Structure of a $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$

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Basic idea: lattice QCD + BO

- Study heavy-heavy-light-light tetraquarks $\bar{b}bqq$ in two steps.
  
  1. Compute potentials of two static quarks $\bar{b}b$ in the presence of two lighter quarks $qq$ ($q \in \{u, d, s, c\}$) using lattice QCD.
  2. Check, whether these potentials are sufficiently attractive to host bound states or resonances ($\rightarrow$ tetraquarks) by using techniques from quantum mechanics and scattering theory.

$((1) + (2) \rightarrow$ Born-Oppenheimer approximation$)$.

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Previous work on $\bar{b}bqq$ tetraquarks

- Lattice QCD static potentials and Born-Oppenheimer approximation.
  

- Full lattice QCD ($b$ quarks with Non Relativistic QCD) [list not complete]:

- Other approaches: quark models, effective field theories, QCD sum rules ... [list not complete]:
Outline

- $\bar{b}\bar{b}qq / BB$ potentials.
- Stable $\bar{b}\bar{b}qq$ tetraquarks.
- Structure of a $\bar{b}\bar{b}qq$ tetraquark with quantum numbers $I(J^P) = 0(−)$ (meson-meson versus diquark-antidiquark structure).

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\[\bar{b}bqq \quad \bar{b}B \quad potentials \ (1)\]

- At large $\bar{b}b$ separation $r$, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Spins of static antiquarks $\bar{b}b$ are irrelevant (they do not appear in the Hamiltonian).
- Compute and study the dependence of $\bar{b}b$ potentials in the presence of $qq$ on
  - the “light” quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the “light” quark spin (the static quark spin is irrelevant),
  - the type of the meson $B, B^*, \text{and/or} B_0^*, B_1^*$ (parity).

→ Many different channels: attractive as well as repulsive, different asymptotic values ...

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\[ \bar{b}bqq / BB \] potentials (2)

- To determine potentials, compute temporal correlation functions of operators

\[ O_{BB, \Gamma} = 2N_{BB} (C\Gamma)_{AB} (C\tilde{\Gamma})_{CD} \left( \bar{Q}^a_C (-r/2) \psi^{(f)a}_A (-r/2) \right) \left( \bar{Q}^b_D (+r/2) \psi^{(f')b}_B (+r/2) \right). \]

- The most attractive potential of a \( B^*B^* \) meson pair has \( (I, |j_z|, P, P_x) = (0, 0, +, -) \):
  - \( C = \gamma_0 \gamma_2 \) (charge conjugation matrix).
  - \( \psi^{(f)} \psi^{(f')} = ud - du \), \( \Gamma \in \{(1 + \gamma_0)\gamma_5, (1 - \gamma_0)\gamma_5\} \).
  - \( \bar{Q}\bar{Q} = \bar{b}b \), \( \tilde{\Gamma} \in \{(1 + \gamma_0)\gamma_5, (1 + \gamma_0)\gamma_j\} \) (irrelevant).

- Parameterize lattice results by

\[ V_{qq,j_z,P,P_x}(r) = -\frac{\alpha}{r} \exp \left( -\left( \frac{r}{d} \right)^p \right) + V_0 \]

(1-gluon exchange at small \( r \); color screening at large \( r \).


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Stable $\bar{b}bqq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}b$ using the previously computed $\bar{b}bqq / BB$ potentials,

$$\left( \frac{1}{m_b} \left( - \frac{d^2}{dr^2} + \frac{L(L+1)}{r^2} \right) + V_{qq,jz,px}(r) - 2m_{sl} \right) R(r) = ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}bqq$ tetraquarks.

- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}b$:
  - Binding energy $-E = 38(18)$ MeV with respect to the $BB^*$ threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$. \\

- No further bound states.

Structure of the $\bar{b}bqq$ tetraquark (1)

- Two types of operators, which probe the same sector:

  - **Meson-meson operator** ($BB$):

    $\mathcal{O}_{BB,\Gamma} = 2N_{BB}(C\Gamma)_{AB}(C\tilde{\Gamma})_{CD}\left(\bar{Q}^a_C(-r/2)\psi_A^{(f)a}(-r/2)\right)\left(\bar{Q}^b_D(+r/2)\psi_B^{(f)b}(+r/2)\right)$

    with $\Gamma \in \{(1 + \gamma)\gamma_5, \gamma_5\}$ ($\rightarrow (j_z, P, P_x) = (0, -, +)$).

  - **Diquark-antidiquark operator** ($Dd$):

    $\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\left(\psi_A^{(f)b}(z)(C\Gamma)_{AB}\psi_B^{(f)c}(z)\right)$

    $\epsilon^{ade}\left(\bar{Q}^f_C(-r/2)U^{fd}(-r/2; z)(C\tilde{\Gamma})_{CD}\bar{Q}^g_D(+r/2)U^{ge}(+r/2; z)\right)$

    with $\Gamma \in \{(1 + \gamma)\gamma_5, \gamma_5\}$ ($\rightarrow (j_z, P, P_x) = (0, -, +)$).

- $\psi^{(f)}\psi^{(f')} = ud - du$ ($\rightarrow I = 0$).

- $\tilde{\Gamma} = (1 + \gamma_0)\gamma_3$ (essentially irrelevant).

- Compute the $4 \times 4$ correlation matrix

  $C_{jk}(t) = \langle \Omega | \mathcal{O}_j^\dagger(t)\mathcal{O}_k(0) | \Omega \rangle$. 
Structure of the $\bar{b}bqq$ tetraquark (2)

- Effective energies corresponding to diagonal elements of the correlation matrix,
  
  $$V_{j}^{\text{eff}}(r, t) = -\frac{1}{a} \log \left( \frac{C_{jj}(t)}{C_{jj}(t-a)} \right) \text{ (no sum over } j).$$

- For large $\bar{b}b$ separations (right plot $r \approx 0.79$ fm), $BB$ effective energies reach plateaus at smaller $t$ separations than $Dd$ effective energies.
  $\rightarrow$ $BB$ dominates at large $r$, $Dd$ not important (energetically disfavored due to flux tube).

- For small $\bar{b}b$ separations (left plot $r \approx 0.16$ fm), $BB$ and $Dd$ effective energies similar.
  $\rightarrow$ More detailed investigation at small $r$ necessary.
Structure of the $\bar{b}bqq$ tetraquark (3)

- Differences of effective energies corresponding to diagonal elements of the correlation matrix at small temporal separation $t = 2a$ as functions of the $\bar{b}b$ separation $r$,

$$V_j^{\text{eff}}(r, t = 2a) - V_k^{\text{eff}}(r, t = 2a).$$

- $BB$ versus $Dd$ (left): $Dd$ dominates for $r \lesssim 3.15a \approx 0.25$ fm, while $BB$ dominates for $r \gtrsim 3.15a \approx 0.25$ fm.

- $BB$ operators (center): $\Gamma = (1 + \gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Expected. Via a Fierz transformation one can show that $\Gamma = (1 + \gamma_0)\gamma_5$ generates exclusively ground state mesons, while $\gamma_5$ also generates parity excitations.)

- $Dd$ operators (right): $\Gamma = (1 + \gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Interesting. In the literature mostly $\gamma_5$ is discussed.)
Structure of the $\bar{b}bqq$ tetraquark (4)

- Optimize trial states

$$|\Phi_{b,d}\rangle = b|\Phi_{BB,(1+\gamma_0)\gamma_5}\rangle + d|\Phi_{Dd,(1+\gamma_0)\gamma_5}\rangle$$

$$(|\Phi_j\rangle = O_j|\Omega\rangle)$$ by minimizing effective energies

$$V_{b,d}^{\text{eff}}(r,t) = -\frac{1}{a} \log \left( \frac{C_{[b,d][b,d]}(t)}{C_{[b,d][b,d]}(t-a)} \right), \quad C_{[b,d][b,d]}(t) = \left( \begin{array}{c} b \\ d \end{array} \right) C_{jk}(t) \left( \begin{array}{c} b \\ d \end{array} \right)_k.$$  

with respect to $b, d \in \mathbb{C}$.

- Since norm and phase of $b$ and $d$ are irrelevant, consider relative weights of $BB$ and $Dd$,

$$w_{BB} = \frac{|b|^2}{|b|^2 + |d|^2}, \quad w_{Dd} = \frac{|d|^2}{|b|^2 + |d|^2} = 1 - w_{BB}.$$  

- For fixed $\bar{b}b$ separation $r$, $w_{BB}$ and $w_{Dd}$ depend only weakly on $t$.  

$\rightarrow w_{BB}$ and $w_{Dd}$ estimate the percentage of $BB$ and of $Dd$. 

Marc Wagner, “Structure of a $\bar{b}bud$ tetraquark”
Structure of the $\bar{b}bqq$ tetraquark (5)

- $w_{BB}$ and $w_{Dd}$ as functions of the $\bar{b}b$ separation $r$ (for two ensembles, $a \approx 0.079\text{ fm}$ and $a \approx 0.063\text{ fm}$).

- $r \lesssim 0.2\text{ fm}$: Clear diquark-antidiquark dominance.

- $0.2\text{ fm} \lesssim r \lesssim 0.3\text{ fm}$: Diquark-antidiquark dominance turns into meson-meson dominance.

- $0.5\text{ fm} \lesssim r$: Essentially a meson-meson system.
Structure of the $\bar{b}bqq$ tetraquark (6)

- Generalized eigenvalue problem (GEVP)

$$C_{jk}(t)v_k^{(n)}(t) = \lambda^{(n)}(t)C_{jk}(t_0)v_k^{(n)}(t) \quad , \quad n = 0, \ldots, N - 1$$

for $t_0/a \geq 1$ and $t/a > t_0/a$ with corresponding effective energies

$$V_{\text{eff},(n)}(r, t) = -\frac{1}{a} \log \left( \frac{\lambda^{(n)}(t)}{\lambda^{(n)}(t-a)} \right).$$

- Eigenvector components $v_j^{(n)}(t)$ (which we always normalize according to $\sum_j |v_j^{(n)}(t)|^2 = 1$) contain information about the relative importance of the operators. For large $t$ and $t_0$,

$$|n\rangle \approx \sum_j v_j^{(n)}(t)|\Phi_j\rangle,$$

where $\approx$ denotes an approximate expansion of the energy eigenstate $|n\rangle$ in terms of the trial states $|\Phi_j\rangle$. 

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• One can show: For $t_0 = t - a$, optimizing trial states by minimizing effective energies (as on previous slides) is equivalent to solving a GEVP, i.e.

$$\begin{align*}
(w_{BB}, w_{Dd}) &= (|v_{BB, (1+\gamma_0)\gamma_5}|^2, |v_{Dd, (1+\gamma_0)\gamma_5}|^2)
\end{align*}$$

(might offer another perspective on GEVP eigenvector components).

$\rightarrow$ Results for $w_{BB}$ and $w_{Dd}$ can also be interpreted as GEVP results.


• In the literature typically small values for $t_0$ are used, e.g. $t_0/a = 1$ (instead of $t_0 = t - a$ as used to obtain $w_{BB}$ and $w_{Dd}$ on previous slides).

• Similar results also for $t_0/a = 1$, when using a $2 \times 2$ correlation matrix (left plot).

• Consistent results, when using a $4 \times 4$ correlation matrix (right plot).
Structure of the $\bar{b}bqq$ tetraquark (8)

- Define the $r$ dependent $BB$ and $Dd$ percentages,

$$ p_{BB}(r) = w_{BB} \quad , \quad p_{Dd}(r) = w_{Dd} $$

and use the probability density of the $\bar{b}b$ separation

$$ p_r(r) = 4\pi |R(r)|^2 $$

obtained from the BO wave function $R(r)/r$, to estimate the total $BB$ and $Dd$ percentages of the $\bar{b}bud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$:

$$ \%BB = \int dr \, p_r(r)p_{BB}(r) \quad , \quad \%Dd = \int dr \, p_r(r)p_{Dd}(r) = 1 - \%BB. $$

- We find $\%BB = 0.59$, $\%Dd = 0.41$.

- Using $|v_{BB,(1+\gamma_0)\gamma_5}^{(0)}|^2, |v_{Dd,(1+\gamma_0)\gamma_5}^{(0)}|^2$ instead of $w_{BB}, w_{Dd}$ we find $\%BB = 0.61$, $\%Dd = 0.39$.

- Results are in agreement with a GEVP result we obtained in a full lattice QCD computation, where the $\bar{b}$ quarks are treated within NRQCD.

[M. Pflaumer, private communications]
Summary

- The hadronically stable $\bar{b}bud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ is neither exclusively a meson-meson system nor a diquark-antidiquark pair.

- $r \lesssim 0.2$ fm: Clear diquark-antidiquark dominance.

- $r \gtrsim 0.3$ fm: Clear meson-meson dominance.

- Total $BB$ and $Dd$ percentages: $\%BB \approx 0.60$, $\%Dd \approx 0.40$. 

![Graph showing the relationship between $r$ and $w_{BB}$, $w_{Dd}$, $w_{BB}$, and $w_{Dd}$ for ensembles B and C.](image)