Tetraquarks with two heavy quarks from lattice QCD

"Deciphering Strong-Interaction Phenomenology through Precision Hadron-Spectroscopy" – MIAPP, Germany

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Part 1: $\bar{b}\bar{b}qq$ tetraquarks

(part 2 is about $\bar{b}b\bar{q}q$ tetraquarks)

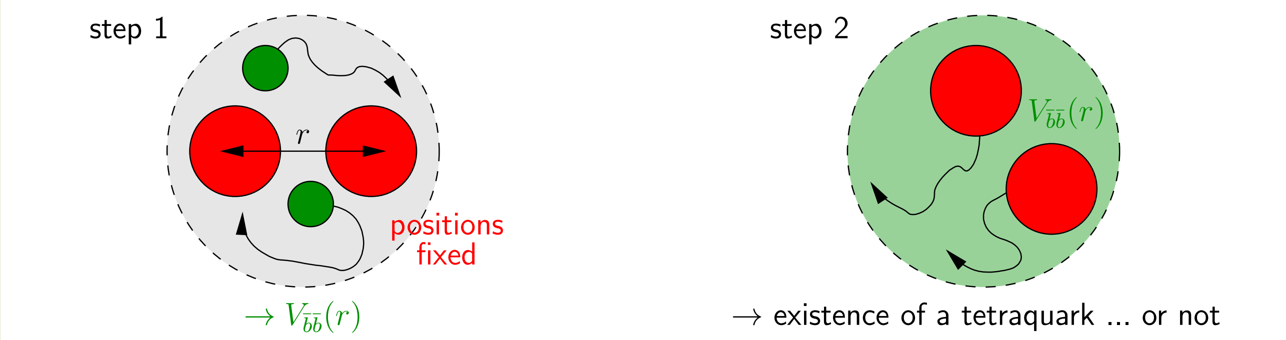
Basic idea: lattice QCD + BO (1)

- Study heavy-heavy-light-light tetraquarks $\bar{b}\bar{b}qq$ in two steps.

  (1) **Compute potentials of two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s, c\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

  ((1) + (2) $\rightarrow$ Born-Oppenheimer approximation).



Basic idea: lattice QCD + BO (2)

- The talk summarizes:
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]
  [P. Bicudo, M. Cardoso, N. Cardoso, M.W. [arXiv:1910.04827]]

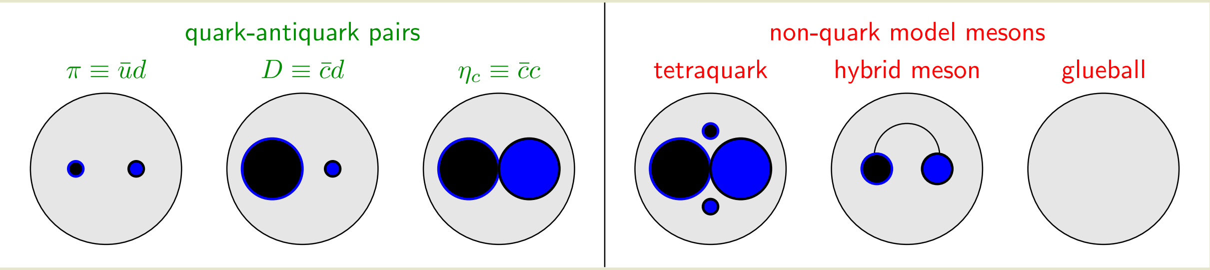
- For recent work from other groups using a similar approach cf. e.g.:
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571]]
  [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953]]
  [S. Prelovsek, H. Bahtiyar and J. Petkovic [arXiv:1909.02356]]

- Related work on quarkonium (non-exotic and exotic):
  [N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Rev. D **63**, 014023 (2001) [hep-ph/0002250]]
  [A. Pineda and A. Vairo, Phys. Rev. D **63**, 054007 (2001) [hep-ph/0009145]]
  [N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77**, 1423 (2005) [hep-ph/0410047]]
  [E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]

- More related work, in particular on heavy hybrid mesons, ...

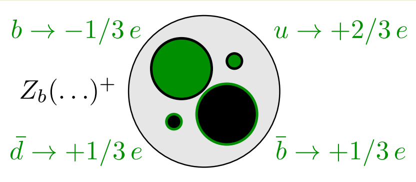
Why are such studies important? (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \ldots$
- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.q. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).
- Certain mesons are poorly understood (e.g. significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
  - **2 quarks and 2 antiquarks (tetraquark)**,
  - **a quark-antiquark pair and gluons (hybrid meson)**,
  - **only gluons (glueball)**.



Why are such studies important? (2)

- Indications for tetraquark structures:
  - Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$:
    - Mass suggests a $b\bar{b}$ pair ...
    - ... but $b\bar{b}$ is electrically neutral ...?
    - **Easy to understand, when assuming a tetraquark structure**: $Z_b(\ldots)^+ \equiv b\bar{b}u\bar{d}$ ($u \rightarrow +2/3\,e$, $\bar{d} \rightarrow -1/3\,e$).



  - Electrically charged Z_c states:
    - Similar to Z_b states.
  - Mass ordering of light scalar mesons:
    - E.g. $m_\kappa > m_{a_0(980)}$ ...?

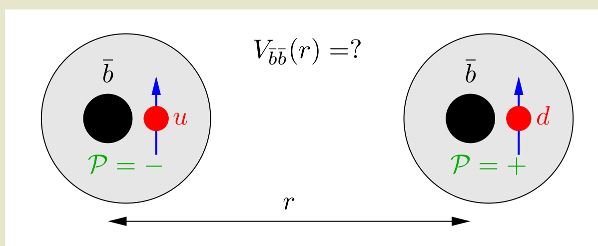
Outline (for part 1)

- $\bar{b}\bar{b}qq$ / BB potentials.
- Lattice setup.
- $\bar{b}\bar{b}qq$ tetraquarks.
- Quantum numbers of the predicted $\bar{b}\bar{b}qq$ tetraquark.
- Inclusion of heavy spin effects.
- $\bar{b}\bar{b}qq$ tetraquark resonances.

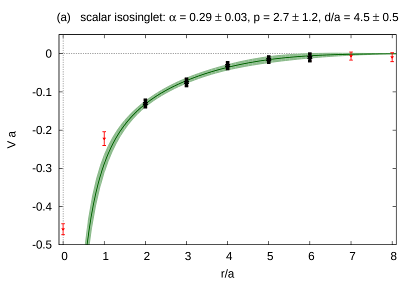
$\bar{b}\bar{b}qq$ / BB potentials (1)

- Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).
- At large $\bar{b}\bar{b}$ separation r, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Consider only pseudoscalar/vector mesons ($j^P = (1/2)^-$, PDG: B, B^*) and scalar/pseudovector mesons ($j^P = (1/2)^+$, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons (j: spin of the light degrees of freedom).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of qq on
  - the "light" quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^* (parity).

$\rightarrow$ Many different channels: attractive as well as repulsive, different asymptotic values ...



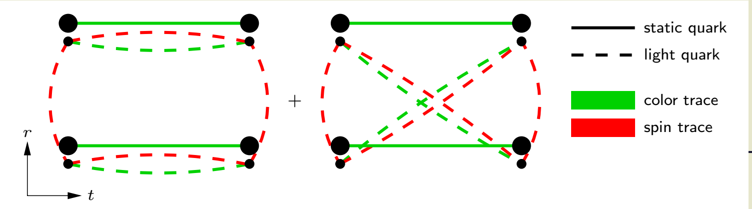
$\bar{b}\bar{b}qq$ / BB potentials (2)



- Rotational symmetry broken by static quarks $\bar{b}\bar{b}$.
- Remaining symmetries and quantum numbers:
  - $j_z \equiv \Lambda$: rotations around the separation axis (e.g. z axis).
  - $P \equiv \eta$: parity.
  - $P_x \equiv \epsilon$: reflection along an axis perpendicular to the separation axis (e.g. x axis).
- To extract the potential(s) of a given sector $(I, I_z, |j_z|, P, P_x)$, compute the temporal correlation function of the trial state(s)

$$\Big(C\Gamma\Big)_{AB}\Big(C\tilde{\Gamma}\Big)_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big)|\Omega\rangle.$$

  - $q^{(1)}q^{(2)} \in \{ud - du\,,\, uu\,,\, dd\,,\, ud + du\,,\, ss\,,\, cc\}$ (isospin I, I_z, flavor).
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity P, P_x).
  - $\tilde{\Gamma} \in \{(1-\gamma_0)\gamma_5\,,\,(1-\gamma_0)\gamma_j\}$ (irrelevant).



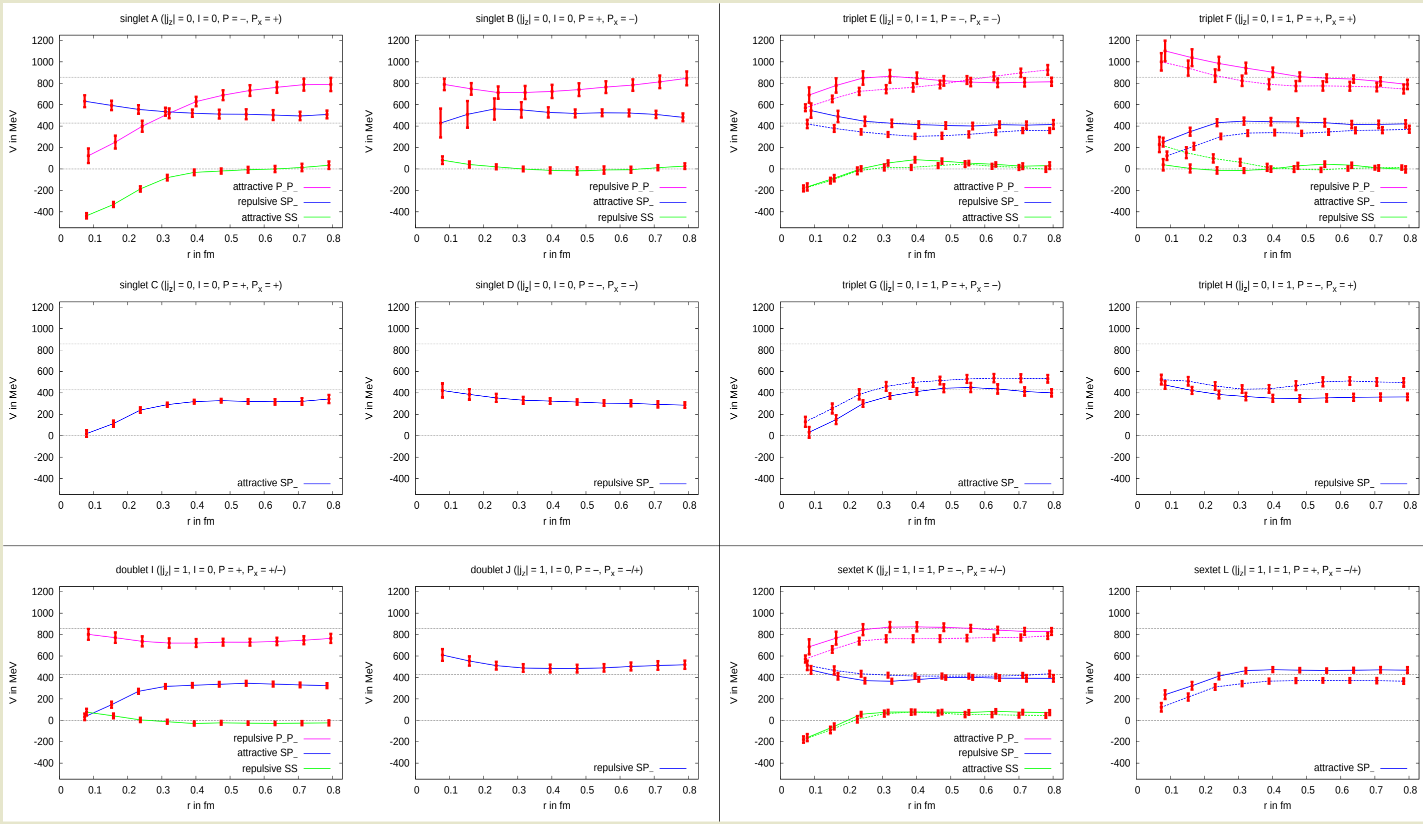
Lattice setup for $\bar{b}\bar{b}qq$ / BB

- ETMC gauge link ensembles:
  - $N_f = 2$ dynamical quark flavors.
  - Lattice spacing $a \approx 0.079\,\text{fm}$.
  - $24^3 \times 48$, i.e. spatial lattice extent $\approx 1.9\,\text{fm}$.
  - Three different pion masses $m_\pi \approx 340\,\text{MeV}$, $m_\pi \approx 480\,\text{MeV}$, $m_\pi \approx 650\,\text{MeV}$.

[R. Baron *et al.* [ETM Collaboration], JHEP **1008**, 097 (2010) [arXiv:0911.5061 [hep-lat]]

$\bar{b}\bar{b}qq$ / BB potentials (3)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



$\bar{b}\bar{b}qq$ / BB potentials (4) to (7)

- **Why are there three different asymtotic values?**
  - They correspond to $B^{(*)}B^{(*)}$ potentials, to $B^{(*)}B^*_{0,1}$ potentials and $B^*_{0,1}B^*_{0,1}$ potentials.
- **Why are certain channels attractive and others repulsive?**
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}qq$ / BB potentials.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}qq$ / BB potentials.
  - Because of the Pauli principle and (assuming) "1-gluon exchange" at small r.
- **24 different (i.e. non-degenerate) $\bar{b}\bar{b}qq$ / BB potentials.**

$\bar{b}\bar{b}qq$ / BB potentials (4)

Why are there three different asymtotic values?

- Differences $\approx 400\,\mathrm{MeV}$, approximately the mass difference of $B^{(*)}$ $(P = -)$ and $B^*_{0,1}$ $(P = +)$.
- Suggests that the three different asymtotic values correspond to $B^{(*)}B^{(*)}$ potentials, to $B^{(*)}B^*_{0,1}$ potentials and $B^*_{0,1}B^*_{0,1}$ potentials.
- Can be checked and confirmed, by rewriting the $\bar{b}\bar{b}qq$ creation operators in terms of meson-meson creation operators (Fierz transformation).
- Example: uu, $\Gamma = \gamma_3$ (attractive, lowest asymptotic value),

$$\Big(C\gamma_3\Big)_{AB}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(u)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(u)}(+\mathbf{r}/2)\Big) \quad \propto$$
$$\propto \quad (B^{(*)})_\uparrow(B^{(*)})_\downarrow + (B^{(*)})_\downarrow(B^{(*)})_\uparrow - (B^*_{0,1})_\uparrow(B^*_{0,1})_\downarrow - (B^*_{0,1})_\downarrow(B^*_{0,1})_\uparrow.$$

- Example: uu, $\Gamma = 1$ (repulsive, medium asymptotic value),

$$\Big(C1\Big)_{AB}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(u)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(u)}(+\mathbf{r}/2)\Big) \quad \propto$$
$$\propto \quad (B^{(*)})_\uparrow(B^*_{0,1})_\downarrow - (B^{(*)})_\downarrow(B^*_{0,1})_\uparrow + (B^*_{0,1})_\uparrow(B^{(*)})_\downarrow - (B^*_{0,1})_\downarrow(B^{(*)})_\uparrow.$$

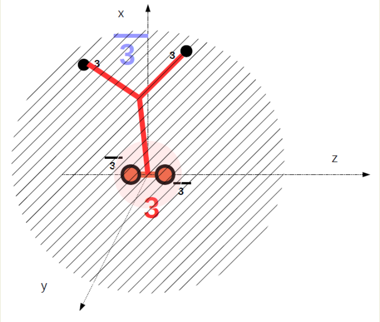
$\bar{b}\bar{b}qq$ / BB potentials (5)

Why are certain channels attractive and others repulsive? (1)

- Fermionic wave function must be antisymmetric (Pauli principle); in quantum field theory/QCD automatically realized.
- qq isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- qq angular momentum/spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- qq color:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e., a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e., a sextet 6.
- The four quarks $\bar{b}\bar{b}qq$ must form a color singlet:
  - qq in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a triplet 3.
  - qq in a color sextet 6 $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a sextet $\bar{6}$.

$\bar{b}\bar{b}qq$ / BB potentials (6)

Why are certain channels attractive and others repulsive? (2)



- Assumption: attractive/repulsive behavior of $\bar{b}\bar{b}$ at small separations r is mainly due to 1-gluon exchange,
  - color triplet 3 is attractive, $V_{\bar{b}\bar{b}}(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V_{\bar{b}\bar{b}}(r) = +\alpha_s/3r$

  (easy to calculate in LO perturbation theory).
- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.
- Expectation consistent with the obtained lattice results.
- **Pauli principle and assuming "1-gluon exchange" at small r explains, why certain channels are attractive and others repulsive.**

$\bar{b}\bar{b}qq$ / BB potentials (7)

- Summary of $\bar{b}\bar{b}qq$ / BB potentials:

$B^{(*)}B^{(*)}$ potentials:	attractive:	$1 \oplus 3 \oplus 6$	(10 states).
	repulsive:	$1 \oplus 3 \oplus 2$	(6 states).
$B^{(*)}B^*_{0,1}$ potentials:	attractive:	$1 \oplus 1 \oplus 3 \oplus 3 \oplus 2 \oplus 6$	(16 states).
	repulsive:	$1 \oplus 1 \oplus 3 \oplus 3 \oplus 2 \oplus 6$	(16 states).
$B^*_{0,1}B^*_{0,1}$ potentials:	attractive:	$1 \oplus 3 \oplus 6$	(10 states).
	repulsive:	$1 \oplus 3 \oplus 2$	(6 states).

  - 2-fold degeneracy due to spin $j_z = \pm 1$.
  - 3-fold degeneracy due to isospin $I = 1$, $I_z = -1, 0, +1$.

$\rightarrow 24$ **different** $\bar{b}\bar{b}qq$ **/** BB **potentials.**

$\bar{b}\bar{b}qq$ / BB potentials (8)

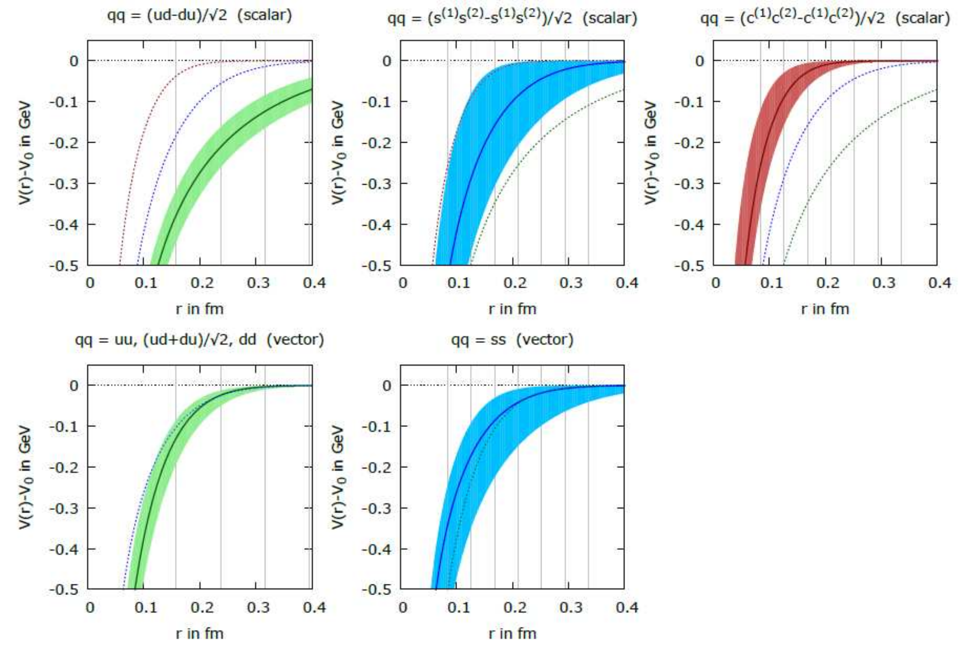
- Focus on the two attractive channels between B and B^*:
  - Scalar isosinglet ($(I = 0, j = 0)$, more attractive): $qq = (ud - du)/\sqrt{2}$, $\Gamma = (1 + \gamma_0)\gamma_5$.
  - Vector isotriplet ($(I = 1, j = 1)$, less attractive): $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$, $\Gamma = (1 + \gamma_0)\gamma_j$.
- Computations for $qq = ll, ss, cc$ ($l \in \{u, d\}$) to study the mass dependence.
- Parameterize lattice potential results by contiuous functions obtained by χ^2 minimizing fits of

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r}\exp\bigg(-\Big(\frac{r}{d}\Big)^p\bigg) + V_0 :$$

  - $1/r$: 1-gluon exchange at small $\bar{b}\bar{b}$ separations.
  - $\exp(-(r/d)^p)$: color screening at large $\bar{b}\bar{b}$ separations due to meson formation.
  - Fit parameters α, d and V_0; $p = 2$ from quark models.

$\bar{b}\bar{b}qq$ / BB potentials (9)

- Potentials for $qq = ll$, $l \in \{u, d\}$ are wider and deeper than potentials for $qq = ss, cc$.
  $\rightarrow$ **Good candidates to find tetraquarks are systems of two very heavy and two very light quarks, i.e., $\bar{b}\bar{b}ll$.**



$\bar{b}\bar{b}qq$ tetraquarks (1)

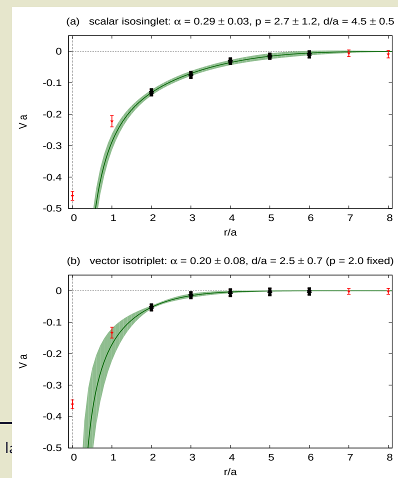
- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

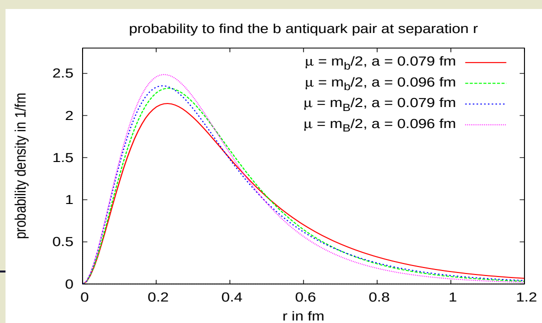
$$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\right)\psi(\mathbf{r}) \;=\; E\psi(\mathbf{r}) \quad , \quad \mu \;=\; m_b/2.$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}\bar{b}qq$ tetraquarks.

- There is a bound state for $qq = (ud - du)/\sqrt{2}$ (i.e., the scalar isosinglet potential) and orbital angular momentum $l = 0$ of $\bar{b}\bar{b}$, binding energy $E = -90^{+43}_{-36}$ MeV with respect to the BB^* threshold, i.e. confidence level $\approx 2\,\sigma$.

- No further bound states, in particular not for $qq = ss, cc$ (i.e., $B_s B_s$, $B_c B_c$).



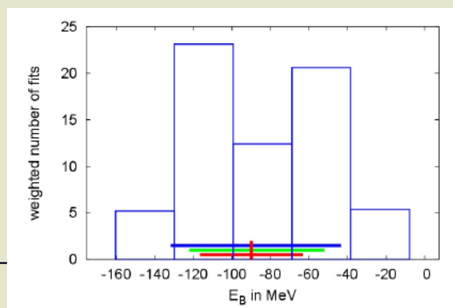


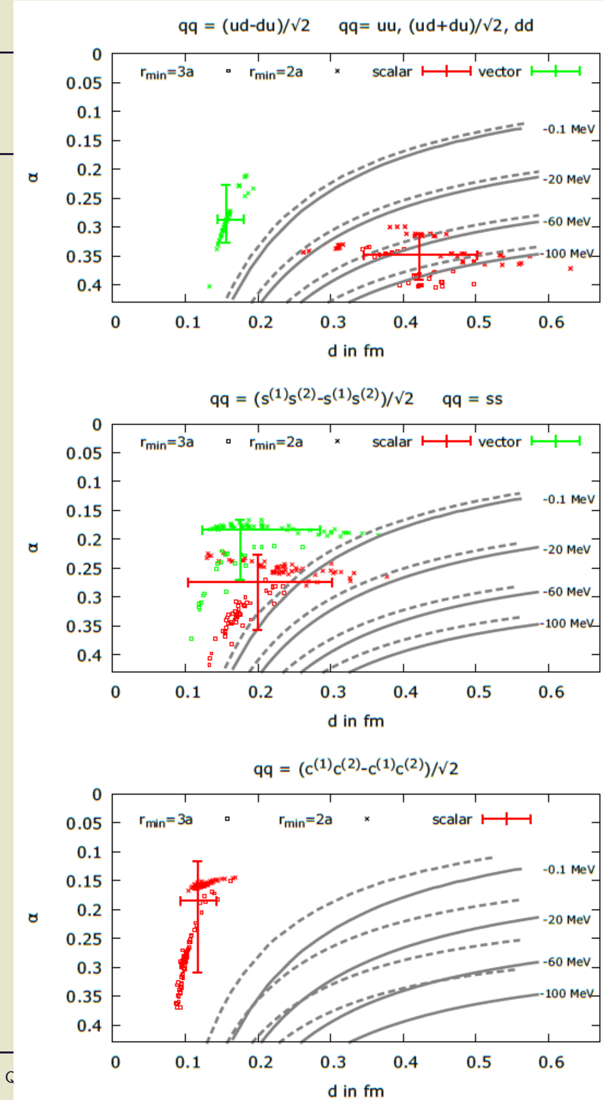
$\bar{b}\bar{b}qq$ tetraquarks (2)

- Estimate the systematic error by varying input parameters:
  - the t fitting range to extract the potential from effective masses,
  - the r fitting range for

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^{p}\right) + V_0.$$

- Right: isoline plots of the binding energy E for $l = 0$.
- Bottom: histogram for the binding energy E for $qq = (ud - du)/\sqrt{2}$ and $l = 0$.





$\bar{b}\bar{b}qq$ tetraquarks (3)

- To quantify "no binding", we list for each channel the factor, by which the reduced mass μ in the Schrödinger equation has to be multiplied, to obtain a tiny but negative energy E (again for $l = 0$).

qq	spin	factor
$(ud - du)/\sqrt{2}$	scalar	0.46
uu, $(ud + du)/\sqrt{2}$, dd	vector	1.49
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	1.20
ss	vector	2.01
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	2.57

  - Factors $\ll 1$ indicate strongly bound states, while for values $\gg 1$ bound states are excluded.
  - Light quarks (u/d) are unphysically heavy (correspond to $m_\pi \approx 340$ MeV); physically light u/d quarks yield similar results.
  - Mass splitting $m(B^*) - m(B) \approx 50$ MeV, neglected at the moment, is expected to weaken binding (will be discussed below).

$\bar{b}\bar{b}qq$ tetraquarks (short version of ...)

- **What are the quantum numbers of the predicted $\bar{b}\bar{b}qq$ tetraquark?**
  - $I(J^P) = 0(1^+)$.
- **Will there still be a bound state, when heavy spin effects are taken into account?**
  - Yes, binding energy $E = -59^{+38}_{-30}$ MeV (without heavy spin effects $E = -90^{+43}_{-36}$ MeV).
  - Tetraquark is approximately a $50\%/50\%$ superposition of BB^* and B^*B^*.
- **Tetraquark resonances can be studied in a similar way using standard methods from scattering theory.**
  - There is a resonance for $qq = (ud - du)/\sqrt{2}$ and $l = 1$.
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma_{\rightarrow B+B} = 112^{+90}_{-103}$ MeV.
  - Quantum numbers $I(J^P) = 0(1^-)$.

Quantum numbers of the $\bar{b}\bar{b}qq$ tetraquark

What are the quantum numbers of the predicted $\bar{b}\bar{b}qq$ tetraquark?

- $I(J^P) = 0(1^+)$.
  - Light scalar isosinglet: $qq = (ud - du)/\sqrt{2}$, $I = 0$, $j = 0$ in a color $\bar{3}$, $\bar{b}\bar{b}$ in a color 3 (antisymmetric) ... as discussed above.
  - Wave function of $\bar{b}\bar{b}$ must also be antisymmetric (Pauli principle).
    - $\bar{b}\bar{b}$ is flavor symmetric.
    - $\bar{b}\bar{b}$ spin must also be symmetric, i.e., $j_b = 1$.

  $\rightarrow$ **The predicted $\bar{b}\bar{b}qq$ tetraquark has isospin $I = 0$, spin $J = 1$.**
  - We study a state, which correspond for large $\bar{b}\bar{b}$ separations to a pair of $B^{(*)}$ mesons in a spatially symmetric s-wave.

  $\rightarrow$ **The predicted $\bar{b}\bar{b}qq$ tetraquark has parity $P = +$** (the product of the parity quantum numbers of the two mesons, which are both negative)**.**

Inclusion of heavy spin effects

- Heavy spin effects have been neglected so far, e.g. mass splitting $m_{B^*} - m_B \approx 46\,\text{MeV}$.
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $E = -90^{+43}_{-36}\,\text{MeV}$.
- Moreover, two competing effects:
  - The attractive $\bar{b}\bar{b}ud$ channel corresponds to a linear combination of BB^* and/or B^*B^*.
  - The BB^* interaction is a superposition of attractive and repulsive $\bar{b}\bar{b}ud$ potentials.
- **Will there still be a bound state, when heavy spin effects are taken into account?**
  - Yes.
  - We include heavy spin effects by solving a coupled channel Schrödinger equation.
    [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  - Binding energy $E = -59^{+38}_{-30}\,\text{MeV}$.
  - Tetraquark is approximately a $50\%/50\%$ superposition of BB^* and B^*B^* (strong attraction more important than light constituents).

$\bar{b}\bar{b}qq$ tetraquark resonances (1)

- Most hadrons are unstable, i.e., resonances.
- If a $\bar{b}\bar{b}qq$ potential $V_{\bar{b}\bar{b}}(r)$ is not sufficiently attractive to host a bound state, there could still be a clear resonance.
- Comparatively easy to investigate within our approach (since we have potentials $V_{\bar{b}\bar{b}}(r)$, no Lüscher method etc. necessary).
- Use standard methods from scattering theory:
  - Solve Schrödinger equation with potential $V_{\bar{b}\bar{b}}(r)$ and appropriate boundary conditions (incident plane wave, emergent spherical wave)
$\rightarrow$ partial wave amplitudes $f_l(E)$.
  - Use partial wave amplitudes $f_l(E)$ to ...
    - ... determine phase shifts and contributions of partial waves to total cross section
$\rightarrow$ peak indicates resonance mass.
    - ... determine poles of the S or the T matrix in the complex energy plane (correspond to poles of $f_l(E)$)
$\rightarrow$ real part of a pole $\equiv$ resonance mass
$\rightarrow$ imaginary part of a pole $\equiv$ resonance width.

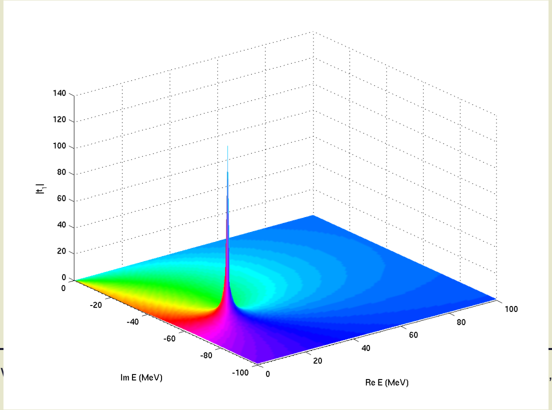
$\bar{b}\bar{b}qq$ tetraquark resonances (2)

- Exploratory study mostly for $qq = (ud - du)/\sqrt{2}$ (i.e., the scalar isosinglet potential) and orbital angular momentum $l = 1$ of $\bar{b}\bar{b}$:

- There is a resonance for $qq = (ud - du)/\sqrt{2}$ and $l = 1$:
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma_{\to B+B} = 112^{+90}_{-103}$ MeV.
  - Quantum numbers $I(J^P) = 0(1^-)$.

  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]

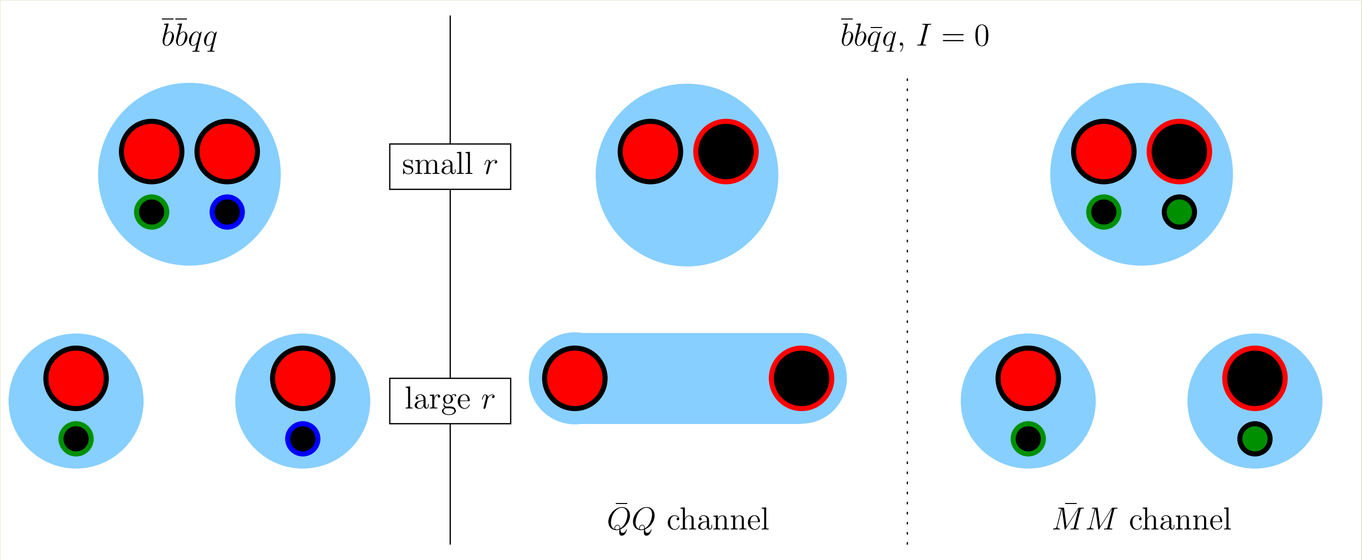
- There do not seem to be resonances in other channels ($l > 1$, vector isotriplet potential, heavier quarks qq).



Part 2: bottomonium, $I = 0$
(superposition of $\bar{b}b$ and $\bar{b}b\bar{q}q$)

Bottomonium, $I = 0$: difference to $\bar{b}\bar{b}qq$

- Now bottomonium with $I = 0$, i.e. $\bar{b}b$ and/or $\bar{b}b\bar{q}q$ (with $\bar{q}q = (\bar{u}u + \bar{d}d)/\sqrt{2}$).
  [P. Bicudo, M. Cardoso, N. Cardoso, M.W. [arXiv:1910.04827]].
- Technically more complicated than $\bar{b}\bar{b}qq$, because there are two channels:
  - Quarkonium channel, $\bar{Q}Q$ (with $Q \equiv b$).
  - Heavy-light meson-meson channel, $\bar{M}M$ (with $M = \bar{Q}q$).



Bottomonium, $I = 0$: coupled channel SE

- Consider only the lightest decay channel to $\bar{B}^{(*)} B^{(*)}$, i.e. at the moment no decays to excited B mesons, e.g. to B_0^* and B_1^*.
- Symmetries and quantum numbers (heavy quark symmetry, S, L, P, C):
  - Spins of the heavy quarks $\bar{Q}$ and Q irrelevant, can be ignored.
    $\rightarrow$ $\bar{Q}Q$ represented by a 1-component wave function $\psi_{\bar{Q}Q}(\mathbf{r})$.
  - $\bar{Q}Q$ (any orbital angular momentum L) can only decay to $\bar{M}M$ with light spin $S_q^{PC} = 1^{--}$ and orbital angular momentum $L \pm 1$.
    $\rightarrow$ $\bar{M}M$ represented by a 3-component wave function $\vec{\psi}_{\bar{M}M}(\mathbf{r})$.
  - Wave function of the coupled channel Schrödinger equation has 4 components, $\psi(\mathbf{r}) = (\psi_{\bar{Q}Q}(\mathbf{r}), \vec{\psi}_{\bar{M}M}(\mathbf{r}))$:

$$\bigg(-\frac{1}{2}\mu^{-1}\bigg(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\mathbf{L}^2}{r^2}\bigg) + V(\mathbf{r}) + 2m_M - E\bigg)\psi(\mathbf{r}) \quad = \quad 0$$

with $\mu^{-1} = \mathrm{diag}(1/\mu_Q, 1/\mu_M, 1/\mu_M, 1/\mu_M)$ and

$$V(\mathbf{r}) \quad = \quad \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\mathrm{mix}}(r)\Big(1 \otimes \mathbf{e}_r\Big) \\ V_{\mathrm{mix}}(r)\Big(\mathbf{e}_r \otimes 1\Big) & V_{\bar{M}M,\parallel}(r)\Big(\mathbf{e}_r \otimes \mathbf{e}_r\Big) + V_{\bar{M}M,\perp}(r)\Big(1 - \mathbf{e}_r \otimes \mathbf{e}_r\Big) \end{pmatrix}.$$

Bottomonium, $I = 0$: potentials (1)

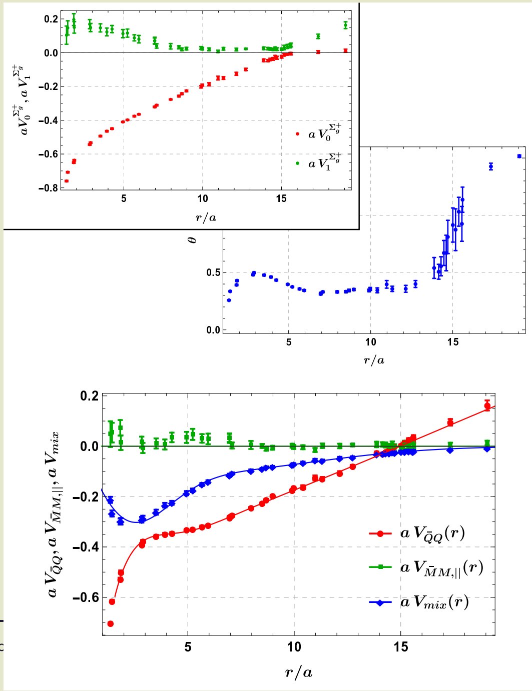
- Use lattice QCD to compute the 4×4 potential matrix

$$V(\mathbf{r}) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\mathrm{mix}}(r)\Big(1 \otimes \mathbf{e}_r\Big) \\ V_{\mathrm{mix}}(r)\Big(\mathbf{e}_r \otimes 1\Big) & V_{\bar{M}M,\parallel}(r)\Big(\mathbf{e}_r \otimes \mathbf{e}_r\Big) + V_{\bar{M}M,\perp}(r)\Big(1 - \mathbf{e}_r \otimes \mathbf{e}_r\Big) \end{pmatrix}.$$

- $V_{\bar{Q}Q}(r)$, $V_{\bar{M}M,\parallel}(r)$ (spin 1 of $\bar{M}M$ parallel to $\mathbf{r}$), $V_{\mathrm{mix}}(r)$:
  - Lattice computation of string breaking with optimized $\bar{Q}Q$ and $\bar{M}M$ operators:
    $\rightarrow V_0^{\Sigma_g^+}(r)$ (ground state), $V_1^{\Sigma_g^+}(r)$ (first excitation), $\theta(r)$ (mixing angle).

$$V_{\bar{Q}Q}(r) = \cos^2(\theta(r)) V_0^{\Sigma_g^+}(r) + \sin^2(\theta(r)) V_1^{\Sigma_g^+}(r)$$
$$V_{\bar{M}M,\parallel}(r) = \sin^2(\theta(r)) V_0^{\Sigma_g^+}(r) + \cos^2(\theta(r)) V_1^{\Sigma_g^+}(r)$$
$$V_{\mathrm{mix}}(r) = \cos(\theta(r)) \sin(\theta(r)) \Big(V_0^{\Sigma_g^+}(r) - V_1^{\Sigma_g^+}(r)\Big).$$

  - We use existing results from:
    [G. S. Bali *et al.* [SESAM Collaboration], Phys. Rev. D **71**, 114513 (2005) [hep-lat/0505012]]



Bottomonium, $I = 0$: potentials (2)

- Use lattice QCD to compute the 4×4 potential matrix

$$V(\mathbf{r}) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r)\Big(1 \otimes \mathbf{e}_r\Big) \\ V_{\text{mix}}(r)\Big(\mathbf{e}_r \otimes 1\Big) & V_{\bar{M}M,\parallel}(r)\Big(\mathbf{e}_r \otimes \mathbf{e}_r\Big) + V_{\bar{M}M,\perp}(r)\Big(1 - \mathbf{e}_r \otimes \mathbf{e}_r\Big) \end{pmatrix} .$$

- $V_{\bar{M}M,\perp}(r)$:
  - Simpler lattice computation with an optimized $\bar{M}M$ operator (no mixing with $\bar{Q}Q$).

Bottomonium, $I = 0$: partial waves

- Ordinary Schrödinger equation (1 channel, no spin), $V(r)$: PDE can be simplified to ODE for radial coordinate r and definite L (scattering: partial wave decomposition).
- Similar here, but technically more complicated (4 components, L and S).
- **Specialize coupled channel Schrödinger equation to** $\widetilde{J}^{PC} = 0^{++}$, which is ...
  ... orbital angular momentum L^{PC} for $\bar{Q}Q$ **(**$\rightarrow S$ **wave bottomonium)** ...
  ... total light angular momentum for $\bar{M}M$:

$$\left(-\frac{1}{2}\begin{pmatrix} 1/\mu_Q & 0 \\ 0 & 1/\mu_M \end{pmatrix}\partial_r^2 + \frac{1}{2r^2}\begin{pmatrix} 0 & 0 \\ 0 & 2/\mu_M \end{pmatrix} + V_0(r) + 2m_M - E\right)\begin{pmatrix} u(r) \\ \chi(r) \end{pmatrix} =$$

$$= -\begin{pmatrix} V_{\text{mix}}(r) \\ V_{\bar{M}M,\parallel}(r) \end{pmatrix} kr j_1(kr) \qquad (1)$$

$$V_0(r) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r) \\ V_{\text{mix}}(r) & V_{\bar{M}M,\parallel}(r) \end{pmatrix},$$

i.e. 2 coupled ODEs (before 4 coupled PDEs).

  - $u(r)$ and $\chi(r)$ are radial wave functions.
  - Right hand side $\propto j_1(kr)$ from boundary conditions for scattering (plane incident wave and radial emergent wave).

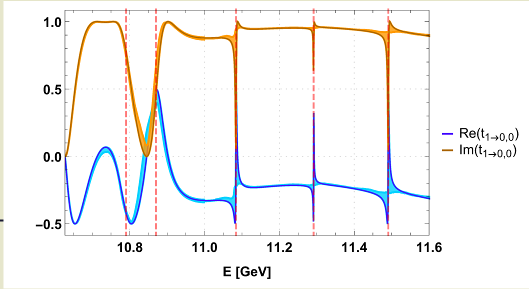
Bottomonium, $I = 0$: bound states

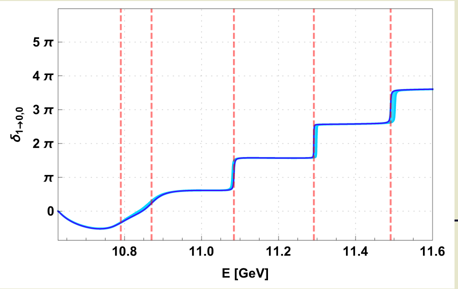
- Solve coupled channel Schrödinger equation (1) for bound states with boundary conditions
  - $u(r) = 0$ for $r \to \infty$ (radial wave function for the $\bar{Q}Q$ channel),
  - $\chi(r) = 0$ for $r \to \infty$ (radial wave function for the $\bar{M}M$ channel).
- Four bound states, correspond to experimentally observed $\eta_b(1S) \equiv \Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$.
- Agreement up to expected precision: static limit, i.e. neglect of the spin of $\bar{b}b$, suggests a systematic error of order $m_{\Upsilon(1S)} - m_{\eta_b(1S)} \approx 60\,\text{MeV}$.

	from poles of $t_{1\to 0,0}$			from experiment		
n	$m = \text{Re}(E)$ [GeV]	$\text{Im}(E)$ [MeV]	Γ [MeV]	name	m [GeV]	Γ [MeV]
1	9.478^{+3}_{-13}	0	–	$\eta_b(1S)$	$9.399(2)$	$10(5)$
				$\Upsilon_b(1S)$	$9.460(0)$	≈ 0
2	9.970^{+0}_{-8}	0	–	$\Upsilon_b(2S)$	$10.023(0)$	≈ 0
3	10.304^{+0}_{-6}	0	–	$\Upsilon_b(3S)$	$10.355(1)$	≈ 0
4	10.578^{+0}_{-5}	0	–	$\Upsilon_b(4S)$	$10.579(1)$	$21(3)$

Bottomonium, $I = 0$: resonances (1)

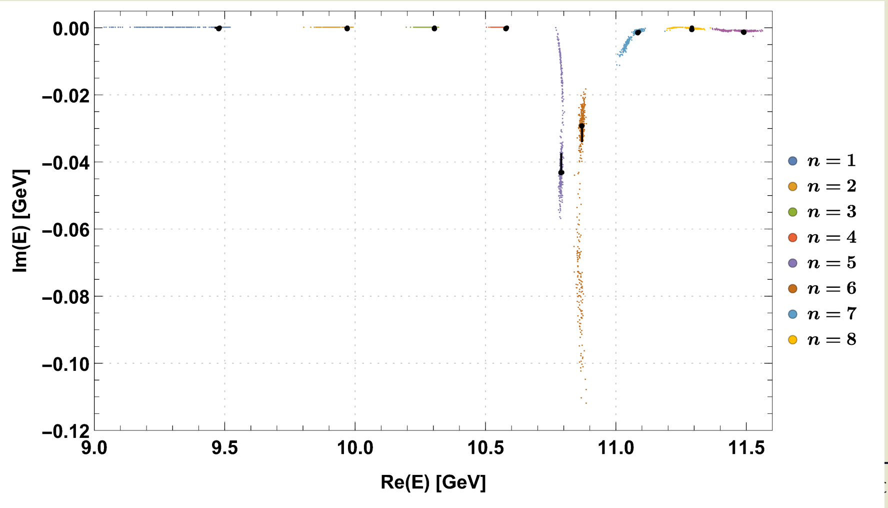
- Solve coupled channel Schrödinger equation (1) for resonances with boundary conditions
  - $u(r) = 0$ for $r \to \infty$ (radial wave function for the $\bar{Q}Q$ channel),
  - $\chi(r) = it_{1\to 0,0} kr h_1^{(1)}(kr)$ for $r \to \infty$ (radial wave function for the emergent wave in the $\bar{M}M$ channel).
    - For a given value of E the boundary condition is fulfilled for a specific corresponding value of $t_{1\to 0,0}$, i.e. $t_{1\to 0,0}$ is a function of E.
    - **Partial wave scattering amplitude**: $t_{1\to 0,0} kr$.
    - **Eigenvalue of the** T **matrix**: $t_{1\to 0,0}$.
    - **Partial wave scattering phase**: $e^{2i\delta_{1\to 0,0}} = 1 + 2it_{1\to 0,0}$.
- $t_{1\to 0,0}$ and $\delta_{1\to 0,0}$ for real energies E:
  - $E \lesssim 11\,\text{GeV}$: clear indentification of resonances not possible.
  - $E \gtrsim 11\,\text{GeV}$: resonances not trustworthy (excited B mesons neglected).

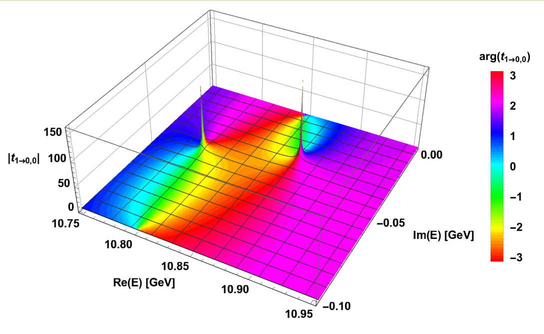




Bottomonium, $I = 0$: resonances (2)

- Find poles of $t_{1\to 0,0}$ in the complex energy plane to identify resonances clearly.
  - Resonance mass: $m = \mathrm{Re}(E)$.
  - Width: $\Gamma = -2\mathrm{Im}(E)$.
  - Four bound states on the real axis ($n = 1, 2, 3, 4$), previous results confirmed.
  - Two resonances, which can decay only to $\bar{B}^{(*)}B^{(*)}$, widths comparable to experimental widths ($n = 5, 6$).
  - Higher resonances not trustworthy, because excited B mesons neglected ($n \geq 7$).





Bottomonium, $I = 0$: resonances (3)

- Resonance with $n = 6$ rather close to experimentally observed $\Upsilon(10860)$.
  $\rightarrow$ Indication that $\Upsilon(10860)$ should be interpreted as $\Upsilon(5S)$.
- No resonance close to experimentally observed $\Upsilon(11020)$.
  $\rightarrow$ Indication that $\Upsilon(11020)$ is not an S wave resonance.
- New resonance close to the $\bar{B}^{(*)}B^{(*)}$ threshold predicted ($n = 5$) with fully dynamical origin (disappears, when reducing the mixing between the $\bar{Q}Q$ and the $\bar{M}M$ channel).

	from poles of $t_{1\rightarrow 0,0}$			from experiment		
n	$m = \mathrm{Re}(E)$ [GeV]	$\mathrm{Im}(E)$ [MeV]	Γ [MeV]	name	m [GeV]	Γ [MeV]
1	9.478^{+3}_{-13}	0	–	$\eta_b(1S)$	$9.399(2)$	$10(5)$
				$\Upsilon_b(1S)$	$9.460(0)$	≈ 0
2	9.970^{+0}_{-8}	0	–	$\Upsilon_b(2S)$	$10.023(0)$	≈ 0
3	10.304^{+0}_{-6}	0	–	$\Upsilon_b(3S)$	$10.355(1)$	≈ 0
4	10.578^{+0}_{-5}	0	–	$\Upsilon_b(4S)$	$10.579(1)$	$21(3)$
5	10.790^{+2}_{-1}	$-42.9^{+5.3}_{-0.0}$	$85.9^{+10.6}_{-0.0}$			
6	10.870^{+1}_{-4}	$-29.0^{+0.0}_{-4.8}$	$58.0^{+9.7}_{-0.0}$	$\Upsilon(10860)$	$10.890(3)$	$51(7)$
7	11.084^{+0}_{-4}	$-1.3^{+0.0}_{-0.2}$	$2.5^{+0.0}_{-0.4}$	$\Upsilon(11020)$	$10.993(1)$	$49(15)$
8	11.292^{+0}_{-6}	$-0.3^{+0.0}_{-0.0}$	$0.5^{+0.1}_{-0.0}$			
9	11.491^{+0}_{-8}	$-1.1^{+0.0}_{-0.0}$	$2.3^{+0.1}_{-0.0}$			

Bottomonium, $I = 0$: outlook

- Work on bottomonium resonances with $I = 0$ just a first step.
- To get a complete and more precise picture of bottomonium resonances with $I = 0$ ...
  - ... study also orbital angular momentum $L = 1, 2, \ldots$ for $\bar{Q}Q$ ...
    (at the moment only $L = 0$, then e.g. investigation of possibly existing X_b [counterpart of $X_c(3872)$])
  - ... include decays to excited B mesons, e.g. to $\bar{B}^{(*)} B^*_{0,1}$ ...
    (at the moment resonances only trustworthy up to $\approx 11.0\,\mathrm{MeV}$, then up to $\approx 11.5\,\mathrm{MeV}$)
  - ... precise lattice QCD computation of all required static potentials with u and d quark mass closer to the physical value and at smaller lattice spacing ...
  - ... include $1/m_b$ corrections.