Heavy hybrid mesons and tetraquarks from lattice QCD

"Interface of Effective Field Theories and Lattice Gauge Theory" – MIAPP, Germany

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Part 1: tetraquarks

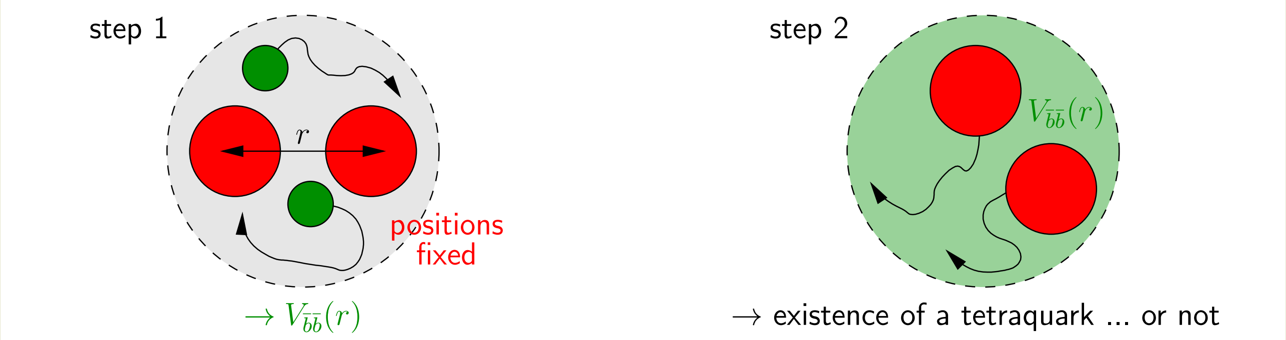
Basic idea to study $bbqq$ tetraquarks (1)

- Study heavy-heavy-light-light tetraquarks $\bar{b}\bar{b}qq$ or $\bar{b}b\bar{q}q$ in two steps.

  (1) **Compute potentials of two static quarks ($\bar{b}\bar{b}$ or $\bar{b}b$) in the presence of two lighter quarks (qq or $\bar{q}q$, $q \in \{u, d, s, c\}$) using lattice QCD.**

  (2) **Explore, whether these potentials are sufficiently attractive to host bound states or resonances** (→ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

  ((1) + (2) → Born-Oppenheimer approximation).



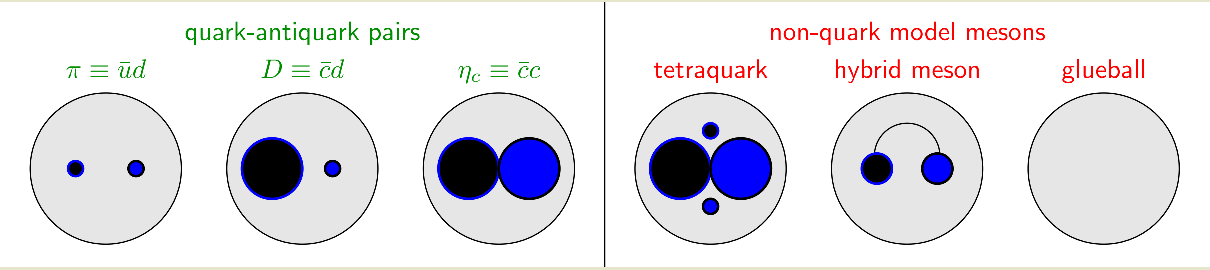
Basic idea to study $bbqq$ tetraquarks (2)

- The talk summarizes
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]

- For recent work from other groups using a similar approach cf. e.g.
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571 [hep-lat]]
  [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953 [hep-lat]]

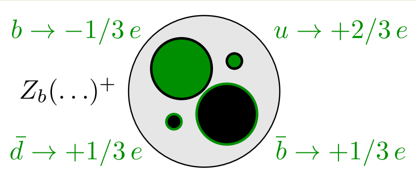
Why are such studies important? (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \ldots$
- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.q. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).
- Certain mesons are poorly understood (significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
  - **2 quarks and 2 antiquarks (tetraquark)**,
  - **a quark-antiquark pair and gluons (hybrid meson)**,
  - **only gluons (glueball)**.



Why are such studies important? (2)

- Indications for tetraquark structures:
  - Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$:
    - Mass suggests a $b\bar{b}$ pair ...
    - ... but $b\bar{b}$ is electrically neutral ...?
    - **Easy to understand, when assuming a tetraquark structure**: $Z_b(\ldots)^+ \equiv b\bar{b}u\bar{d}$ ($u \rightarrow +2/3\,e$, $\bar{d} \rightarrow -1/3\,e$).



  - Electrically charged Z_c states:
    - Similar to Z_b.
  - Mass ordering of light scalar mesons:
    - E.g. $m_\kappa > m_{a_0(980)}$ ...?

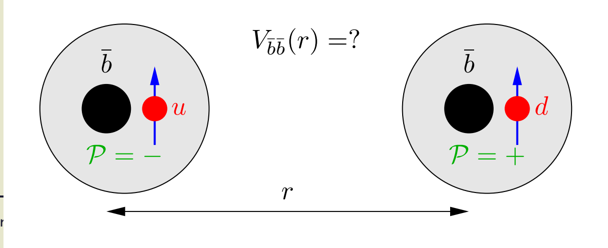
Outline

- $\bar{b}\bar{b}qq$ / BB potentials.
- Lattice setup.
- $\bar{b}\bar{b}qq$ tetraquarks.
- Inclusion of heavy spin effects.
- $\bar{b}b\bar{q}q$ / $\bar{B}B$ potentials

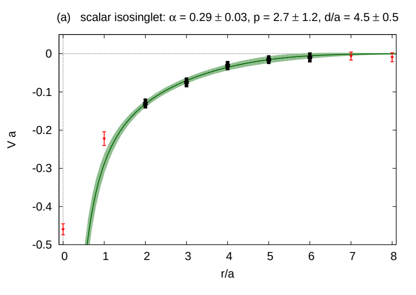
$\bar{b}\bar{b}qq$ / BB potentials (1)

- From now on $\bar{b}\bar{b}qq$ ($\bar{b}b\bar{q}q$ technically more difficult, will be discussed at the end of this talk).
- Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).
- At large $\bar{b}\bar{b}$ separation r, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Consider only pseudoscalar/vector mesons ($j^P = (1/2)^-$, PDG: B, B^*) and scalar/pseudovector mesons ($j^P = (1/2)^+$, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons (j: spin of the light degrees of freedom).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of qq on
  - the "light" quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^* (parity).

$\rightarrow$ Many different channels: attractive versus repulsive, different asymptotic values ...



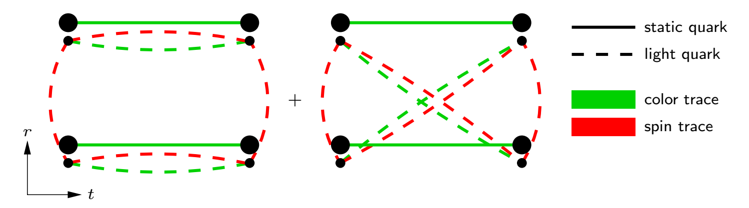
$\bar{b}\bar{b}qq$ / BB potentials (2)



- Rotational symmetry broken by static quarks $\bar{b}\bar{b}$.
- Remaining symmetries and quantum numbers:
  - Rotations around the separation axis (e.g. z axis), quantum number $j_z \equiv \Lambda$.
  - P.
  - $P_x \equiv \epsilon$ (reflection along an axis perpendicular to the separation axis, e.g. x axis).
- To extract the potential(s) of a given sector $(I, I_z, |j_z|, P, P_x)$, compute the temporal correlation function of the trial state

$$\Big(C\Gamma\Big)_{AB}\Big(C\tilde{\Gamma}\Big)_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big)|\Omega\rangle.$$

  - $q^{(1)}q^{(2)} \in \{ud - du\,,\, uu\,,\, dd\,,\, ud + du\,,\, ss\,,\, cc\}$ (isospin I, I_z, flavor).
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity P, P_x).
  - $\tilde{\Gamma} \in \{(1-\gamma_0)\gamma_5\,,\, (1-\gamma_0)\gamma_j\}$ (irrelevant).



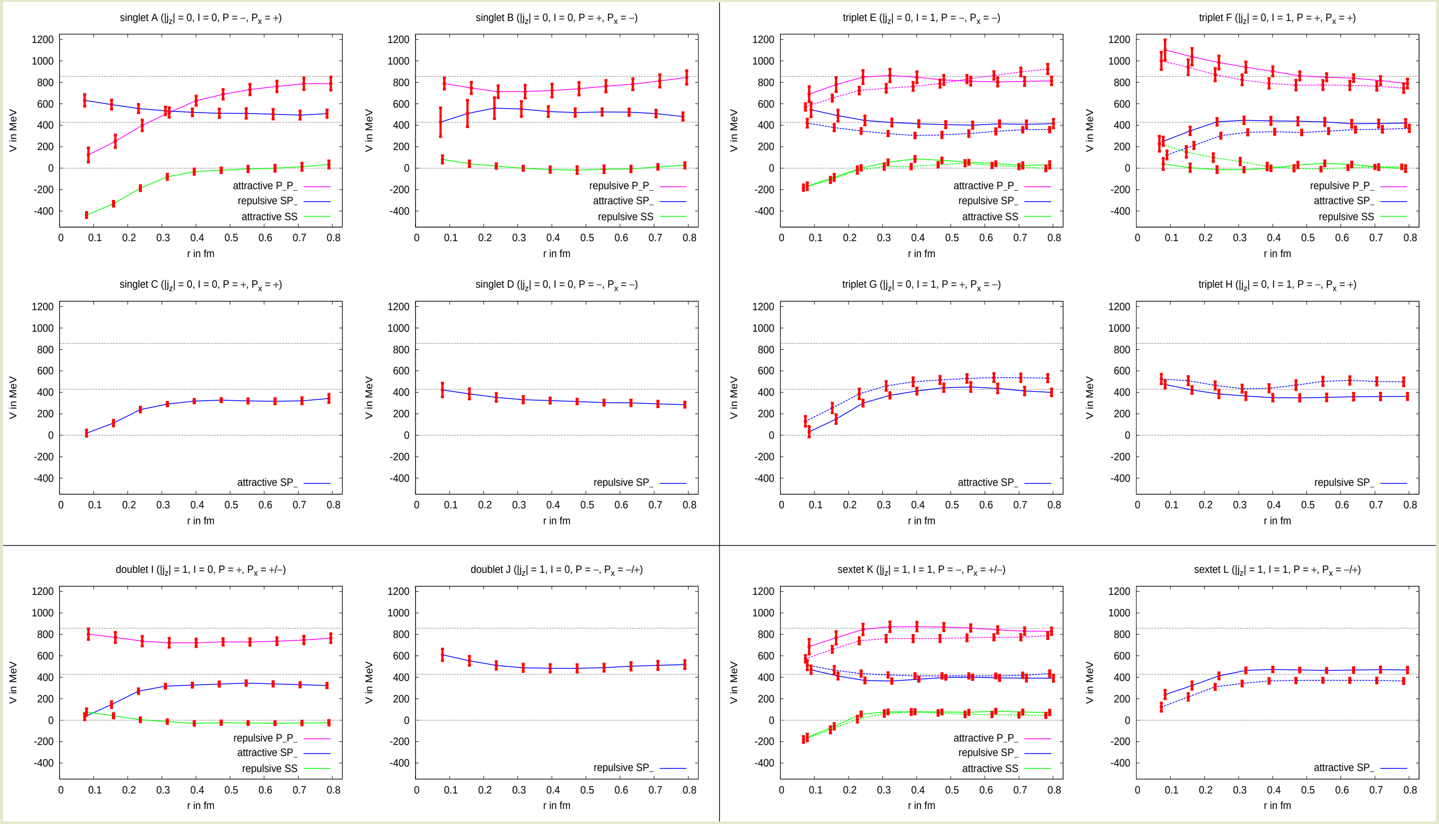
Lattice setup

- ETMC gauge link ensembles:
  - $N_f = 2$ dynamical quark flavors.
  - Lattice spacing $a \approx 0.079\,\text{fm}$.
  - $24^3 \times 48$, i.e. spatial lattice extent $\approx 1.9\,\text{fm}$.
  - Three different pion masses $m_\pi \approx 340\,\text{MeV}$, $m_\pi \approx 480\,\text{MeV}$, $m_\pi \approx 650\,\text{MeV}$.

[R. Baron *et al.* [ETM Collaboration], JHEP **1008**, 097 (2010) [arXiv:0911.5061 [hep-lat]]

$\bar{b}\bar{b}qq$ / BB potentials (3)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



$\bar{b}\bar{b}qq$ / BB potentials (4) to (7)

- **Why are there three different asymtotic values?**
  - They correspond to $B^{(*)}B^{(*)}$ potentials, to $B^{(*)}B^*_{0,1}$ potentials and $B^*_{0,1}B^*_{0,1}$ potentials.
- **Why are certain channels attractive and others repulsive?**
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}qq$ / BB potentials.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}qq$ / BB potentials.
  - Because of the Pauli principle and (assuming) "1-gluon exchange" at small r.
- **24 different (i.e. non-degenerate) $\bar{b}\bar{b}qq$ / BB potentials.**

$\bar{b}\bar{b}qq$ / BB potentials (4)

Why are there three different asymtotic values?

- Differences $\approx 400\,\text{MeV}$, approximately the mass difference of $B^*_{0,1}$ ($P = +$) and $B^{(*)}$ ($P = -$).
- Suggests that the three different asymtotic values correspond to $B^{(*)} B^{(*)}$ potentials, to $B^{(*)} B^*_{0,1}$ potentials and $B^*_{0,1} B^*_{0,1}$ potentials.
- Can be checked and confirmed, by rewriting the $\bar{b}\bar{b}qq$ creation operators in terms of meson-meson creation operators (Fierz transformation).
- Example: uu, $\Gamma = \gamma_3$ (attractive, lowest asymptotic value),

$$\Big(C\gamma_3\Big)_{AB}\Big(\bar{Q}_C(-\mathbf{r}/2) q^{(u)}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2) q^{(u)}_B(+\mathbf{r}/2)\Big) \quad \propto$$
$$\propto \quad (B^{(*)})_\uparrow (B^{(*)})_\downarrow + (B^{(*)})_\downarrow (B^{(*)})_\uparrow - (B^*_{0,1})_\uparrow (B^*_{0,1})_\downarrow - (B^*_{0,1})_\downarrow (B^*_{0,1})_\uparrow .$$

- Example: uu, $\Gamma = 1$ (repulsive, medium asymptotic value),

$$\Big(C1\Big)_{AB}\Big(\bar{Q}_C(-\mathbf{r}/2) q^{(u)}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2) q^{(u)}_B(+\mathbf{r}/2)\Big) \quad \propto$$
$$\propto \quad (B^{(*)})_\uparrow (B^*_{0,1})_\downarrow - (B^{(*)})_\downarrow (B^*_{0,1})_\uparrow + (B^*_{0,1})_\uparrow (B^{(*)})_\downarrow - (B^*_{0,1})_\downarrow (B^{(*)})_\uparrow .$$

$\bar{b}\bar{b}qq$ / BB potentials (5)

Why are certain channels attractive and others repulsive? (1)

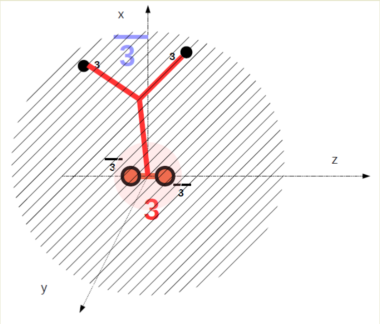
- Fermionic wave function must be antisymmetric (Pauli principle); in quantum field theory/QCD automatically realized.
- qq isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- qq angular momentum/spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- qq color:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e., a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e., a sextet 6.
- The four quarks $\bar{b}\bar{b}qq$ must form a color singlet:
  - qq in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a triplet 3.
  - qq in a color sextet 6 $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a sextet $\bar{6}$.

$\bar{b}\bar{b}qq$ / BB potentials (6)

Why are certain channels attractive and others repulsive? (2)

- Assumption: attractive/repulsive behavior of $\bar{b}\bar{b}$ at small separations r is mainly due to 1-gluon exchange,
  - color triplet 3 is attractive, $V_{\bar{b}\bar{b}}(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V_{\bar{b}\bar{b}}(r) = +\alpha_s/3r$

  (easy to calculate in LO perturbation theory).



- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.
- Expectation consistent with the obtained lattice results.
- **Pauli principle and assuming "1-gluon exchange" at small r explains, why certain channels are attractive and others repulsive.**

$\bar{b}\bar{b}qq$ / BB potentials (7)

- Summary of $\bar{b}\bar{b}qq$ / BB potentials:

$B^{(*)}B^{(*)}$ potentials:	attractive:	$1 \oplus 3 \oplus 6$	(10 states).
	repulsive:	$1 \oplus 3 \oplus 2$	(6 states).
$B^{(*)}B^*_{0,1}$ potentials:	attractive:	$1 \oplus 1 \oplus 3 \oplus 3 \oplus 2 \oplus 6$	(16 states).
	repulsive:	$1 \oplus 1 \oplus 3 \oplus 3 \oplus 2 \oplus 6$	(16 states).
$B^*_{0,1}B^*_{0,1}$ potentials:	attractive:	$1 \oplus 3 \oplus 6$	(10 states).
	repulsive:	$1 \oplus 3 \oplus 2$	(6 states).

  - 2-fold degeneracy due to spin $j_z = \pm 1$.
  - 3-fold degeneracy due to isospin $I = 1$, $I_z = -1, 0, +1$.

$\rightarrow 24$ **different $\bar{b}\bar{b}qq$ / BB potentials.**

$\bar{b}\bar{b}qq$ / BB potentials (8)

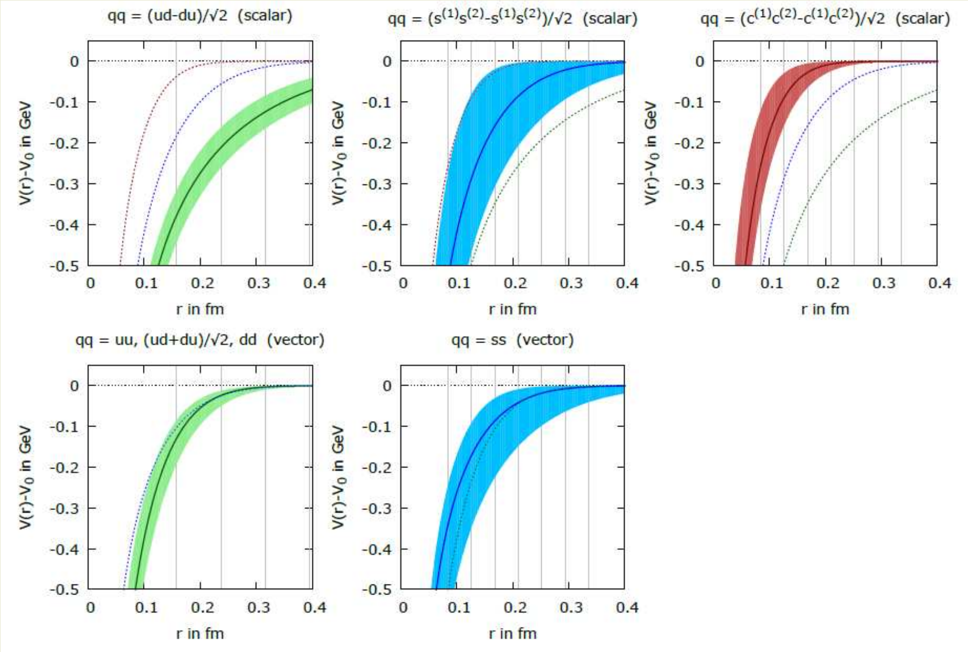
- Focus on the two attractive channels between B and B^*:
  - Scalar isosinglet ($(I=0, j=0)$, more attractive):
    $qq = (ud - du)/\sqrt{2}$, $\Gamma = (1+\gamma_0)\gamma_5$.
  - Vector isotriplet ($(I=1, j=1)$, less attractive):
    $qq \in \{uu, (ud+du)/\sqrt{2}, dd\}$, $\Gamma = (1+\gamma_0)\gamma_j$.
- Computations for $qq = ll, ss, cc$ ($l \in \{u, d\}$) to study the mass dependence.
- Parameterize lattice potential results by contiuous functions obtained by χ^2 minimizing fits of

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r} \exp\bigg(-\bigg(\frac{r}{d}\bigg)^p\bigg) + V_0 :$$

  - $1/r$: 1-gluon exchange at small $\bar{b}\bar{b}$ separations.
  - $\exp(-(r/d)^p)$: color screening at large $\bar{b}\bar{b}$ separations due to meson formation.
  - Fit parameters α, d and V_0; $p = 2$ from quark models.

$\bar{b}\bar{b}qq$ / BB potentials (9)

- Potentials for $qq = ll$, $l \in \{u, d\}$ are wider and deeper than potentials for $qq = ss, cc$.
  $\rightarrow$ **Good candidates to find tetraquarks are systems of two very heavy and two very light quarks, i.e., $\bar{b}\bar{b}ll$.**

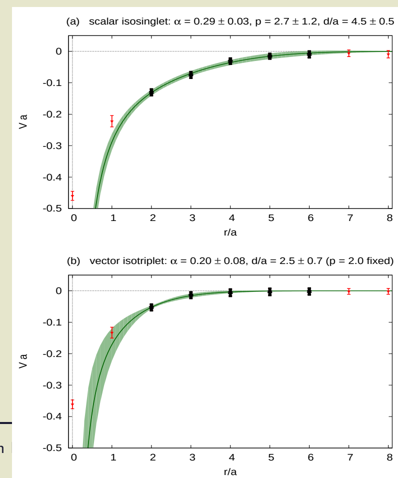


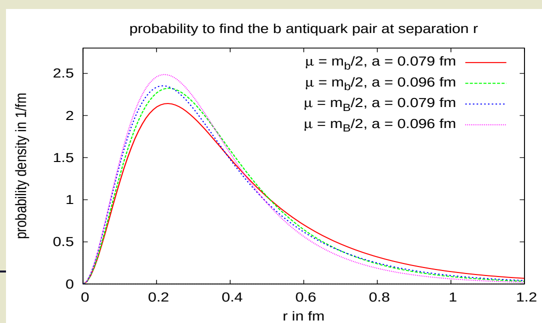
$\bar{b}\bar{b}qq$ tetraquarks (1)

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

$$\Big(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\Big)\psi(\mathbf{r}) \;=\; E\psi(\mathbf{r}) \quad , \quad \mu \;=\; m_b/2.$$

- Possibly existing bound states, i.e., $E < 0$, indicate stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for $qq = (ud - du)/\sqrt{2}$ (i.e., the scalar isosinglet potential) and orbital angular momentum $l = 0$ of $\bar{b}\bar{b}$, binding energy $E = -90^{+43}_{-36}$ MeV with respect to the BB^* threshold, i.e. confidence level $\approx 2\,\sigma$.
- No further bound states, in particular not for $qq = ss, cc$ (i.e., $B_s B_s$, $B_c B_c$).





from

$\bar{b}\bar{b}qq$ tetraquarks (2) to ...

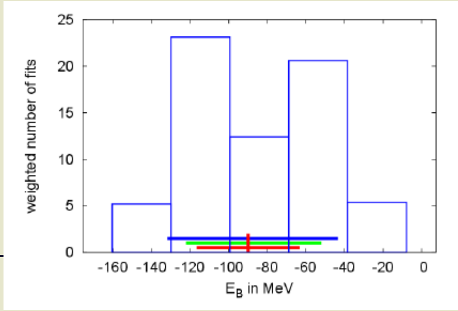
- **What are the quantum numbers of the predicted $\bar{b}\bar{b}qq$ tetraquark?**
  - $I(J^P) = 0(1^+)$.
- **Will there still be a bound state, when heavy spin effects are taken into account?**
  - Yes, binding energy $E = -59^{+38}_{-30}$ MeV (without heavy spin effects $E = -90^{+43}_{-36}$ MeV).
  - Tetraquark is approximately a $50\%/50\%$ superposition of BB^* and B^*B^*.
- **Tetraquark resonances can be studied in a similar way using standard methods from scattering theory.**
  - There is a resonance for $qq = (ud - du)/\sqrt{2}$ and $l = 1$.
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma_{\rightarrow B+B} = 112^{+90}_{-103}$ MeV.
  - Quantum numbers $I(J^P) = 0(1^-)$.
- **Exploring the existence of $\bar{b}b\bar{q}q$ tetraquarks in the same way is more difficult.**
  - $\bar{b}b\bar{q}q$ can decay into $\bar{b}b + \bar{q}q$ ("bottomonium + pion").
  - A potential can be just a $\bar{b}b$ potential shifted by the mass of a $\bar{q}q$ meson.

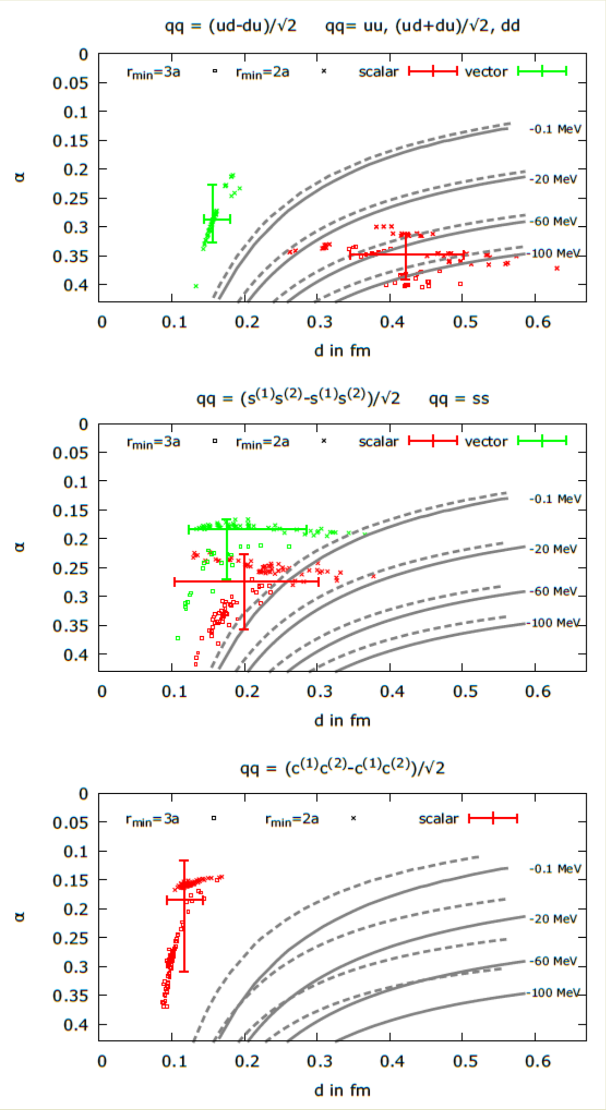
$\bar{b}\bar{b}qq$ tetraquarks (2)

- Estimate the systematic error by varying input parameters:
  - the t fitting range to extract the potential from effective masses,
  - the r fitting range for

$$V_{\bar{b}\bar{b}}(r) \quad = \quad -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0 .$$

- Right: isoline plots of the binding energy E for $l = 0$.
- Bottom: histogram for the binding energy E for $qq = (ud - du)/\sqrt{2}$ and $l = 0$.





$\bar{b}\bar{b}qq$ tetraquarks (3)

- To quantify "no binding", we list for each channel the factor, by which the reduced mass μ in the Schrödinger equation has to be multiplied, to obtain a tiny but negative energy E (again for $l = 0$).

qq	spin	factor
$(ud - du)/\sqrt{2}$	scalar	0.46
uu, $(ud + du)/\sqrt{2}$, dd	vector	1.49
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	1.20
ss	vector	2.01
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	2.57

  - Factors $\ll 1$ indicate strongly bound states, while for values $\gg 1$ bound states are essentially excluded.
  - Light quarks (u/d) are unphysically heavy (correspond to $m_\pi \approx 340\,\text{MeV}$); physically light u/d quarks yield similar results.
  - Mass splitting $m(B^*) - m(B) \approx 50\,\text{MeV}$, neglected at the moment, is expected to weaken binding (will be discussed below).

$\bar{b}\bar{b}qq$ tetraquarks (4)

What are the quantum numbers of the predicted $\bar{b}\bar{b}qq$ tetraquark?

- $I(J^P) = 0(1^+)$.
  - Light scalar isosinglet: $qq = (ud - du)/\sqrt{2}$, $I = 0$, $j = 0$ in a color $\bar{3}$, $\bar{b}\bar{b}$ in a color 3 (antisymmetric) ... as discussed above.
  - Wave function of $\bar{b}\bar{b}$ must also be antisymmetric (Pauli principle).
    - $\bar{b}\bar{b}$ is flavor symmetric.
    - $\bar{b}\bar{b}$ spin must also be symmetric, i.e., $j_b = 1$.

  $\rightarrow$ **The predicted $\bar{b}\bar{b}qq$ tetraquark has isospin $I = 0$, spin $J = 1$.**
  - We study a state, which correspond for large $\bar{b}\bar{b}$ separations to a pair of $B^{(*)}$ mesons in a spatially symmetric s-wave.

  $\rightarrow$ **The predicted $\bar{b}\bar{b}qq$ tetraquark has parity $P = +$** (the product of the parity quantum numbers of the two mesons, which are both negative)**.**

Inclusion of heavy spin effects

- Heavy spin effects have been neglected so far, e.g. mass splitting $m_{B^*} - m_B \approx 46\,\text{MeV}$.
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $E = -90^{+43}_{-36}\,\text{MeV}$.
- Moreover, two competing effects:
  - The attractive $\bar{b}\bar{b}ud$ channel corresponds to a linear combination of BB^* and/or B^*B^*.
  - The BB^* interaction is a superposition of attractive and repulsive $\bar{b}\bar{b}ud$ potentials.
- **Will there still be a bound state, when heavy spin effects are taken into account?**
  - Yes.
  - We include heavy spin effects by solving a coupled channel Schrödinger equation.
    [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  - Binding energy $E = -59^{+38}_{-30}\,\text{MeV}$.
  - Tetraquark is approximately a $50\%/50\%$ superposition of BB^* and B^*B^* (strong attraction more important than light constituents).

$\bar{b}\bar{b}qq$ tetraquark resonances (1)

- Most hadrons are unstable, i.e., resonances.
- If a $\bar{b}\bar{b}qq$ potential $V_{\bar{b}\bar{b}}(r)$ is not sufficiently attractive to host a bound state, there could still be a clear resonance.
- Comparatively easy to investigate within our approach (since we have potentials $V_{\bar{b}\bar{b}}(r)$, no Lüscher method etc. necessary).
- Use standard methods from scattering theory:
  - Solve Schrödinger equation with potential $V_{\bar{b}\bar{b}}(r)$ and appropriate boundary conditions (incident plane wave, outgoing spherical wave)
    $\rightarrow$ partial wave amplitudes $f_l(E)$.
  - Use partial wave amplitudes $f_l(E)$ to ...
    - ... determine phase shifts and contributions of partial waves to total cross section
      $\rightarrow$ peak indicates resonance mass.
    - ... determine poles of the S or the T matrix in the complex energy plane (correspond to poles of $f_l(E)$)
      $\rightarrow$ real part of a pole $\equiv$ resonance mass
      $\rightarrow$ imaginary part of a pole $\equiv$ resonance width.

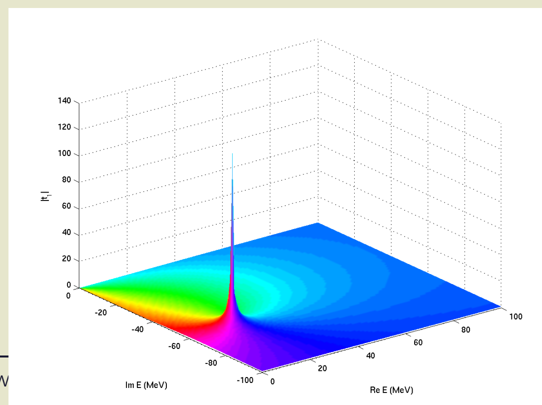
$\bar{b}\bar{b}qq$ tetraquark resonances (2)

- Exploratory study mostly for $qq = (ud - du)/\sqrt{2}$ (i.e., the scalar isosinglet potential) and orbital angular momentum $l = 1$ of $\bar{b}\bar{b}$:

- There is a resonance for $qq = (ud - du)/\sqrt{2}$ and $l = 1$:
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma_{\rightarrow B+B} = 112^{+90}_{-103}$ MeV.
  - Quantum numbers $I(J^P) = 0(1^-)$.

  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., arXiv:1704.02383].

- There do not seem to be resonances in other channels ($l > 1$, vector isotriplet potential, heavier quarks qq).



$\bar{b}b\bar{q}q$ / $\bar{B}B$ potentials

- Exploring the existence of $\bar{b}b\bar{q}q$ tetraquarks in the same way is more difficult:
  - $\bar{b}\bar{b}qq$ (discussed on pevious slides) can decay into:
    - $\bar{B} + \bar{B}$.
      "Easy" ... on the level of the Schrödinger equation for the relative coordinate of the two $\bar{b}$ quarks (step (2) of the BO approximation).
  - $\bar{b}b\bar{q}q$ can decay into:
    - $\bar{B} + B$.
      "Easy" ... on the level of the Schrödinger equation for the relative coordinate of the $\bar{b}$ quark and the b quark (step (2) of the BO approximation).
    - $\bar{b}b + \bar{q}q$ ("bottomonium + pion").
      "Rather hard" ... on the level of lattice QCD, when computing the $\bar{b}b$ potentials in the presence of $\bar{q}q$ (step (1) of the BO approximation).
      - A potential can be relevant for a $\bar{b}b\bar{q}q$ tetraquark (if $\bar{q}q$ is close to $\bar{b}b$) ...
      - ... or just a $\bar{b}b$ potential shifted by the mass of a $\bar{q}q$ meson.
- Work in progress.
  [A. Peters, P. Bicudo, L. Leskovec, S. Meinel and M.W., PoS LATTICE **2016**, 104 (2016) [arXiv:1609.00181]]
  [A. Peters, P. Bicudo and M.W., EPJ Web Conf. **175**, 14018 (2018) [arXiv:1709.03306]]

Summary and outlook

- Computation of 3×24 different $\bar{b}\bar{b}qq$ / BB potentials.
- Prediction of a stable $\bar{b}\bar{b}qq$, $qq = (ud - du)/\sqrt{2}$ tetraquark.
  - Quantum numbers $I(J^P) = 0(1^+)$.
  - Binding energy $E = -59^{+38}_{-30}$ MeV with respect to the BB^* threshold.
- Prediction of a $\bar{b}\bar{b}qq$, $qq = (ud - du)/\sqrt{2}$ tetraquark resonance.
  - Quantum numbers $I(J^P) = 0(1^-)$.
  - Resonance mass $E = +17^{+4}_{-4}$ MeV above the BB threshold.
  - Decay width $\Gamma_{\rightarrow B+B} = 112^{+90}_{-103}$ MeV.
- Future plans:
  - Explore $\bar{b}\bar{b}qq$ tetraquark resonances in detail.
  - Investigate the structure of the predicted $I(J^P) = 0(1^+)$ tetraquark ... is it a mesonic molecule or rather a diquark-antidiquark?
  - Study $\bar{b}b\bar{q}q$ / BB, which is experimentally more relevant ($Z_b(10610)^+$, $Z_b(10650)^+$, ...), but theoretically much harder.

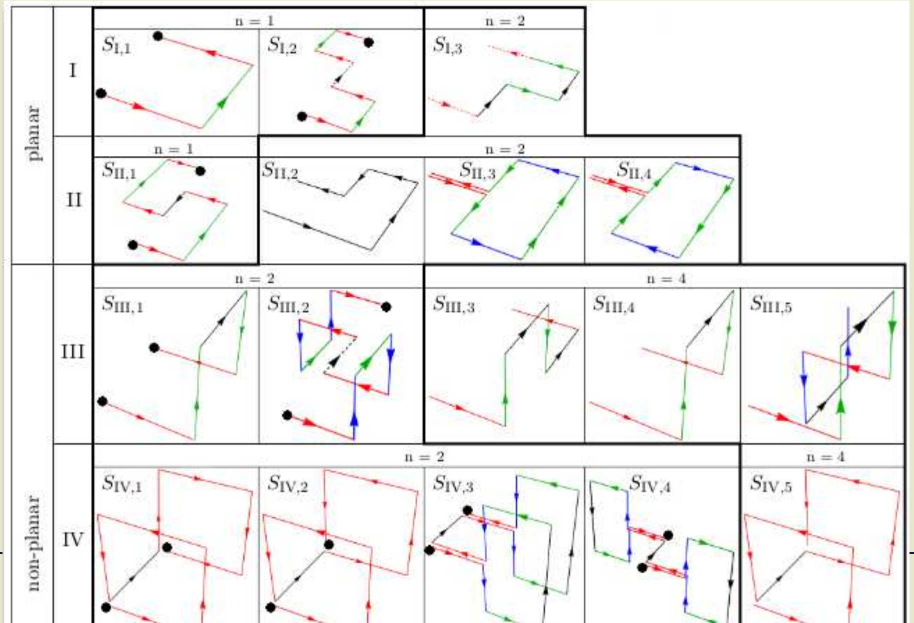
Part 2: hybrid mesons

Heavy hybrid mesons (1)

- Same idea as for heavy-heavy-light-light tetraquarks.
- Extract hybrid static potentials from correlation functions of trial states

$$|\Psi_{\text{hybrid}}\rangle_{S;\Lambda_\eta^\epsilon} \quad = \quad \bar{Q}(-r/2)a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2)Q(+r/2)|\Omega\rangle,$$

where $a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2)$ is a non-straight path of links generating quantum numbers Λ_η^ϵ.



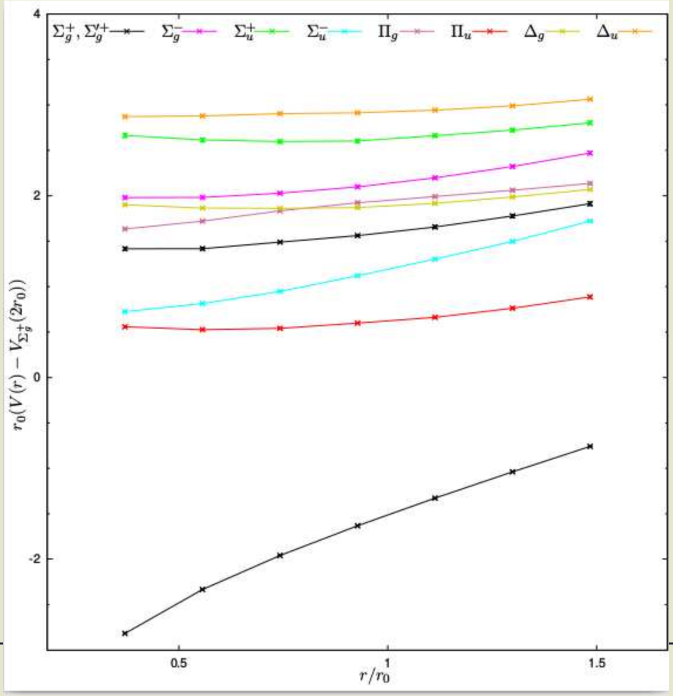
Heavy hybrid mesons (2)

- The talk summarizes
  [C. Reisinger, S. Capitani, O. Philipsen, M.W., EPJ Web Conf. **175**, 05012 (2018) [arXiv:1708.05562]]
  [L. Müller, M.W., Acta Phys. Polon. Supp. **11**, 551 (2018) [arXiv:1803.11124]]
  [C. Reisinger, S. Capitani, L. Müller, O. Philipsen, M.W., arXiv:1810.13284]
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, arXiv:1811.00452]

- For recent work from other groups using a similar approach cf. e.g.
  [K. J. Juge, J. Kuti, C. J. Morningstar, Nucl. Phys. Proc. Suppl. **63**, 326 (1998) [hep-lat/9709131]
  [C. Michael, Nucl. Phys. A **655**, 12 (1999) [hep-ph/9810415]
  [G. S. Bali *et al.* [SESAM and TχL Collaborations], Phys. Rev. D **62**, 054503 (2000) [hep-lat/0003012]
  [K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]
  [C. Michael, Int. Rev. Nucl. Phys. **9**, 103 (2004) [hep-lat/0302001]
  [G. S. Bali, A. Pineda, Phys. Rev. D **69**, 094001 (2004) [hep-ph/0310130]

Hybrid static potentials

- E.g. useful for effective field theory studies and predictions of heavy hybrid meson masses.
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]
  [R. Oncala, J. Soto, Phys. Rev. D **96**, 014004 (2017) [arXiv:1702.03900]]
  [N. Brambilla, G. Krein, J. Tarrus Castella, A. Vairo, Phys. Rev. D **97**, 016016 (2018)[arXiv:1707.09647]]
  [N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, arXiv:1805.07713]



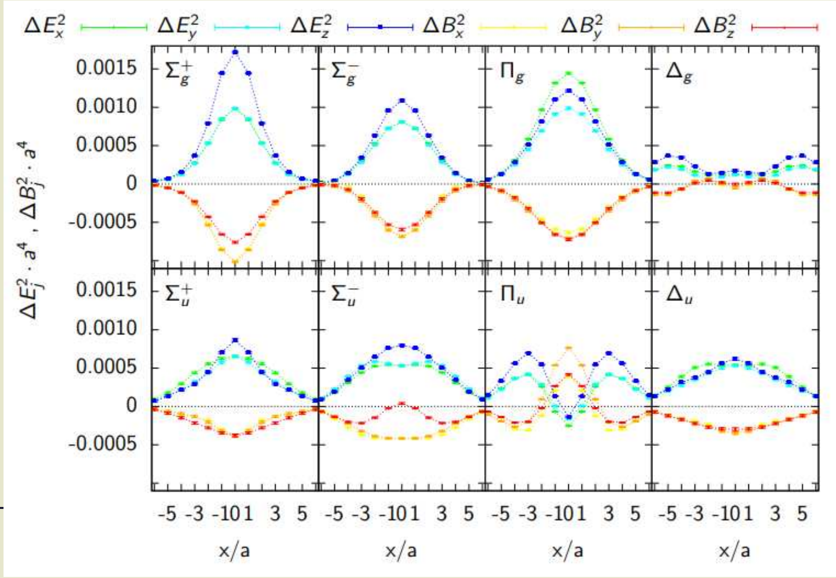
Hybrid static potential flux tubes (1)

- Compute the squared field strength components for hybrid static potentials via

$$\Delta E_j^2 \equiv \left\langle E_j(\mathbf{x})^2 \right\rangle_{Q\bar{Q}} - \left\langle E_j^2 \right\rangle_{\text{vac}} \propto \left(\frac{\langle W \cdot P_{0j}(t/2,\mathbf{x}) \rangle}{\langle W \rangle} - \langle P_{0j} \rangle \right)$$

$$\Delta B_j^2 \equiv \left\langle B_j(\mathbf{x})^2 \right\rangle_{Q\bar{Q}} - \left\langle B_j^2 \right\rangle_{\text{vac}} \propto \left(\langle P_{kl} \rangle - \frac{\langle W \cdot P_{kl}(t/2,\mathbf{x}) \rangle}{\langle W \rangle} \right)$$

(plots correspond to the mediator axis).



Hybrid static potential flux tubes (2)

- Differences of hybrid static potential flux tubes to that of the ordinary static potential: $\Delta E^2_{j,\Lambda^\epsilon_\eta} - \Delta E^2_{j,\Sigma^+_g}$ and $\Delta B^2_{j,\Lambda^\epsilon_\eta} - \Delta B^2_{j,\Sigma^+_g}$.