- $r \lesssim 0.2$ fm: Clear diquark-antidiquark dominance.
- $0.2\,\text{fm} \lesssim r \lesssim 0.3$ fm: Diquark-antidiquark dominance turns into meson-meson dominance.
- $0.5\,\text{fm} \lesssim r$: Essentially a meson-meson system.



Structure of the $\bar{b}\bar{b}qq$ tetraquark (6)

- Generalized eigenvalue problem (GEVP)

$$C_{jk}(t)v_k^{(n)}(t) \quad = \quad \lambda^{(n)}(t)C_{jk}(t_0)v_k^{(n)}(t) \quad , \quad n \quad = \quad 0,\ldots,N-1$$

for $t_0/a \geq 1$ and $t/a > t_0/a$ with corresponding effective energies

$$V^{\mathrm{eff},(n)}(r,t) \quad = \quad -\frac{1}{a}\log\left(\frac{\lambda^{(n)}(t)}{\lambda^{(n)}(t-a)}\right).$$

- Eigenvector components $v_j^{(n)}(t)$ (which we always normalize according to $\sum_j |v_j^{(n)}(t)|^2 = 1$) contain information about the relative importance of the operators. For large t and t_0,

$$|n\rangle \approx \sum_j v_j^{(n)}(t)|\Phi_j\rangle,$$

where $\approx$ denotes an approximate expansion of the energy eigenstate $|n\rangle$ in terms of the trial states $|\Phi_j\rangle$.

Structure of the $\bar{b}\bar{b}qq$ tetraquark (7)

- One can show: For $t_0 = t - a$, optimizing trial states by minimizing effective energies (as on previous slides) is equivalent to solving a GEVP, i.e.

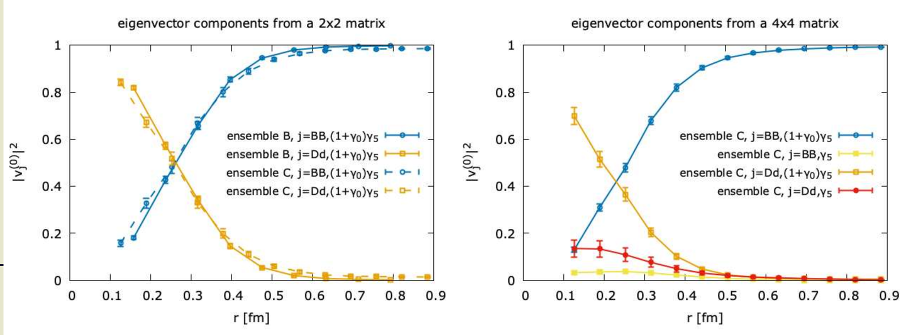
$$(w_{BB}, w_{Dd}) \quad = \quad (|v^{(0)}_{BB,(1+\gamma_0)\gamma_5}|^2, |v^{(0)}_{Dd,(1+\gamma_0)\gamma_5}|^2)$$

  (might offer another perspective on GEVP eigenvector components).
  $\rightarrow$ Results for w_{BB} and w_{Dd} can also be interpreted as GEVP results.
  [P. Bicudo, A. Peters, S. Velten, M.W., arXiv:2101.00723]

- In the literature typically small values for t_0 are used, e.g. $t_0/a = 1$ (instead of $t_0 = t - a$ as used to obtain w_{BB} and w_{Dd} on previous slides).

- Similar results also for $t_0/a = 1$, when using a 2×2 correlation matrix (left plot).

- Consistent results, when using a 4×4 correlation matrix (right plot).



Structure of the $\bar{b}\bar{b}qq$ tetraquark (8)

- Define the r dependent BB and Dd percentages,

  $$p_{BB}(r) = w_{BB} \quad , \quad p_{Dd}(r) = w_{Dd}$$

  and use the probability density of the $\bar{b}\bar{b}$ separation

  $$p_r(r) \quad = \quad 4\pi |R(r)|^2$$

  obtained from the BO wave function $R(r)/r$, to estimate the total BB and Dd percentages of the $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$:

  $$\%BB = \int dr\, p_r(r) p_{BB}(r) \quad , \quad \%Dd = \int dr\, p_r(r) p_{Dd}(r) = 1 - \%BB.$$

- We find $\%BB = 0.59$, $\%Dd = 0.41$.
- Using $|v^{(0)}_{BB,(1+\gamma_0)\gamma_5}|^2, |v^{(0)}_{Dd,(1+\gamma_0)\gamma_5}|^2$ instead of w_{BB}, w_{Dd} we find $\%BB = 0.61$, $\%Dd = 0.39$.
- Results are in agreement with a GEVP result we obtained in a full lattice QCD computation, where the $\bar{b}$ quarks are treated within NRQCD.
  [L. Leskovec, S. Meinel, M. Pflaumer and M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197]]
  [M. Pflaumer, private communications]

Summary

- The hadronically stable $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ is neither exclusively a meson-meson system nor a diquark-antdiquark pair.
- $r \lesssim 0.2$ fm: Clear diquark-antidiquark dominance.
- $r \gtrsim 0.3$ fm: Clear meson-meson dominance.
- Total BB and Dd percentages: $\%BB \approx 0.60$, $\%Dd \approx 0.40$.