Spectrum, decays and structure of mesons from lattice QCD

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Outline

(1) Introduction to lattice computations, QCD and **lattice QCD**
($\gtrsim$ 10 minutes).

(2) Selected research results: **spectrum, decays and structure of mesons**
($\lesssim$ 20 minutes).

- Lattice meson spectroscopy (D, D_s, B, B_s and charmonium).
- Semileptonic decays $B^{(*)} \rightarrow D^{**}$ (1/2 versus 3/2 puzzle).
- Structure of scalar mesons, possible interpretation in terms of mesonic molecules or tetraquarks.
- BB interactions, formation of stable $bb\bar{q}\bar{q}$ tetraquark states.

Part 1: Introduction to lattice computations, QCD and lattice QCD.

Lattice computations in QM (1)

- Introduce the basic principle of lattice computations via a simple example, the 1-dimensional harmonic oscillator in quantum mechanics.
- (Euclidean) action of the harmonic oscillator:

$$S[x] \quad = \quad \int dt\,\Big(\frac{m}{2}\dot{x}(t)^2 + \frac{m\omega^2}{2}x(t)^2\Big).$$

- Goal: compute the average quadratic oscillation x^2 for the ground state $|0\rangle$, i.e. $\langle 0|x^2|0\rangle$, by means of a lattice computation.
- Starting point: path integral formulation (equivalent to Schrödinger's equation),

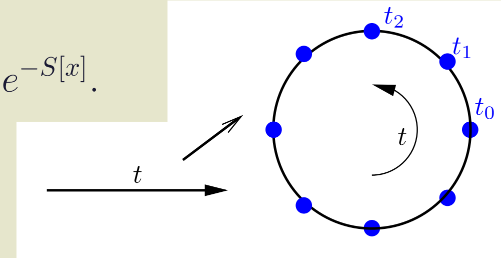
$$\langle 0|x^2|0\rangle \quad = \quad \frac{1}{Z}\int Dx\, x^2 e^{-S[x]} \quad , \quad Z \quad = \quad \int Dx\, e^{-S[x]}.$$

  - $\int Dx$: integral over all possible paths $x(t)$, i.e. an integral over a function space ($=$ "integral over infinitely many variables").
  - $e^{-S[x]}$: weight factor containing the action of the harmonic oscillator.

Lattice computations in QM (2)

- Starting point: path integral formulation (equivalent to Schrödinger's equation),

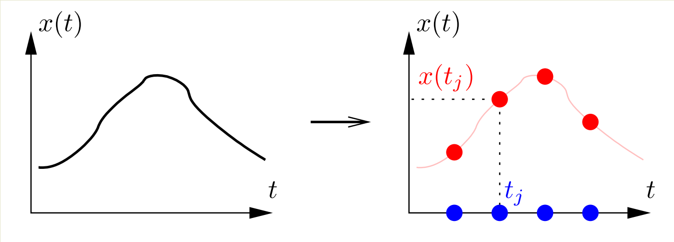
$$\langle 0|x^2|0\rangle = \frac{1}{Z}\int Dx\, x^2 e^{-S[x]} \quad , \quad Z = \int Dx\, e^{-S[x]}.$$



- Discretize and compactify time:
$t \in \mathbb{R} \quad \rightarrow \quad t_j = j \times \Delta t \quad , \quad j = 0, 1, \ldots, N-1$
$\rightarrow$ path integral reduced to an ordinary multi-dimensional integral, i.e.

$$\int Dx\, e^{-S[x]} \quad \rightarrow \quad \int \Big(\prod_{j=0}^{N-1} dx(t_j)\Big) e^{-S[x(t_0),\ldots,x(t_{N-1})]}.$$

- Solve this multi-dimensional by means of a (high performance) computer.



QCD (quantum chromodynamics)

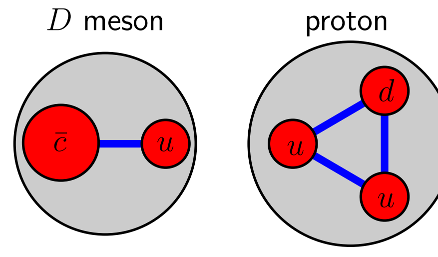
- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (mesons $= q\bar{q}$, baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD by means of an action simple:

$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu \Big) + m^{(f)} \Big) \psi^{(f)} + \frac{1}{2g^2} \text{Tr} \Big(F_{\mu\nu} F_{\mu\nu} \Big) \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$



- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).

Lattice QCD (1)

- Goal: compute QCD observables, e.g. hadron masses, from first principles with controllable systematic error.

- Use the path integral formulation of QCD,

$$\Big\langle \mathcal{O}(\psi^{(f)},\bar{\psi}^{(f)},A_\mu)\Big\rangle \;=\; \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,\mathcal{O}(\psi^{(f)},\bar{\psi}^{(f)},A_\mu)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

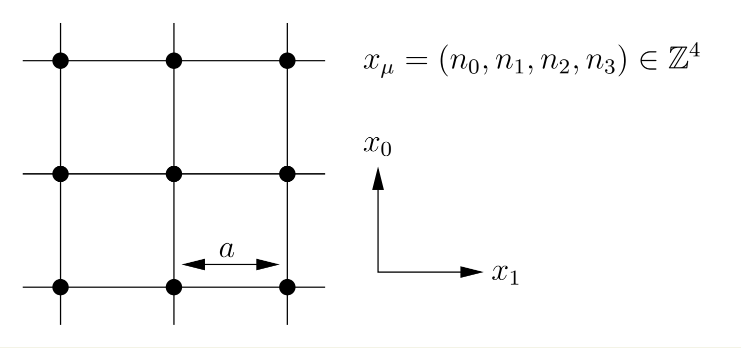
  - $\langle\ldots\rangle$: ground state/vacuum expectation value.
  - $\mathcal{O}(\psi^{(f)},\bar{\psi}^{(f)},A_\mu)$: function of the quark and gluon fields, which can be related to an observable, e.g. a specific meson/baryon mass.
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x},t)$ and $A_\mu(\mathbf{x},t)$.
  - $e^{-S[x]}$: weight factor containing the QCD action.

Note that this path integral is analogous to the quantum mechanical example,

$$\langle 0|x^2|0\rangle \;=\; \frac{1}{Z}\int Dx\,x^2 e^{-S[x]}.$$

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite size effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi \, D\bar{\psi} \, DA \, \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu) \, d\bar{\psi}(x_\mu) \, dU(x_\mu) \bigg) \ldots$$

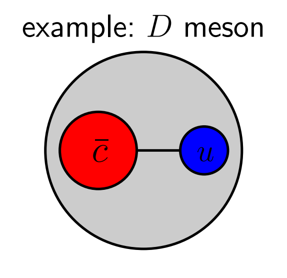
  - Typical present-day dimensionality of a discretized QCD path integral:
    * x_μ: $32^4 \approx 10^6$ lattice sites.
    * $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    * $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    * In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

$\rightarrow$ standard approaches for numerical integration not applicable

$\rightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

European Twisted Mass Collaboration

Part 2: Selected research results: spectrum, decays and structure of mesons.



Mesons

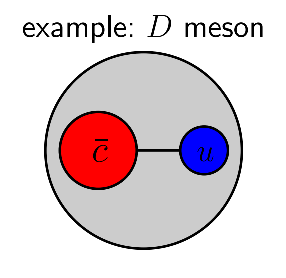
- Mesons:
  - Most mesons are predominantly bound quark-antiquark pairs.
  - Similar to the hydrogen atom, in particular **heavy-light mesons**: a light particle (u, d, or s quark) orbits a heavy particle ($\bar{c}$ or $\bar{b}$ quark).
  - In addition to their quark content states are characterized by
    * Total angular momentum/spin J,
    * Parity P,

    i.e. labeled by J^P.

- In the following, lattice computation of masses of
  - D mesons ($\bar{c}u$, $\bar{c}d$), D_s mesons ($\bar{c}s$),
  - B mesons ($\bar{b}u$, $\bar{b}d$), B_s mesons ($\bar{b}s$),
  - Charmonium ($\bar{c}c$).

quark	mass in MeV/c^2
up	$1.5 \ldots 3.3$
down	$3.5 \ldots 6$
strange	104^{+26}_{-34}
charm	1270^{+70}_{-11}
bottom	4200^{+170}_{-70}
top	170900 ± 1800

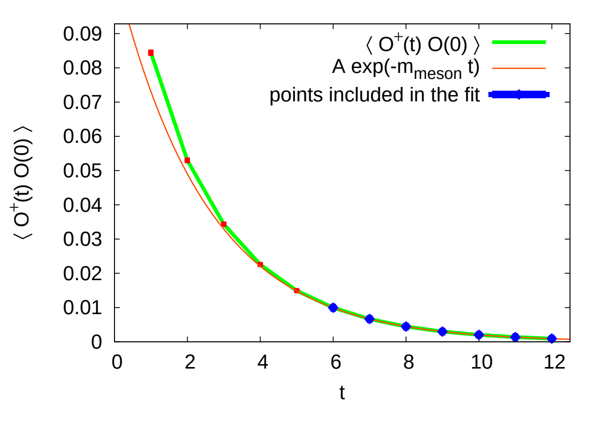
Lattice spectroscopy of mesons (1)



- Proceed as follows:
  (1) Compute the temporal correlations of mesonic $q\bar{q}$ operators $\mathcal{O}$.
  (2) Determine the meson mass of interest from the asymptotic exponential decay in time.
- Example: D meson mass m_D (valence quarks $\bar{c}$ and u, $J^P = 0^-$),

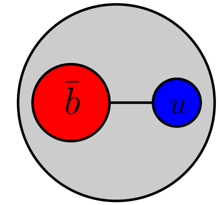
$$\mathcal{O} \equiv \int d^3r \, \bar{c}(\mathbf{r})\gamma_5 u(\mathbf{r})$$

$$\langle \Omega | \mathcal{O}^\dagger(t) \mathcal{O}(0) | \Omega \rangle \overset{t \rightarrow \infty}{\propto} \exp\Big(- m_D t \Big).$$

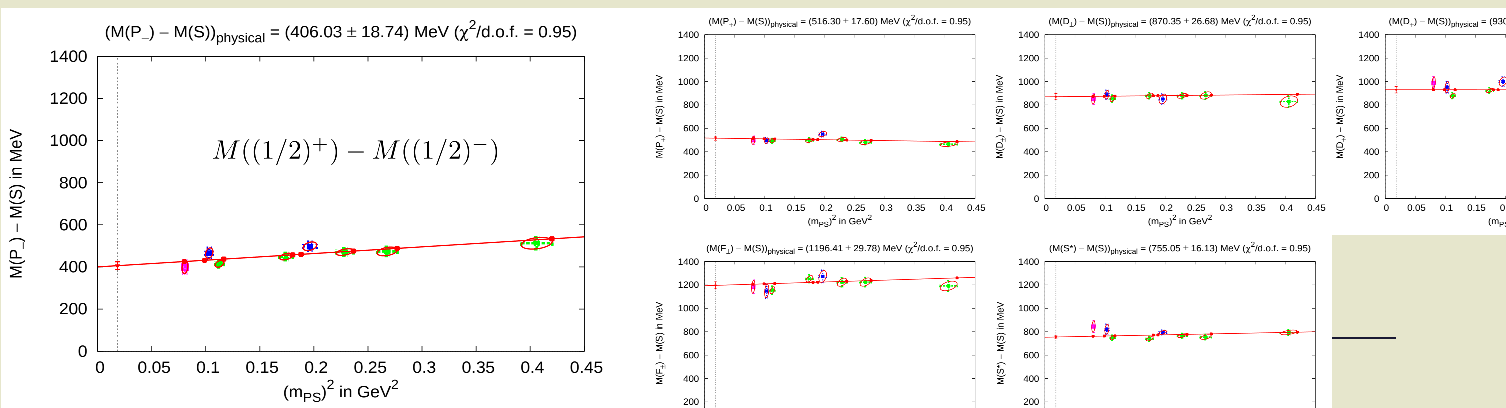


Lattice spectroscopy of mesons (2)

example: B meson



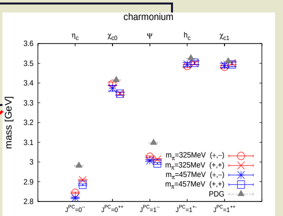
- To be able to remove systematic errors via controlled extrapolations one typically needs a number of independent computations with e.g.
  - different lattice spacings $\longrightarrow$ continuum extrapolation,
  - different quark masses $\longrightarrow$ extrapolation to the "physical point".
- Examplary plots (B meson spectrum):
  different lattice spacings $\longleftrightarrow$ different colors,
  different light quark masses $\longleftrightarrow$ horizontal axis.

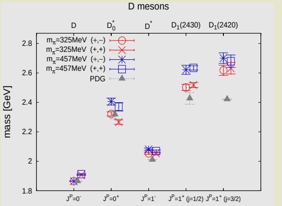


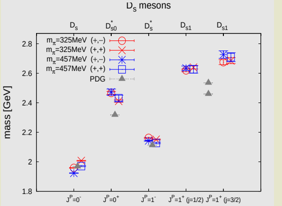
Current status of meson spectra (1)

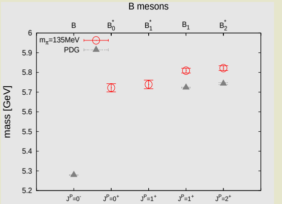
- Quantitative agreement (within errors) between lattice results (red, blue) and experimental results (gray) for some states.
- Some other states disagree by $\lesssim 5\%$; possible reasons include:
  - no continuum limit yet for D, D_s and charmonium; ...

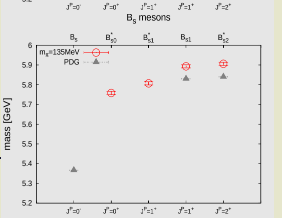
[M. Kalinowski and M.W. [ETM Collaboration], **PoS Confinement10**, 303 (2012)]
[M. Kalinowski and M.W. [ETM Collaboration], submitted to Acta Phys. Polon. (2013)]

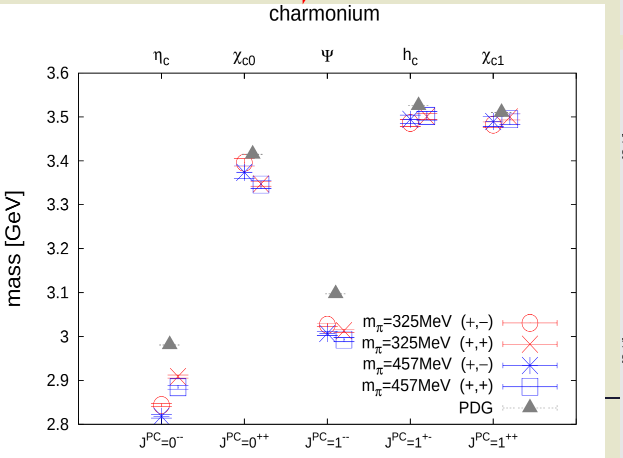




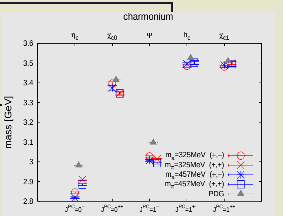




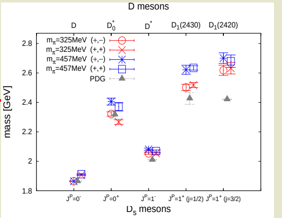


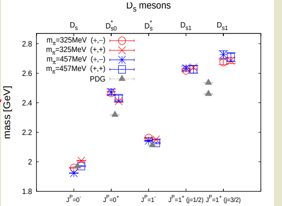


Current status of meson spectra (2)



- Lattice results (red, blue), experimental results (gray).
- Some other states disagree by $\lesssim 5\%$; possible reasons include:
  - certain states (e.g. D_{s0}^*, D_{s1}) might be tetraquarks, i.e. predominantly not $q\bar{q}$;
  - hadronic decays are possible (e.g. $D_{s0}^* \to D + \pi$), contamination by two-particle states (excluded for B, B_s); ...





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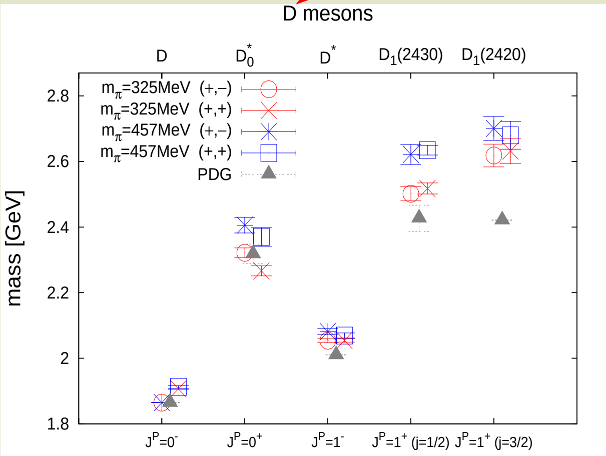
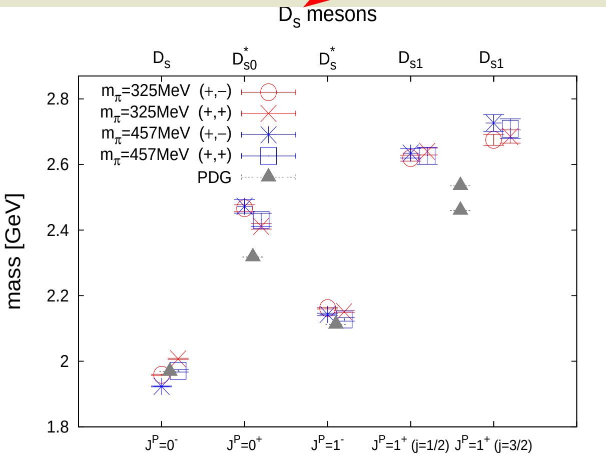
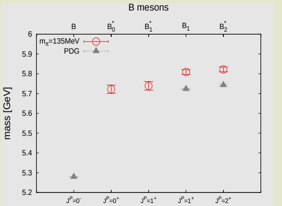
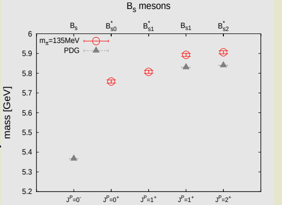




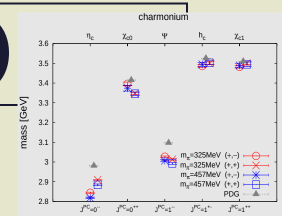


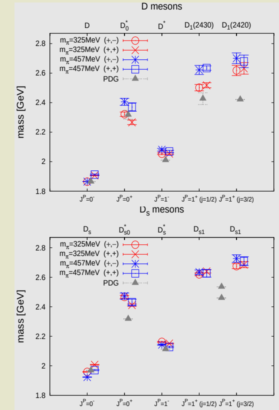


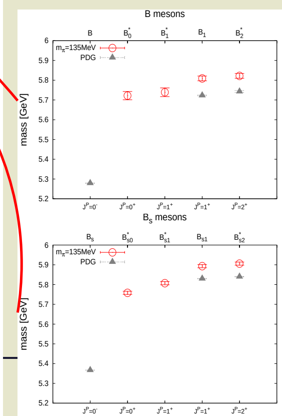
Current status of meson spectra (3)

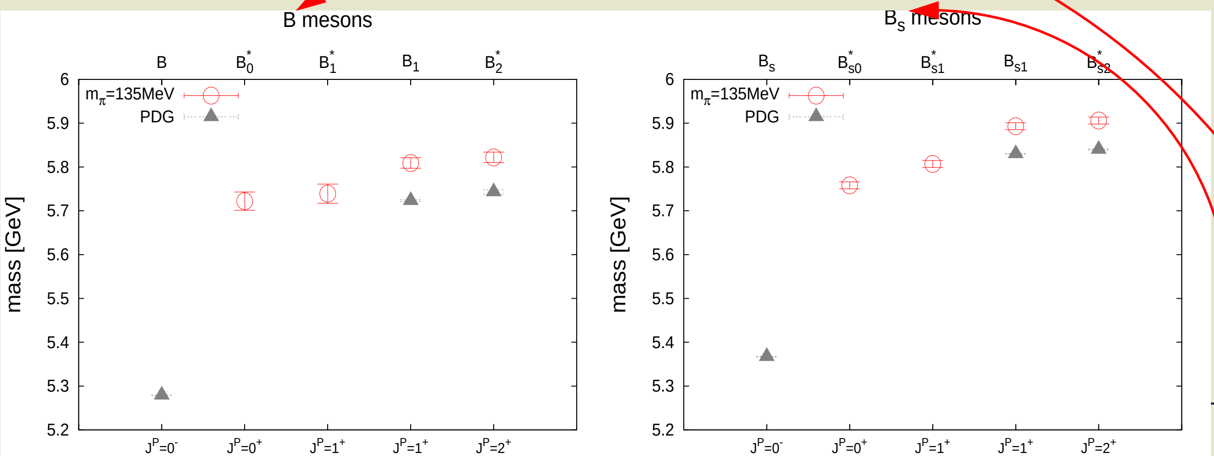
- Lattice results (red), experimental results (gray).
- Some other states disagree by $\lesssim 5\%$; possible reasons include:
  - neglect of $1/m_b^2$ corrections (HQET treatment of $\bar{b}$ quarks);
  - scale setting issues (major collaborations differ in simple quantities like r_0 by up to 20%); ...

[K. Jansen, C. Michael, A. Shindler and M.W. [ETM Collaboration], PoS **LATTICE2008**, 122 (2008)]
[K. Jansen, C. Michael, A. Shindler and M.W. [ETM Collaboration], JHEP **0812**, 058 (2008)]
[C. Michael, A. Shindler and M.W. [ETM Collaboration], JHEP **1008**, 009 (2010)]



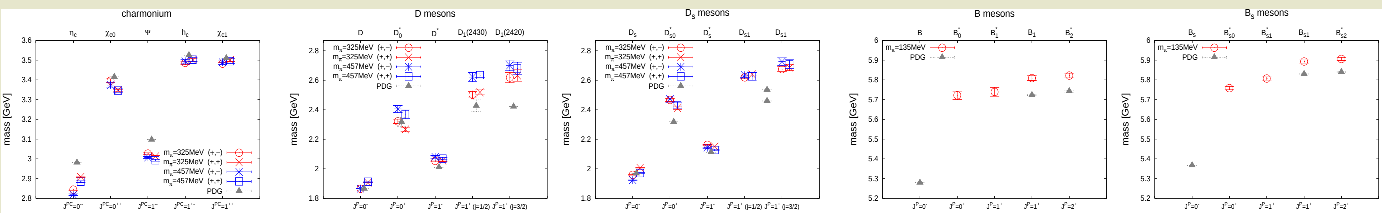






Why lattice spectroscopy of mesons? (1)

- **Reproduction of the major part of the experimentally measured meson spectra** (a minimal requirement for any general non-perturbative first principles computational method).
- **Validate QCD up to a certain level of precision/search for new physics** (in principle): requires excellent experimental and theoretical precision; currently no clear indication for new physics; there are probably more promising quantities for this, e.g. $g-2$, ...
- **Predict experimentally not yet observed mesons**: rather simple; might be important input for future experiments; allows to check/rule out certain phenomenological models (e.g. "Is the ordering of the P_- and P_+ B mesons reversed?"; our lattice results clearly say "No!").



Why lattice spectroscopy of mesons? (2)

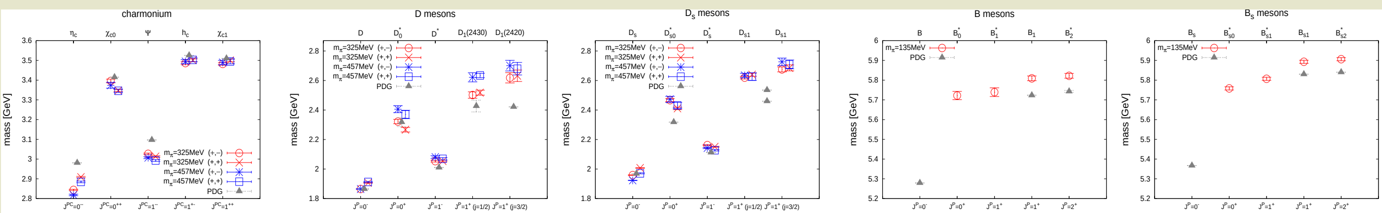
- **Spectroscopy is a necessary first step for most lattice QCD projects**: e.g. the computation of decays usually requires masses and suitable mesonic trial states, which one obtains, while doing spectroscopy.

- **Starting point to resolve puzzles, to answer open questions** (three examples in the following):

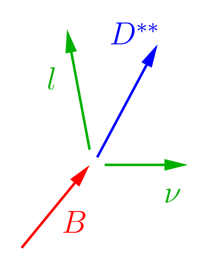
  (A) Semileptonic decays $B^{(*)} \rightarrow D^{**}$ (1/2 versus 3/2 puzzle).

  (B) Structure of scalar mesons, possible interpretation in terms of mesonic molecules or tetraquarks.

  (C) BB interactions, formation of stable $bb\bar{q}\bar{q}$ tetraquark states.



(A) Semileptonic decays $B \to D^{**}$ (1)

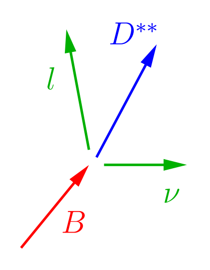


- The weak interactions change quark flavor, e.g. $b \to c + l + \nu$.
- Consider the specific weak decays

$$B \quad \to \quad D^{**} + l + \nu .$$

  - B: the lightest B meson (e.g. $\bar{b}u$), $J^P = 0^-$.
  - D^{**}:
    - An orbitally excited D meson (e.g. $\bar{c}u$) with parity $P = +$.
    - Coupling of angular momentum $L = 1$ ("P wave") and the light and the heavy quark spin yields four possible states:
      - Two $1/2$ D^{**} ($L = 1$ and light quark spin $1/2$ are coupled to total angular momentum $j = 1/2$): D_0^*, $D_1(2430)$.
      - Two $3/2$ D^{**} ($L = 1$ and light quark spin $1/2$ are coupled to total angular momentum $j = 3/2$): $D_1(2420)$, D_2^*.
  - $l + \nu$: lepton and corresponding neutrino.

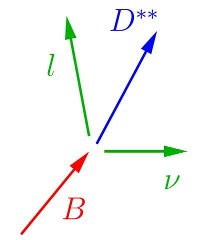
(A) Semileptonic decays $B \to D^{**}$ (2)



- There is a conflict between theory and experiment:
  - **Theory** (operator product expansion, sum rules):
    - Decay of B to $3/2$ D^{**} is more likely.
    - However:
      - Statements only hold in the static limit $m_B, m_D \to \infty$.
      - Assumption: excited states can be neglected in sum rules.
      - Statements apply only for the "zero recoil situation".
    - Supported by model calculations at finite m_B, m_D beyond zero recoil.
  - **Experiment**:
    - Decay of B to $1/2$ D^{**} is more likely.
    - However:
      - The measured signal for $1/2$ D^{**} is extremely weak.
      - Assumption: no contributions of states "above D^{**}".

$\to$ Lattice computations can help, to resolve this conflict.

(A) Semileptonic decays $B \to D^{**}$ (3)



- Computation of the decay probabilities based on Fermi's golden rule, $\mathcal{M}_{fi} = \langle D^{**}_{1/2|3/2}\, l\, \nu | \mathcal{H}_\text{weak} | B \rangle$.

- The hadronic part can be computed by means of lattice QCD ("Isgur-Wise functions" $\tau_{1/2}$ and $\tau_{3/2}$ in differential decay rates); requires meson masses and the corresponding states, which are a byproduct of lattice meson spectroscopy.

- Lattice result: $\tau_{1/2} = 0.30(3)$, $\tau_{3/2} = 0.53(2)$
("$|\tau_{1/2,3/2}|^2$ is proportional to the decay probability to $1/2, 3/2$ D^{**}") ... however, with infinitely heavy b and c quarks.
[B. Blossier, M.W. and O. Pene [ETM Collaboration], JHEP **0906**, 022 (2009)]
[B. Blossier, M.W. and O. Pene [ETM Collaboration], PoS **LATTICE2009**, 253 (2009)]

- A fully dynamical lattice computation is ongoing; the computation of the masses of D_0^*, $D_1(2430)$, $D_1(2420)$, D_2^*, in particular a clear separation of the two D_1 states into a $j = 1/2$ and a $j = 3/2$ state, is an essential ingredient.

(B) Studying tetraquark candidates (1)

- Certain mesons, in particular scalar/pseudovector mesons, do not seem to be ordinary quark-antiquark states, could be four-quark states (tetraquarks) ...?
- A prominent tetraquark candidate is the nonet of light scalar mesons,
  - $\sigma \equiv f_0(500)$, $I = 0$, $400 \ldots 550$ MeV,
  - $\kappa \equiv K_0^*(800)$, $I = 1/2$, 682 ± 29 MeV,
  - $f_0(980)$, $a_0(980)$, $I = 0, 1$, 990 ± 20 MeV, 980 ± 20 MeV

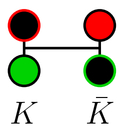
  ($J^P = 0^+$), which is poorly understood:
  - All nine states are unexpectedly light.
  - Their ordering is inverted compared to expectation.

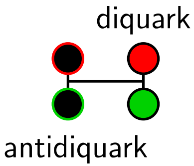
  $\longrightarrow$ Can naturally be explained, when assuming a tetraquark structure.

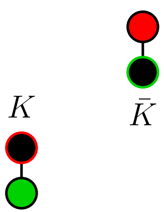
$\longrightarrow$ Study these states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.

(B) Studying tetraquark candidates (2)

- Tetraquark operators for $a_0(980)$ (quantum numbers $I(J^P) = 1(0^+)$):







  - Needs two light quarks due to $I = 1$, e.g. $u\bar{d}$.
  - $a_0(980)$ decays to $K\bar{K}$ ... suggests an $s\bar{s}$ component.
  - Molecule type (models a bound $K\bar{K}$ state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \int d^3x \, \underbrace{\Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)}_{\equiv \bar{K}(\mathbf{x})} \underbrace{\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big)}_{\equiv K(\mathbf{x})}.$$

  - Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \int d^3x \, \underbrace{\Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)}_{\equiv \text{antidiquark}(\mathbf{x})} \underbrace{\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big)}_{\equiv \text{diquark}(\mathbf{x})}.$$

  - Two particle $K + \bar{K}$ type:

$$\mathcal{O}_{a_0(980)\text{ quantum numbers}}^{K+\bar{K}\text{ two-particle}} = \bigg(\int d^3x \, \overbrace{\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})}^{\equiv \bar{K}(\mathbf{x})}\bigg)\bigg(\int d^3y \, \overbrace{\bar{d}(\mathbf{y})\gamma_5 s(\mathbf{y})}^{\equiv K(\mathbf{y})}\bigg).$$

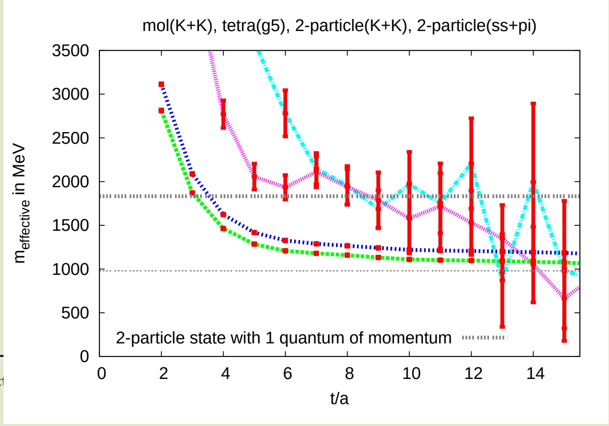
  - Two particle $\eta + \pi$ type: ...

(B) Studying tetraquark candidates (3)

- Current status of our computations:
  - There are two states around the expected $a_0(980)$ mass 980 ± 20 MeV:
    - one has $\gtrsim 95\%$ operator content "two-particle $\eta + \pi$",
    - the other has $\gtrsim 95\%$ operator content "two-particle $K + \bar{K}$".
  - Higher states have energies $\gtrsim 1700\,\mathrm{MeV}$ (consistent with two-particle $\eta + \pi$ and $K + \bar{K}$ excitations with one relative quantum of momentum).
- **Conclusions: $a_0(980)$ is not a strongly bound four-quark state ... maybe of $\bar{q}q$ type ... probably a rather unstable resonance.**

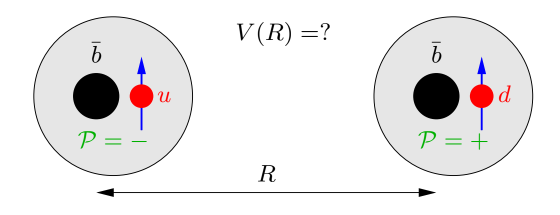
  [C. Alexandrou, M.W. *et al.* [ETM Collaboration], accepted by JHEP (2013)]

- Similar results for κ.
- Investigation of $a_0(980)$ as a resonance ongoing (challenging in lattice QCD, needs volume-dependent two-particle spectrum).



(C) Heavy-heavy-light-light tetraquarks (1)

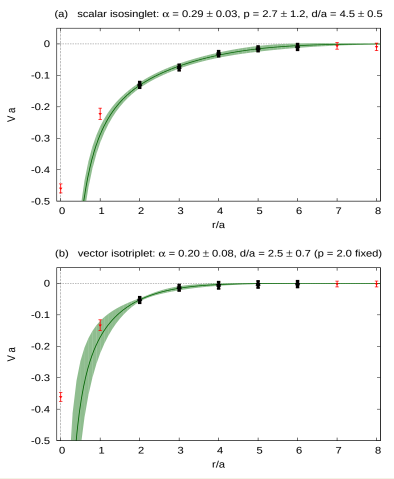
- Study possibly existing $QQ\bar{q}\bar{q}$ (heavy-heavy-light-light) tetraquark states:
  - Use the static approximation for the heavy quarks QQ (reduces the necessary computation time significantly).
  - Most appropriate for $QQ \equiv bb$.
  - Could also yield information for $QQ \equiv cc$.



- Proceed in two steps:

(1) Compute the potential of two heavy quarks QQ in the background of two light antiquarks $\bar{q}\bar{q}$ by means of lattice QCD
$\longrightarrow$ many different channels/quantum numbers.
[M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538 [hep-lat]]]
[M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147 [hep-lat]]]

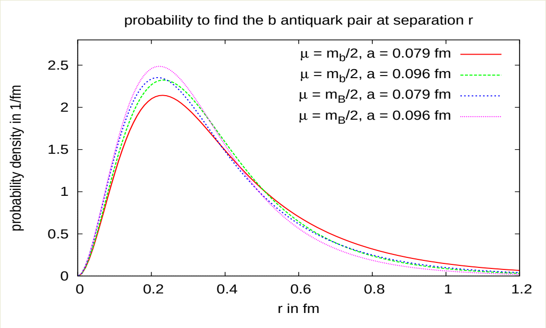
(2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks QQ.



(C) Heavy-heavy-light-light tetraquarks (2)

- Clear indication for a bound state for $QQ \equiv bb$ in a specific channel:
  - Quantum numbers: $I(J^P) = 0(0^+)\,,\, 0(1^+)$ (degeneracy with respect to the heavy quark spin).
  - Binding energy: $E \approx -50\,\mathrm{MeV}$.

  [P. Bicudo, M.W., submitted to Phys. Rev. D (2013)]



- No four-quark binding in other channels.
- Next steps:
  - Extend these investigations to the experimentally more interesting case of $Q\bar{Q}$ (instead of QQ).
  - Statements about $QQ = cc$ and $Q\bar{Q} = c\bar{c}$ (instead of $QQ = bb$ and $Q\bar{Q} = b\bar{b}$).

Conclusions

- Most of the presented lattice results for mesons, tetraquark candidates, specific decays, etc. are:
  - Preliminary
    - $\longrightarrow$ certain systematic errors need to be studied and quantified, e.g. lattice discretization errors, unphysically heavy u/d quark masses;
    - $\longrightarrow$ statistical errors need to be reduced.
  - Promising
    - $\longrightarrow$ contact to experimental results established (e.g. meson spectra)
    - $\longrightarrow$ first statements about states and decays, which are presently not well understood (e.g. the tetraquark candidate $a_0(980)$, the decay $B \rightarrow D^{**}$).
- **Long-term goal: meson spectroscopy/structure from first principles (QCD) free of systematic errors, which is relevant and important in the context of current/future experiments.**