$\Lambda_{\overline{\mathrm{MS}}}$ from the static potential for QCD with $n_f = 2$ dynamical quark flavors

Liverpool High Energy Theory Group Seminar

Marc Wagner

Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik

mwagner@th.physik.uni-frankfurt.de

http://th.physik.uni-frankfurt.de/∼mwagner/

in collaboration with Karl Jansen, Felix Karbstein, Attila Nagy

[K. Jansen, F. Karbstein, A. Nagy, M. Wagner [ETM Collaboration], arXiv:1110.6859 [hep-ph]]

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Motivation (1)

- $\Lambda_{\overline{\mathrm{MS}}}$ defines the scale for dimensionful perturbative results (calculated in the $\overline{\mathrm{MS}}$ scheme).

- r_0 is a typical scale for dimensionful lattice results (definition: $|F_{Q\bar{Q}}(r_0)|r_0^2 = 1.65$).

- Goal: determine $r_0\Lambda_{\overline{\mathrm{MS}}}$ for $n_f = 2$ QCD
$\longrightarrow$ relate lattice and perturbative results
$\longrightarrow$ read off $\Lambda_{\overline{\mathrm{MS}}}$ in MeV, if r_0 is known in fm
$\longrightarrow$ read off r_0 in fm, if $\Lambda_{\overline{\mathrm{MS}}}$ is known in MeV.

- To determine $r_0\Lambda_{\overline{\mathrm{MS}}}$, one needs an observable, which can reliably be determined by both a lattice computation and a perturbative calculation
$\longrightarrow$ we use the $Q\bar{Q}$ static potential V at separations $r \approx 0.10\,\mathrm{fm} \ldots 0.25\,\mathrm{fm}$.

Motivation (2)

- Why the $Q\bar{Q}$ static potential V?
  - Lattice computation of V is simple (no quark propagators needed), results are rather precise; moreover, ensembles with dynamical quarks and finer and finer lattice spacing are generated such that matching with perturbative results seems feasible.
  - The perturbative expansion of V has recently been calculated up to $\mathcal{O}(\alpha_s^4)$ and terms $\sim \alpha_s^4 \ln \alpha_s$ (NNNLO).

    [C. Anzai, Y. Kiyo and Y. Sumino, Phys. Rev. Lett. **104**, 112003 (2010) [arXiv:0911.4335 [hep-ph]]]

    [A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **104**, 112002 (2010) [arXiv:0911.4742 [hep-ph]]]

- Previous work ($n_f = 0$):

  [C. Michael, Phys. Lett. B **283**, 103 (1992) [arXiv:hep-lat/9205010]]

  [N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo, Phys. Rev. Lett. **105**, 212001 (2010) [arXiv:1006.2066 [hep-ph]]]

Outline

- Matching the lattice and the perturbative static $Q\bar{Q}$ potential.
- Lattice computation of the static $Q\bar{Q}$ potential.
- Perturbation theory for the static $Q\bar{Q}$ potential.
- Determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$ and $\Lambda_{\overline{\mathrm{MS}}}$ and the associated errors.
- Conclusions, future plans.

Matching lattice and perturbative V

- Lattice QCD:
  - Nowadays typical fine lattice spacing: $a \approx 0.05\,\textrm{fm}$.
  - Simulations at even finer lattice spacings expensive/problematic: small physical volumes, frozen topology, ...
  - Severe lattice discretization errors associated with the lattice static potential $V^{(\textrm{lattice})}(r)$ for $Q\bar{Q}$ separations $r \lesssim 2a \ldots 3a$.
- Perturbation theory:
  - Severe errors associated with the perturbative static potential $V^{(\textrm{perturbative})}(r)$ for $Q\bar{Q}$ separations $r \gtrsim 0.20\,\textrm{fm} \ldots 0.25\,\textrm{fm}$ (details later).
- To determine $r_0 \Lambda_{\overline{\textrm{MS}}}$, match $V^{(\textrm{lattice})}(r)$ and $V^{(\textrm{perturbative})}(r)$ in a small window of separations $r_\textrm{min} \ldots r_\textrm{max} \approx 0.10\,\textrm{fm} \ldots 0.25\,\textrm{fm}$.

Lattice computation of V (1)

- Lattice static potential computations on various $n_f = 2$ QCD ensembles generated by ETMC (European Twisted Mass Collaboration):

β	a in fm	$(L/a)^3 \times T/a$	m_{PS} in MeV	# gauges
3.90	0.079(3)	$24^3 \times 48$	340(13)	168
4.05	0.063(2)	$32^3 \times 64$	325(10) 449(14) 517(16)	71 100 92
4.20	0.0514(8)	$24^3 \times 48$ $48^3 \times 96$	284(5)	123 46
4.35	0.0420(17)	$32^3 \times 64$	352(22)	146

Allows to exclude/remove unwanted discretization, finite volume and finite quark mass effects (perturbative V only available for $m_q = 0$).

Lattice computation of V (2)

- V is extracted from the exponential decay of the correlator

  $$\langle Q\bar{Q}(t=T)|Q\bar{Q}(t=0)\rangle \quad \propto \quad e^{-V(r)T} \quad \text{(for very large } T\text{)},$$

  where

  $$|Q\bar{Q}\rangle \quad = \quad \bar{Q}(\mathbf{x})\gamma_5\Big(P\exp\Big(i\int_{\mathbf{x}}^{\mathbf{y}} d\mathbf{z}\mathbf{A}(\mathbf{z})\Big)\Big)Q(\mathbf{y})|\Omega\rangle \quad , \quad r \quad = \quad |\mathbf{x}-\mathbf{y}|.$$

- $\langle Q\bar{Q}(t=T)|Q\bar{Q}(t=0)\rangle$ is proportional to a Wilson loop, i.e.

  $$\langle Q\bar{Q}(t=T)|Q\bar{Q}(t=0)\rangle \quad \propto \quad \Big\langle W(r,T)\Big\rangle \quad = \quad \Big\langle \mathrm{Tr}\Big(P\exp\Big(i\oint_C dz_\mu A_\mu(z)\Big)\Big)\Big\rangle,$$

  where C is a rectangular closed path with extension r and T in spatial and temporal direction respectively.

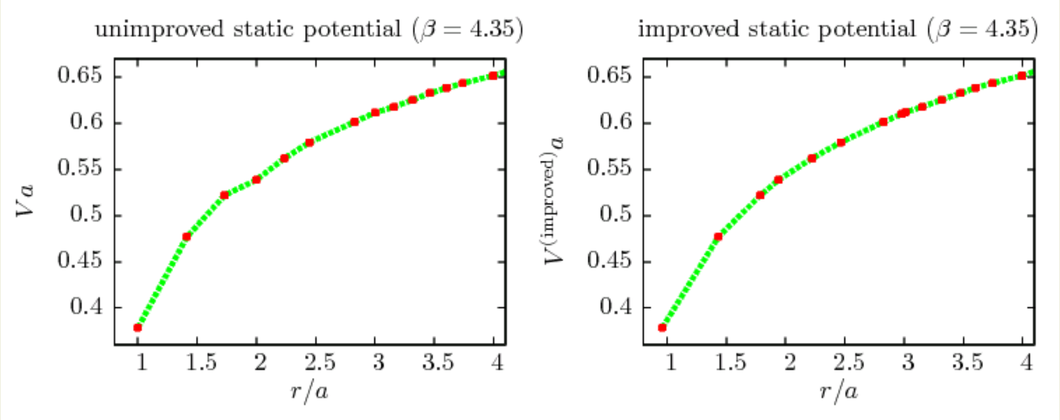
- On the lattice $W(r,t)$ is an ordinary product of link variables and, therefore, rather simple to compute.

Lattice computation of V (3)

- Lattice details (1):
  - APE smearing of spatial links, to "optimize" the ground state overlap of the trial state $|Q\bar{Q}\rangle$.
  - No smearing of temporal links: UV fluctuations are important and may not be removed, because we are interested at small $Q\bar{Q}$ separations.
  - Off-axis spatial $Q\bar{Q}$ separations, to obtain a fine resolution of the static potential (significantly finer than the lattice spacing a).
  - Automatic $\mathcal{O}(a)$ improvement of the static potential, due to twisted mass lattice QCD (the specific quark discretization we are using).

Lattice computation of V (4)

- Lattice details (2):
  - Additional tree-level improvement of the static potential; particularly helpful at small $Q\bar{Q}$ separations (cf. plot).



Perturbation theory for V (1)

- The static potential in momentum space:

$$\tilde{V}(p) = -\frac{16\pi}{3p^2}\tilde{\alpha}_V[\alpha_s(\mu), L(\mu,p)]$$

$$\tilde{\alpha}_V[\alpha_s(\mu), L(\mu,p)] = \alpha_s(\mu)\bigg\{1 + \frac{\alpha_s(\mu)}{4\pi}P_1(L) + \Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^2 P_2(L) + \Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^3\Big[P_3(L) + a_{3\mathrm{ln}}\ln\alpha_s(\mu)\Big] + \ldots\bigg\}.$$

  - Requirement: $\mu \gg \Lambda_{\overline{\mathrm{MS}}}$
    $\longrightarrow$ $\alpha_s(\mu) \ll 1$.
  - $P_n(L)$: polynomials in $L = \ln(\mu^2/p^2)$ of degree n.
  - $\alpha_s^{n+1}(\mu)$ is multiplied to terms up to order L^n
    $\longrightarrow$ expansion not only in $\alpha_s(\mu)$, but also in L
    $\longrightarrow$ requirement: $L \lesssim 1$
    $\longrightarrow$ μ and p must be of the same order.

Perturbation theory for V (2)

- Fourier transform of the static potential in momentum space yields the static potential in position space:

$$V(r) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{r}} \tilde{V}(p) = -\frac{4\alpha_s(\mu)}{3r}\bigg\{1 + \frac{\alpha_s(\mu)}{4\pi}\tilde{P}_1(L') + \bigg(\frac{\alpha_s(\mu)}{4\pi}\bigg)^2 \tilde{P}_2(L') + \bigg(\frac{\alpha_s(\mu)}{4\pi}\bigg)^3 \Big[\tilde{P}_3(L') + a_{3\mathrm{ln}} \ln \alpha_s(\mu)\Big] + \ldots\bigg\}.$$

  - Requirement: $\mu \gg \Lambda_{\overline{\mathrm{MS}}}$
    $\rightarrow \alpha_s(\mu) \ll 1$.
  - $\tilde{P}_n(L')$: polynomials in $L' = \ln(\mu^2 r^2) + 2\gamma_E$ of degree n.
  - $\alpha_s^{n+1}(\mu)$ is multiplied to terms up to order L'^n
    $\rightarrow$ expansion not only in $\alpha_s(\mu)$, but also in L'
    $\rightarrow$ requirement: $L' \lesssim 1$
    $\rightarrow$ μ and $1/r$ must be of the same order.

Perturbation theory for V (3)

- The scale $\Lambda_{\overline{\mathrm{MS}}}$ and its relation to the coupling $\alpha_s(\mu)$:

$$\Lambda_{\overline{\mathrm{MS}}} \equiv \mu\left(\frac{\beta_0\alpha_s(\mu)}{4\pi}\right)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left\{-\frac{2\pi}{\beta_0\alpha_s(\mu)} - \int_0^{2\sqrt{\pi\alpha_s(\mu)}} \frac{d\alpha_s}{\alpha_s}\left[\frac{1}{\beta(\alpha_s)} + \frac{2\pi}{\beta_0\alpha_s} - \frac{\beta_1}{2\beta_0^2}\right]\right\}$$

$$\beta(\alpha_s) = -\frac{\alpha_s}{2\pi}\sum_{n=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \beta_n$$

($\beta[\alpha_s]$: QCD β-function, known up to $n = 3$ [4-loop order]).

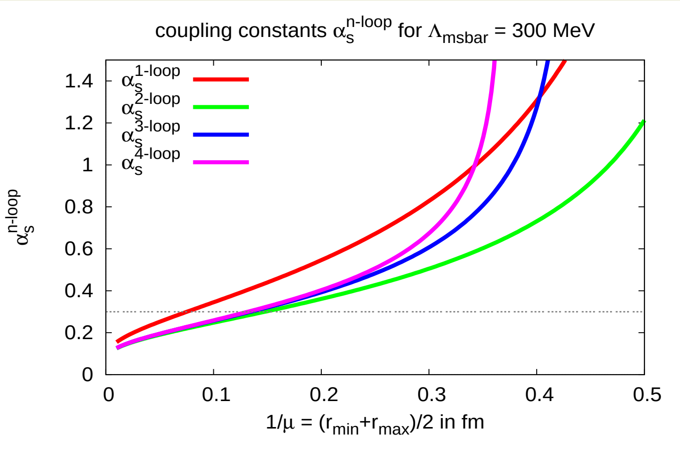
- Expression simple at 1-loop order:

$$\Lambda_{\overline{\mathrm{MS}}} \equiv \mu \exp\left\{-\frac{2\pi}{\beta_0\alpha_s(\mu)}\right\}.$$

  - $\mu = \Lambda_{\overline{\mathrm{MS}}} \quad \rightarrow \quad \alpha_s(\mu) = \infty$,
    i.e. perturbation theory only valid, if $\mu \gg \Lambda_{\overline{\mathrm{MS}}}$.

Perturbation theory for V (4)

- Setting the scale μ:
  - Requirement: μ and $1/r$ must be of the same order
    $\longrightarrow$ since we need a reliable perturbative expansion of V in a range of $Q\bar{Q}$ separations $r_{\text{min}} \leq r \leq r_{\text{max}}$, choose $1/\mu \approx (r_{\text{max}} - r_{\text{min}})/2$.
  - Requirement: $\mu \gg \Lambda_{\overline{\text{MS}}} \approx 300\,\text{MeV} \approx 1/0.67\,\text{fm}$
    $\longrightarrow$ choose $r_{\text{max}} \lesssim 0.25\,\text{fm}$, to assure $\alpha_s(\mu) \ll 1$ (cf. plot).



Perturbation theory for V (5)

- Summary of perturbation theory for V:

$$V(r) = -\frac{4\alpha_s(\mu)}{3r}\Big\{1 + \frac{\alpha_s(\mu)}{4\pi}\tilde{P}_1(L') + \Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^2\tilde{P}_2(L') + \Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^3\Big[\tilde{P}_3(L') + a_{3\mathrm{ln}}\ln\alpha_s(\mu)\Big] + \ldots\Big\}$$

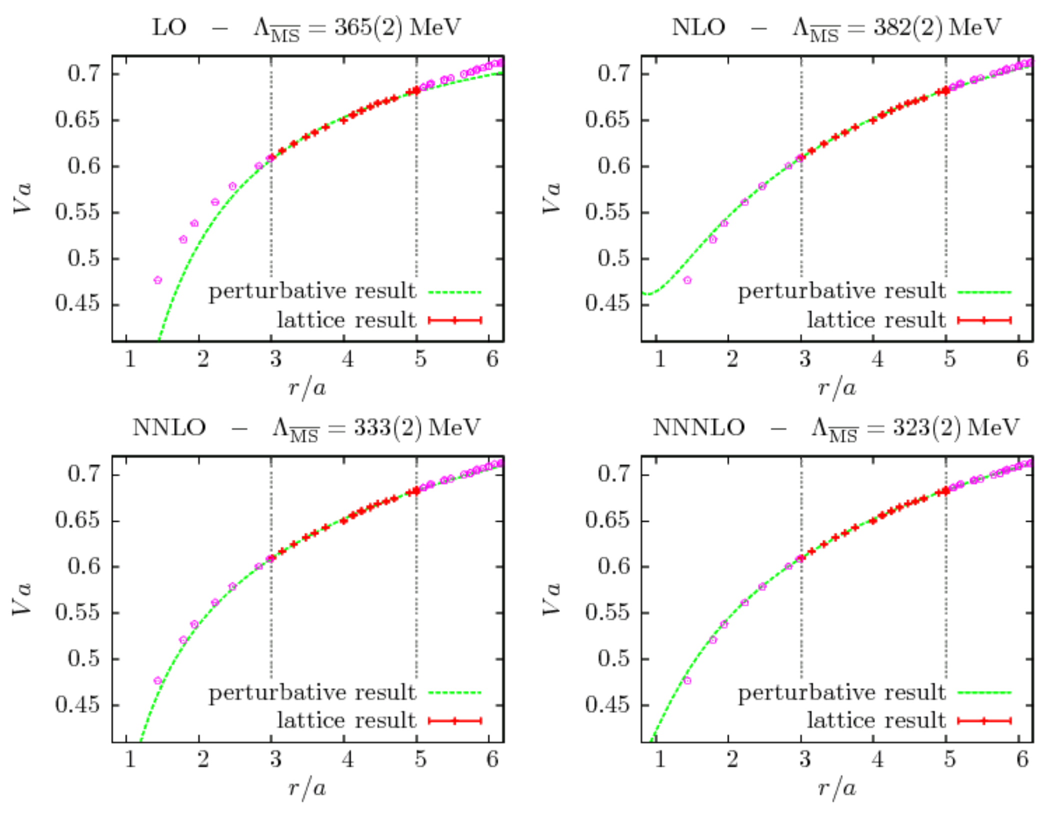
  - Four perturbative orders available:
    - LO, i.e. $\mathcal{O}(\alpha_s(\mu))$.
    - NLO, i.e. $\mathcal{O}(\alpha_s^2(\mu))$.
    - NNLO, i.e. $\mathcal{O}(\alpha_s^3(\mu))$.
    - NNNLO, i.e. $\mathcal{O}(\alpha_s^4(\mu))$ and $\sim \alpha_s^4(\mu)\ln\alpha_s(\mu)$.
  - $\Lambda_{\overline{\mathrm{MS}}}$ enters via $\alpha_s(\mu)$.
  - To get a reliable perturbative expression, choose
    - $1/\mu \approx (r_\mathrm{max} - r_\mathrm{min})/2$,
    - $r_\mathrm{max} \lesssim 0.25\,\mathrm{fm}$.

Determination of $\Lambda_{\overline{\mathrm{MS}}}$ (1)

- Perform an uncorrelated χ^2-minimizing fit of the perturbative static potentials (LO, NLO, NNLO, NNNLO) to the lattice results (various ensembles), fitting range $[r_{\textrm{min}}, r_{\textrm{max}}]$:
  - $r_{\textrm{min}}$ restricted by lattice results; vary in $[2a, 4a]$ (corresponding to $0.08\,\textrm{fm} \ldots 0.17\,\textrm{fm}$ for our smallest lattice spacing).
  - $r_{\textrm{max}}$ restricted by perturbative results; vary in $[4a, 6a]$ (corresponding to $0.17\,\textrm{fm} \ldots 0.25\,\textrm{fm}$ for our smallest lattice spacing).
  - $1/\mu$ should be of the same order as the $Q\bar{Q}$ separation r; vary in $[3a, 5a]$.

Determination of $\Lambda_{\overline{MS}}$ (2)

- Exemplary fits for $a = 0.042$ fm, $r_{\text{min}} = 0.13$ fm, $r_{\text{max}} = 0.21$ fm, $1/\mu = 0.17$ fm:



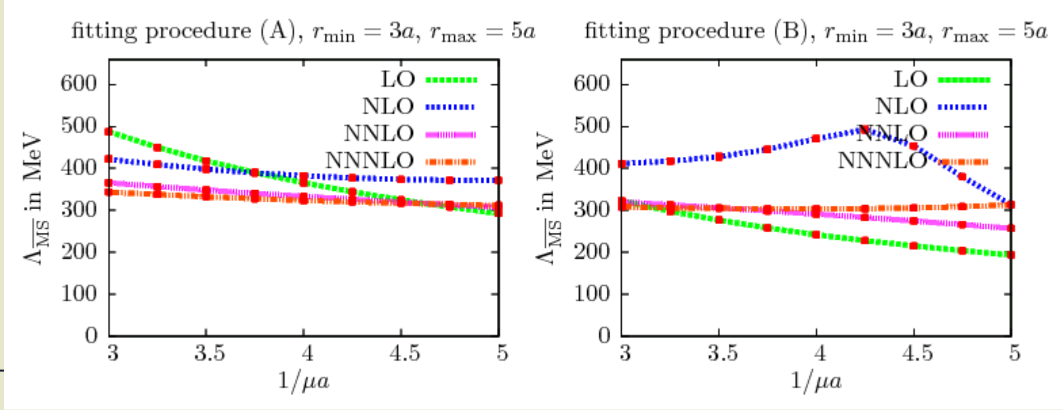
The perturbative error of $\Lambda_{\overline{\mathrm{MS}}}$ (1)

- Use two different sets of formulae for the perturbative static potential:

  (A) Always use the 4-loop result, when expressing $\alpha_s(\mu)$ in V in terms of $\Lambda_{\overline{\mathrm{MS}}}$.

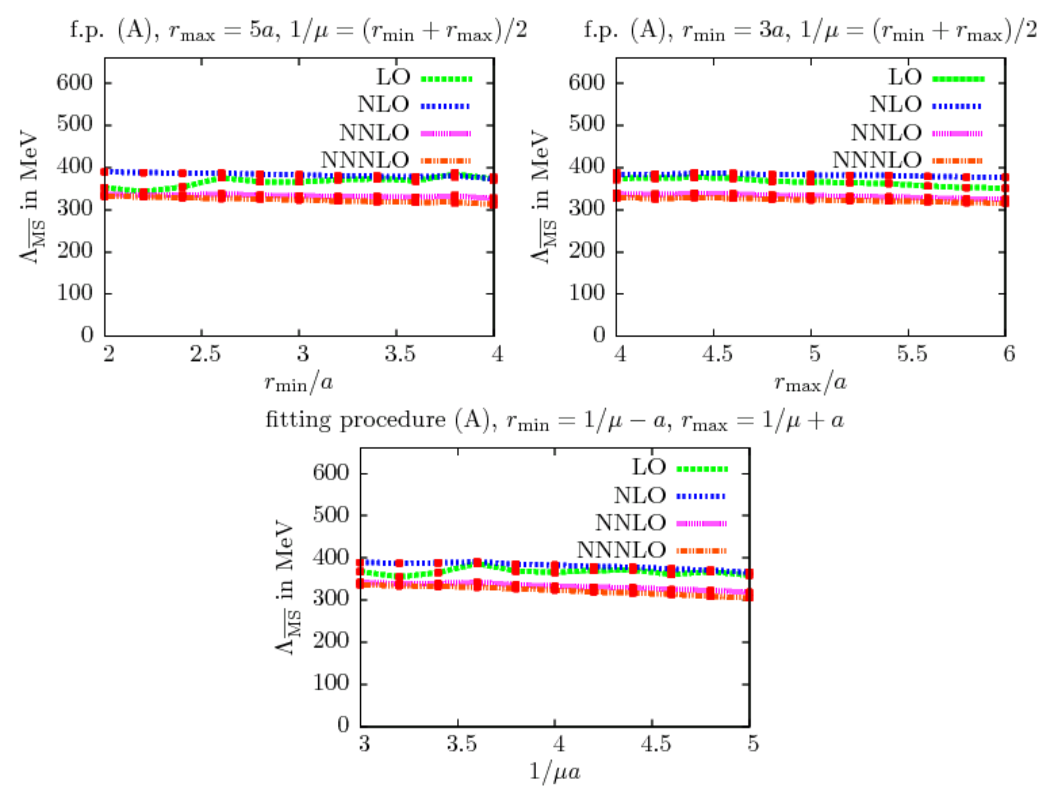
  (B) Use also lower (minimal) loop results, such that the resulting expression for V is "just" LO, NLO, NNLO and NNNLO.

  (A) and (B) fit formulae differ in higher orders; a comparison of (A) and (B) might give an indication of the effect caused by not fully considering these higher orders.



The perturbative error of $\Lambda_{\overline{\mathrm{MS}}}$ (2)

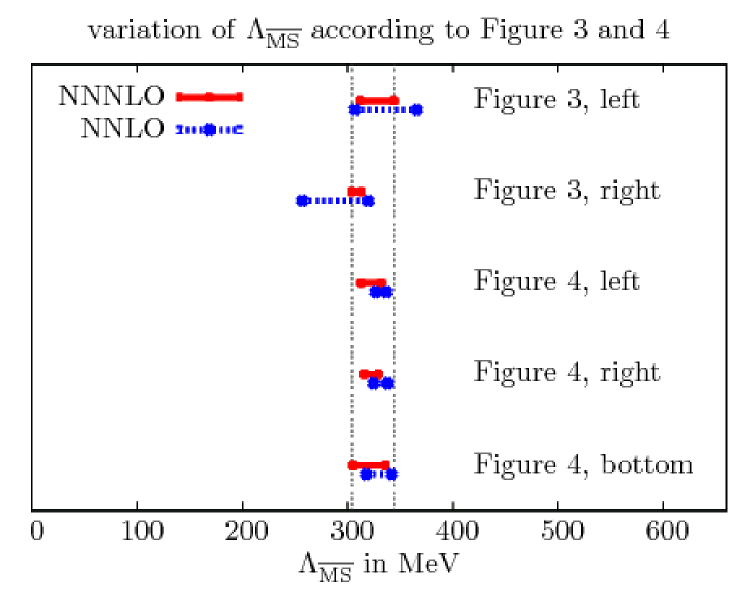
- Vary r_min, r_max and $1/\mu$.



The perturbative error of $\Lambda_{\overline{\mathrm{MS}}}$ (3)

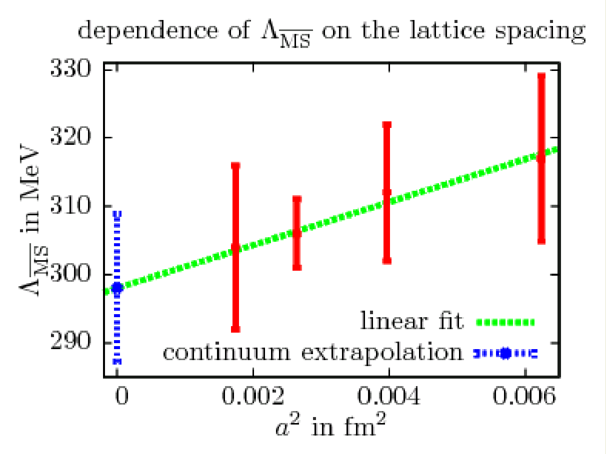
- Summary of $\Lambda_{\overline{\mathrm{MS}}}$ variations (previous two slides): cf. plot.
- Determine the combined error by simultaneous random variations of the fit formulae (NNLO and NNNLO, (A) and (B)) and the parameters r_{min}, r_{max} and $1/\mu$:

$$\Lambda_{\overline{\mathrm{MS}}} \quad = \quad 315(26)\,\mathrm{MeV}.$$



The lattice error of $\Lambda_{\overline{\mathrm{MS}}}$ (1)

- Use various ensembles to investigate all sources of lattice systematic errors:
  - Lattice discretization errors:
    $\rightarrow \Delta\Lambda_{\overline{\mathrm{MS}}} = \pm 6\,\mathrm{MeV}$ (cf. plot).
  - Finite volume effects:
    $\rightarrow$ negligible.
  - Non-vanishing light quark mass (perturbative V only available for $m_q = 0$):
    $\rightarrow$ negligible.



Final results for $r_0 \Lambda_{\overline{\mathrm{MS}}}$ and $\Lambda_{\overline{\mathrm{MS}}}$

- Combine
  - perturbative errors,
  - lattice errors,
  - uncertainties associated with r_0 and the lattice spacing a

  by adding them in quadrature.
- $r_0 \Lambda_{\overline{\mathrm{MS}}}$ and $\Lambda_{\overline{\mathrm{MS}}}$ for $n_f = 2$ QCD with massless quarks:

  $$r_0 \Lambda_{\overline{\mathrm{MS}}} \ = \ 0.658(55) \quad , \quad \Lambda_{\overline{\mathrm{MS}}} \ = \ 315(30)\,\mathrm{MeV}.$$

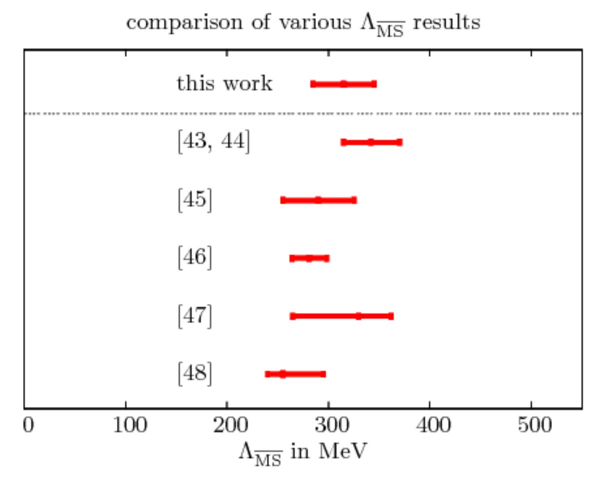
Conclusions (1)

- We have determined $r_0 \Lambda_{\overline{\mathrm{MS}}}$ and $\Lambda_{\overline{\mathrm{MS}}}$ for $n_f = 2$ QCD with massless quarks:

$$r_0 \Lambda_{\overline{\mathrm{MS}}} = 0.658(55) \quad , \quad \Lambda_{\overline{\mathrm{MS}}} = 315(30)\,\mathrm{MeV}.$$

- All sources of systematic errors have been investigated and taken into account.

Conclusions (2)



- Our results compare well with other results obtained by employing different methods (Schrödinger functional; r_0 and a boosted coupling; Landau-gauge gluon and ghost correlations; Taylor running coupling constant; variationally optimized perturbation, combined with renormalization group properties).

[M. Della Morte, R. Frezzotti, J. Heitger, J. Rolf, R. Sommer and U. Wolff [ALPHA Collaboration], Nucl. Phys. B **713**, 378 (2005) [arXiv:hep-lat/0411025]]

[B. Leder *et al.* [ALPHA Collaboration], PoS **LATTICE2010**, 233 (2010) [arXiv:1012.1141 [hep-lat]]]

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Future plans

- Determine $r_0\Lambda_{\overline{\mathrm{MS}}}$ and $\Lambda_{\overline{\mathrm{MS}}}$ for QCD with four quark flavors (ETMC is currently generating $n_f = 2 + 1 + 1$ gauge link configurations).