Static-light mesons in twisted mass QCD

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European Twisted Mass Collaboration

- **Cyprus**: University of Nikosia.
- **France**: University of Paris Sud, LPSC Grenoble.
- **Germany**: Humboldt University Berlin, University of Münster, DESY Hamburg, DESY Zeuthen.
- **Great Britain**: University of Glasgow, University of Liverpool.
- **Italy**: University of Rome I, University of Rome II, University of Rome III, ECT* Trento.
- **Netherlands**: University of Groningen.
- **Spain**: University of Valencia.
- **Switzerland**: University of Zürich.

Introduction

- **Static-light meson**: a bound state of an infinitely heavy quark and a light quark ("a B-meson in leading order").
- Static-light mesons can be classified according to certain quantum numbers:
  - Total angular momentum $F = 0, 1, 2, 3, \ldots$
  - Parity $P = \pm$.
- **Goal**: compute static-light meson masses for low lying states (ground state, first excited state) for different quantum numbers F and P.

Outline

- Basic principle.
- (Twisted mass) lattice QCD.
- Static-light meson creation operators on the lattice.
- Simulation setup and numerical results.
- The static quark antiquark potential.
- Summary and outlook.

Basic principle (1)

- Let $\mathcal{O}(\mathbf{x})$ be a suitable "static-light meson creation operator", i.e. an operator such that $\mathcal{O}(\mathbf{x})|\Omega\rangle$ is a state containing a static-light meson at position $\mathbf{x}$ ($|\Omega\rangle$: vacuum).

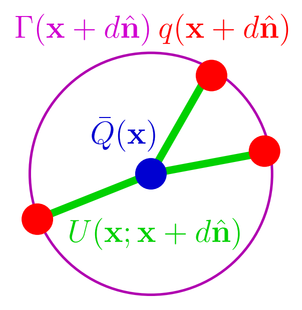
- Determine the ground state mass of the static-light meson from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times T:

$$\begin{aligned}\mathcal{C}(T) \quad &= \quad \langle\Omega|\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger \mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \langle\Omega|e^{+HT}\Big(\mathcal{O}(\mathbf{x},0)\Big)^\dagger e^{-HT}\mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \sum_n \Big|\langle n|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)T\Big) \quad \approx \quad (\text{for } T \gg 1) \\ &\approx \quad \Big|\langle n|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{\text{meson mass}}T\Big).\end{aligned}$$

Basic principle (2)

- To compute the static-light spectrum, i.e. meson masses for different quantum numbers, consider extended meson creation operators with different spatial structure and different spin structure yielding well defined total angular momentum F.
- Static-light meson masses are degenerate with respect to the static spin.
- Therefore, it is more appropriate to label static-light mesons by $J = L \pm 1/2$, where L is the angular momentum quantum number and $\pm$ describes the coupling of the light spin.
- Parity P is also a good quantum number.
- Since static-light mesons are made from non-identical quarks, charge conjugation is not a useful quantum number (static-light meson masses are degenerate with respect to charge conjugation).

Basic principle (3)



- **General form of a static-light meson creation operator**:

$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

  - $\bar{Q}(\mathbf{x})$ creates an infinitely heavy i.e. static antiquark at position $\mathbf{x}$.
  - $q(\mathbf{x} + d\hat{\mathbf{n}})$ creates a light quark at position $\mathbf{x} + d\hat{\mathbf{n}}$ separated by a distance d from the static antiquark.
  - The spatial parallel transporter

$$U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) \quad = \quad P \left\{ \exp\left(+i \int_{\mathbf{x}}^{\mathbf{x}+d\hat{\mathbf{n}}} dz_j\, A_j(\mathbf{z}) \right) \right\}$$

    connects the antiquark and the quark in a gauge invariant way via gluons.
  - The integration over the unit sphere $\int d\hat{\mathbf{n}}$ combined with a suitable weight factor $\Gamma(\hat{\mathbf{n}})$ yields well defined total angular momentum J and parity P ($\Gamma(\hat{\mathbf{n}})$ is a combination of spherical harmonics [$\rightarrow$ angular momentum] and γ-matrices [$\rightarrow$ spin]; Wigner-Eckart theorem).

Basic principle (4)

- **General form of a static-light meson creation operator**:

$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **List of operators** (L: angular momentum; S: total spin; F: total angular momentum; J: angular momentum and light spin; P: parity):

common notation	$\Gamma(\mathbf{x})$	L^P	S^P	F^P	J^P
S	γ_5	0^+	0^-	0^-	$(1/2)^-$
P_-	1	0^+	0^+	0^+	$(1/2)^+$
	$\gamma_j x_j$	1^-	1^-		
P_+	$\gamma_1 x_1 - \gamma_2 x_2$	1^-	1^-	2^+	$(3/2)^+$
D_-	$\gamma_5(\gamma_1 x_1 - \gamma_2 x_2)$	1^-	1^+	2^-	$(3/2)^-$
D_+	$\gamma_1 x_2 x_3 + \gamma_2 x_3 x_1 + \gamma_3 x_1 x_2$	2^+	1^-	3^-	$(5/2)^-$
F_-	$\gamma_5(\gamma_1 x_2 x_3 + \gamma_2 x_3 x_1 + \gamma_3 x_1 x_2)$	2^+	1^+	3^+	$(5/2)^+$

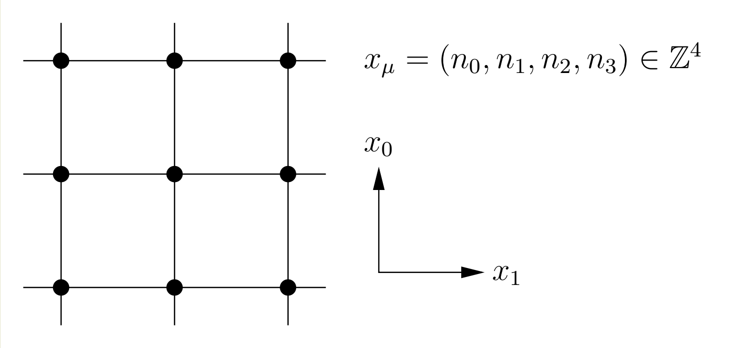
(Twisted mass) lattice QCD (1)

- Compute the correlation functions

$$\begin{aligned} \mathcal{C}(T) &= \langle\Omega|\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger \mathcal{O}(\mathbf{x},0)|\Omega\rangle = \\ &= \frac{1}{\mathcal{Z}}\int D\psi\, D\bar{\psi} \int DA\, \Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger \mathcal{O}(\mathbf{x},0) e^{-S[\psi,\bar{\psi},A]} \end{aligned}$$

  by means of lattice QCD.

- Spacetime is discretized and considered to be periodic.



(Twisted mass) lattice QCD (2)

- **Gluonic fields**:
  - Continuum action:

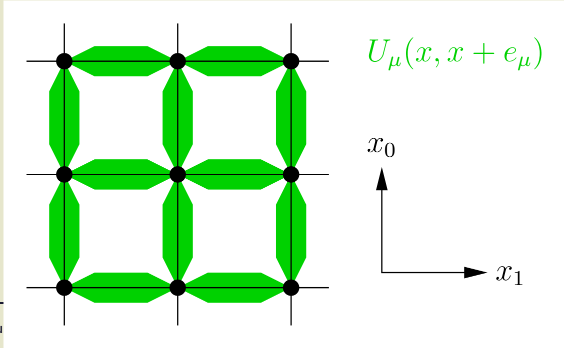
$$S_{\text{gauge}} = \frac{1}{2g^2} \int d^4x \, \text{Tr}(F_{\mu\nu} F_{\mu\nu})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

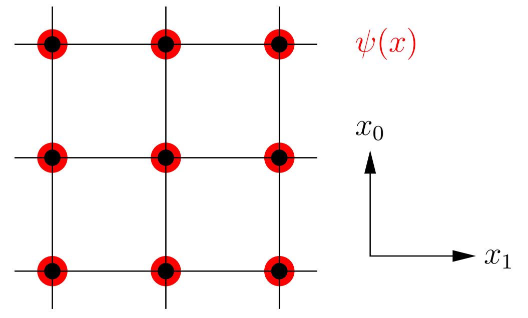
  - To maintain gauge invariance, gluonic fields A_μ are represented via links ("small parallel transporters" connecting neighboring lattice sites):

$$U_\mu(x; x + e_\mu) = P\left\{\exp\left(-i \int_x^{x+e_\mu} dz_\mu \, A_\mu(z)\right)\right\}.$$

  - Lattice formulas are straightforward.
  - Numerically cheap.



(Twisted mass) lattice



- **Quark fields (1)**:
  - Continuum action:

$$S_{\text{fermionic}} = \int d^4x\, \bar{\psi}\Big(\gamma_\mu D_\mu + m\Big)\psi \quad , \quad D_\mu = \partial_\mu - iA_\mu.$$

  - A naive discretization of the fermionic action fails (fermion doubling problem, H. Nielsen and M. Ninomiya, 1981).
  - Different approaches to overcome this problem exist.
  - ETMC: twisted mass formulation with two degenerate flavors.
    * Lattice action ("continuum version"):

$$S_{\text{fermionic}} = \int d^4x\, \bar{\chi}\left(\gamma_\mu D_\mu + m + i\mu\gamma_5\tau_3 - \frac{a}{2}\Box\right)\chi.$$

    * χ: quark fields in the twisted basis, i.e. $\psi = e^{i\omega\gamma_5\tau_3/2}\chi$.
    * μ: twisted mass.
    * τ_3: third Pauli matrix acting in flavor space.
    * a: lattice spacing.

(Twisted mass) lattice QCD (4)

- **Quark fields (2)**:
  - Lattice action ("continuum version"):

$$S_{\mathrm{fermionic}} = \int d^4x\, \bar{\chi} \left(\gamma_\mu D_\mu + m + i\mu\gamma_5\tau_3 - \frac{a}{2}\Box\right) \chi$$
$$\psi = e^{i\omega\gamma_5\tau_3/2}\chi.$$

  - Advantages of twisted mass:
    * Automatic $\mathcal{O}(a)$ improvement, when tuned to maximal twist ($\omega = \pi/2$).
    * "Numerically cheap", i.e. large lattices and/or small lattice spacings are possible.
  - However: explicit breaking of parity and flavor symmetry.

Meson operators on the lattice (1)

- **Static-light meson creation operator in the continuum**:

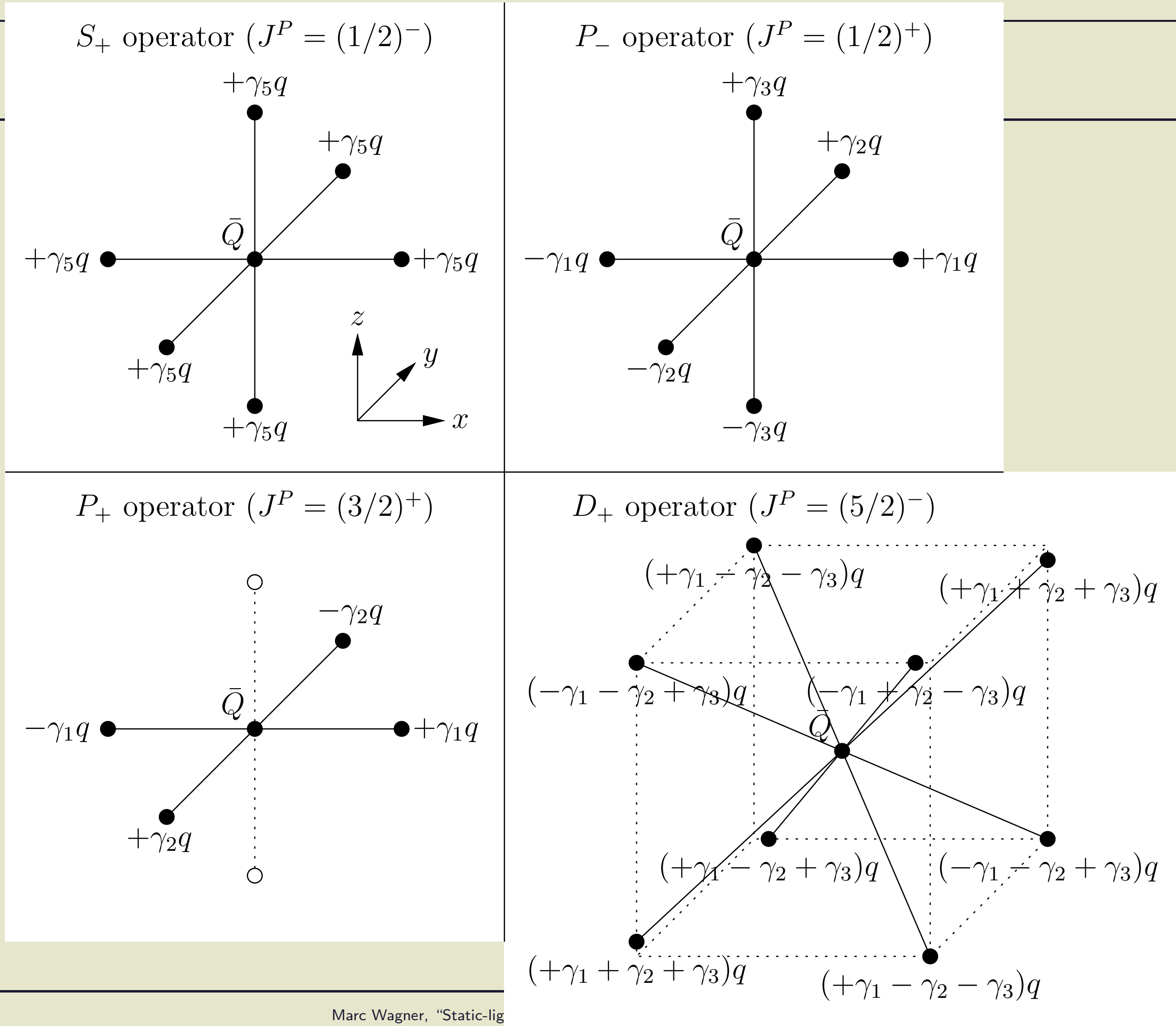
$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **Static-light meson creation operators on the lattice**:

$$\mathcal{O}^{\text{6-path}}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}}=\pm\mathbf{e}_1,\pm\mathbf{e}_2,\pm\mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+$$

$$\mathcal{O}^{\text{8-path}}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}}=\pm\mathbf{e}_1\pm\mathbf{e}_2\pm\mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+.$$

- **Main difference**:
  - The integrations over spheres $\int d\hat{\mathbf{n}}$ are replaced by finite sums $\sum_{\hat{\mathbf{n}}}$.
  - Spherical harmonics contained in Γ are approximated by six or eight points respectively.


S_+ operator ($J^P = (1/2)^-$)
$+\gamma_5 q$
$+\gamma_5 q$
$\bar{Q}$
$+\gamma_5 q$
$+\gamma_5 q$
$+\gamma_5 q$
$+\gamma_5 q$
z
y
x
P_- operator ($J^P = (1/2)^+$)
$+\gamma_3 q$
$+\gamma_2 q$
$\bar{Q}$
$-\gamma_1 q$
$+\gamma_1 q$
$-\gamma_2 q$
$-\gamma_3 q$
P_+ operator ($J^P = (3/2)^+$)
$-\gamma_2 q$
$\bar{Q}$
$-\gamma_1 q$
$+\gamma_1 q$
$+\gamma_2 q$
D_+ operator ($J^P = (5/2)^-$)
$(+\gamma_1 - \gamma_2 - \gamma_3)q$
$(+\gamma_1 + \gamma_2 + \gamma_3)q$
$(-\gamma_1 - \gamma_2 + \gamma_3)q$
$(-\gamma_1 + \gamma_2 - \gamma_3)q$
$\bar{Q}$
$(+\gamma_1 - \gamma_2 + \gamma_3)q$
$(-\gamma_1 - \gamma_2 + \gamma_3)q$
$(+\gamma_1 + \gamma_2 + \gamma_3)q$
$(+\gamma_1 - \gamma_2 - \gamma_3)q$

Meson operators on the lattice (2)

- To determine the total angular momentum quantum numbers of lattice meson creation operators, expand them in terms of spherical harmonics:
  - Expansions are always infinite sums.
  - Lattice operators have no well defined total angular momentum; they always create an infinite superposition of total angular momentum eigenstates.
  - In contrast to the continuum, where there is an infinite number of fixed angular momentum representations (continuous rotation group SO(3)), on the lattice there are only five different representations (discrete rotation group O_h):

$$\begin{aligned} A_1 &\rightarrow L = 0, 4, 6, 8, \ldots \\ A_2 &\rightarrow L = 3, 6, 7, 9, \ldots \\ E &\rightarrow L = 2, 4, 5, 6, \ldots \\ T_1 &\rightarrow L = 1, 3, 4, 5(2\times), \ldots \\ T_2 &\rightarrow L = 2, 3, 4, 5, \ldots \end{aligned}$$

Further lattice techniques (1)

- **Quark propagators**:
  - The computation of the static quark propagator is numerically cheap (just an ordered product of links).
  - The computation of the light quark propagator is numerically challenging (for every spacetime point an inversion of the Dirac matrix is required).
  - Use stochastic propagators with $\mathcal{Z}4$-spin diluted timeslice sources:
    - Only four inversions of the Dirac matrix for every gauge field configuration.
    - Nevertheless, the information contained in a whole timeslice can be accessed, i.e. the gauge configurations can be exploited more fully compared to using point sources, when performing the same number of inversions.

  $\longrightarrow$ Statistical noise is significantly reduced, when making use of translational invariance.

  $\longrightarrow$ Spatial smearing is easy.

Further lattice techniques (2)

- To obtain an acceptable signal-to-noise ratio smearing techniques are indispensable:
  - HYP2 smearing of links in time direction to reduce the static quark self energy.
  - Jacobi smearing of light quark operators and APE smearing of spatial links to enhance the ground state overlap of the "created meson" $\mathcal{O}(\mathbf{x})|\Omega\rangle$ (create a state resembling the lightest meson with the corresponding quantum numbers).
- To extract excited static-light meson states, use correlation matrices instead of correlation functions.

Simulation setup

- $24^3 \times 48$ lattice.
- Twisted mass Dirac operator with two degenerate flavors,

$$Q^{(\chi)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\Box \quad , \quad m + 4 = \frac{1}{2\kappa},$$

  with $\kappa = 0.160856$ and $\mu = 0.0040$.
- Tree-level Symanzik improved gauge action with $\beta = 3.9$.
- Identifying r_0 with $0.5\,\mathrm{fm}$ sets the physical scale yielding a lattice spacing $a \approx 0.097\,\mathrm{fm}$ and a spatial lattice extension $24 \times a \approx 2.32\,\mathrm{fm}$.
- The pseudoscalar meson mass is $m_\mathrm{ps} \approx 278\,\mathrm{MeV}$.
- At the moment static-light meson correlations have been computed on 500 gauge field configurations (≈ 2200 will be included in the final results, i.e. error bars will shrink by a factor of ≈ 2).

Results (1)

- To compute ground states and excited states, consider 6×6 and 7×7 correlation matrices

$$\mathcal{C}_{jk}(T) \quad = \quad \langle \Omega | \Big(\mathcal{O}_j(\mathbf{x}, T) \Big)^\dagger \mathcal{O}_k(\mathbf{x}, 0) | \Omega \rangle .$$

  - Different smearing levels, i.e. different meson extensions.
  - Operators with parity $P = +$ and $P = -$ in the same correlation matrix, because of parity violation of the twisted mass Dirac operator.
  - Fixed total angular momentum J for each correlation matrix.

Results (2)

- Determine effective masses by solving the generalized eigenvalue problem

$$\mathcal{C}(T)\mathbf{v}^{(n)}(T) \quad = \quad \mathcal{C}(T-1)\mathbf{v}^{(n)}(T)\lambda^{(n)}(T) \quad , \quad \lambda^{(n)}(T) \quad \approx \quad e^{-m_{\textrm{effective}}^{(n)}(T)}$$

  (visualization of static-light meson masses and their statistical accuracy).

- Perform a least squares fit to the correlation matrix with the ansatz

$$\mathcal{O}_k(\mathbf{x},0)|\Omega\rangle \quad = \quad \sum_n a_n^{(k)} e^{-E_n T}$$

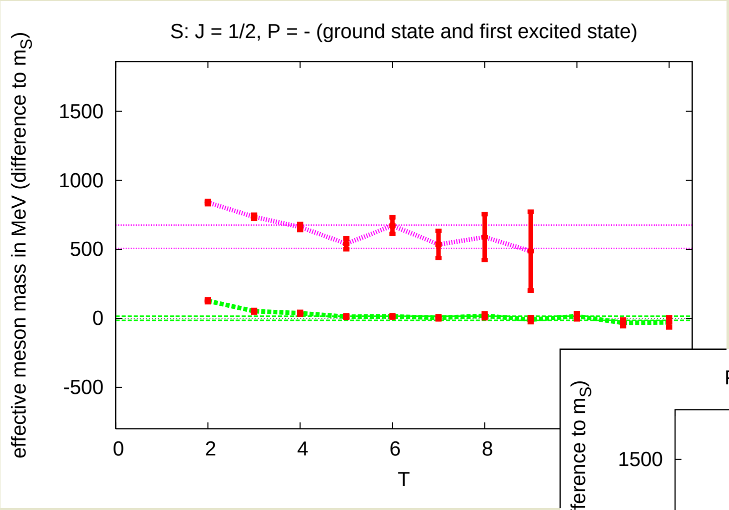
  (numerical values and statistical errors for static-light meson masses).

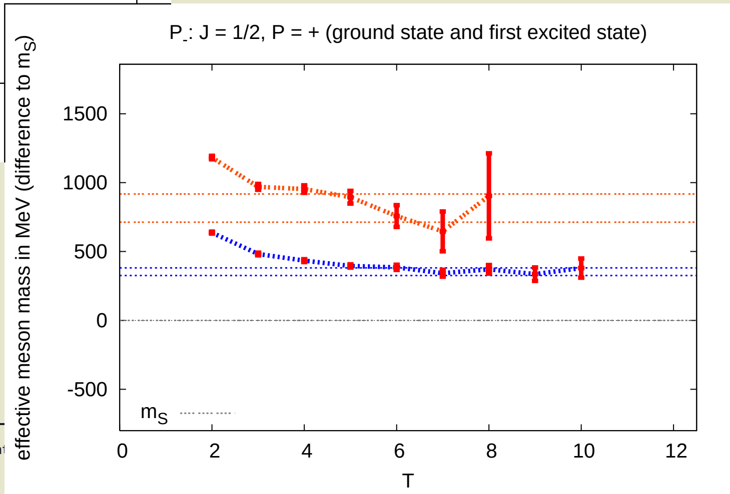
- Both approaches yield consistent results.

Results (3)

- $J = 1/2$: S $(P = -)$ and P_- $(P = +)$.

state	J^P	$m - m_S$ in MeV
S^*	$(1/2)^-$	$590(99)$
P_- P_-^*	$(1/2)^+$	$353(42)$ $815(117)$
P_+ P_+^*	$(3/2)^+$	
D_- $D-^*$	$(3/2)^-$	
D_+	$(5/2)^-$	
F_-	$(5/2)^+$	

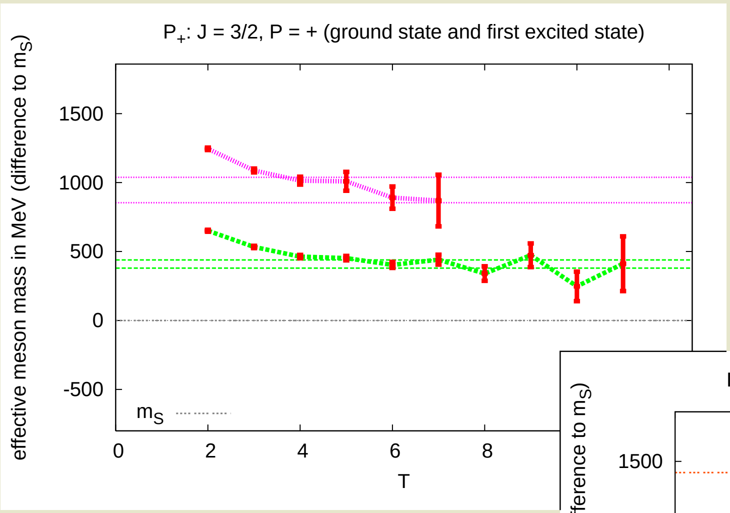


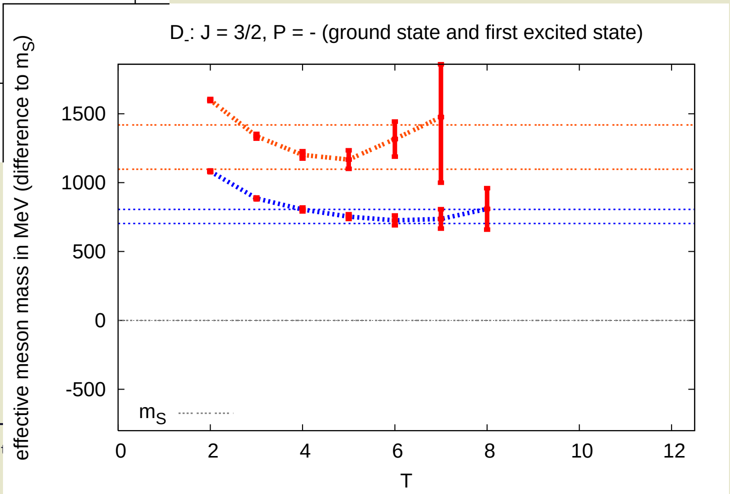


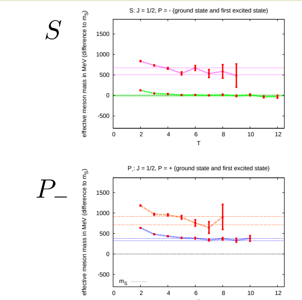
Results (4)

- $J = 3/2$: P_+ ($P = +$) and D_- ($P = -$).

state	J^P	$m - m_S$ in MeV
S^*	$(1/2)^-$	590(99)
P_- P_-^*	$(1/2)^+$	353(42) 815(117)
P_+ P_+^*	$(3/2)^+$	409(44) 946(107)
D_- $D-^*$	$(3/2)^-$	754(66) 1258(176)
D_+	$(5/2)^-$	
F_-	$(5/2)^+$	



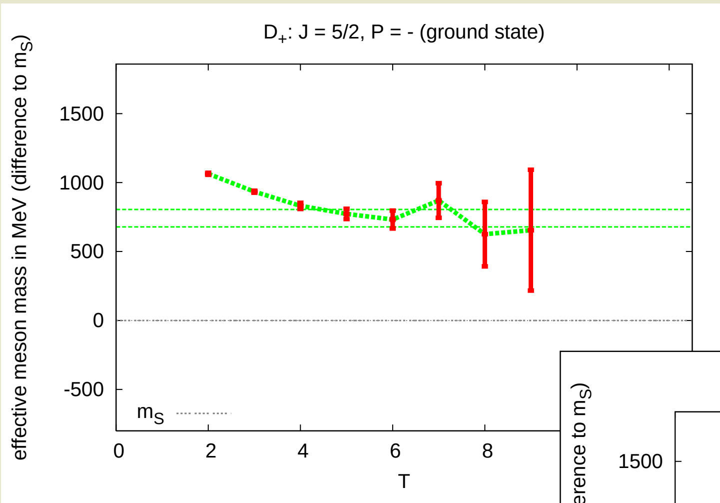


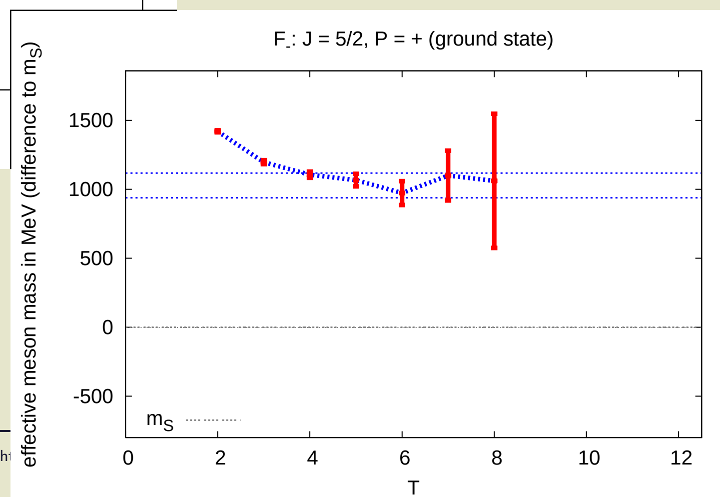


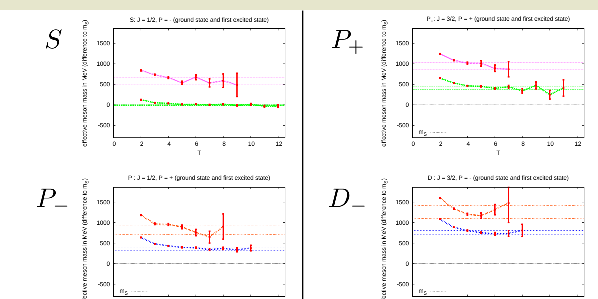
Results (5)

state	J^P	$m - m_S$ in MeV
S^*	$(1/2)^-$	$590(99)$
P_- P_-^*	$(1/2)^+$	$353(42)$ $815(117)$
P_+ P_+^*	$(3/2)^+$	$409(44)$ $946(107)$
D_- $D_-{}^*$	$(3/2)^-$	$754(66)$ $1258(176)$
D_+	$(5/2)^-$	$742(78)$
F_-	$(5/2)^+$	$1028(104)$

- $J = 5/2$: D_+ $(P = -)$ and F_- $(P = +)$.

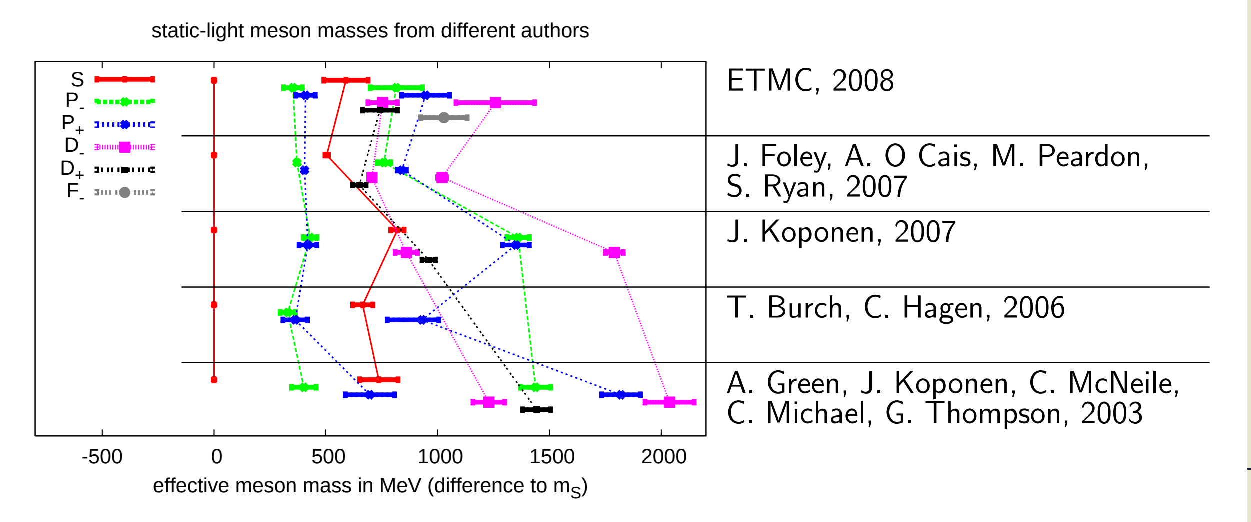






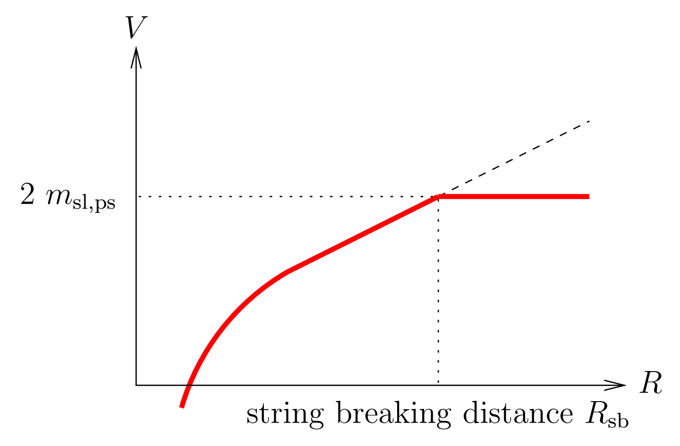
Results (6)

- **ETMC**: $24^3 \times 48$ lattice, $a \approx 0.097\,\mathrm{fm}$, $m_{\mathrm{ps}} \approx 278\,\mathrm{MeV}$.
- **Foley**: $12^3 \times 80$ anisotropic lattice, $a \approx 0.17\,\mathrm{fm}$, $m_{\mathrm{ps}} \approx 400\,\mathrm{MeV}$.
- **Koponen**: $16^3 \times 32$ lattice, $a \approx 0.11\,\mathrm{fm}$, light quark $\approx$ strange quark.
- **Burch**: $12^3 \times 24$ lattice, $a \approx 0.115\,\mathrm{fm}$, $m_{\mathrm{ps}} \approx 500\,\mathrm{MeV}$.
- **Green**: $16^3 \times 32$ lattice, $a \approx 0.105\,\mathrm{fm}$, $m_{\mathrm{ps}} \approx 550\,\mathrm{MeV}$.



The static quark antiquark potential (1)

- **Static quark antiquark potential** $V(R)$:
  - The energy of the lowest state containing an infinitely heavy quark and an infinitely heavy antiquark, separated by a distance R.
  - Expectation:
    - Linear for intermediate separations ($R < R_{\mathrm{sb}}$).
    - Constant for large separations ($R > R_{\mathrm{sb}}$).



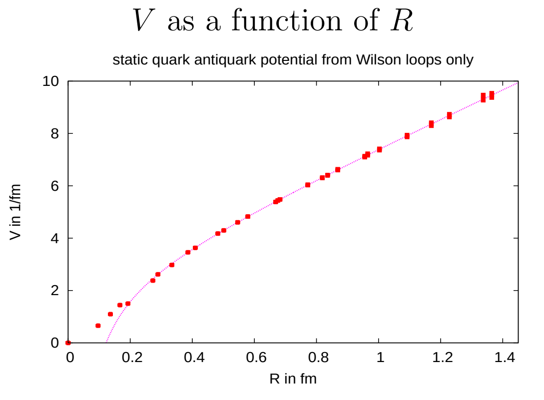
The static quark antiquark potential (2)

- In principle $V(R)$ can be computed via Wilson loops $W_{(R,T)}$:

$$\langle\Omega|\Big(\bar{Q}(\mathbf{x},T)U(\mathbf{x},T;\mathbf{y},T)Q(\mathbf{y},T)\Big)^\dagger \bar{Q}(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)Q(\mathbf{y},0)|\Omega\rangle \quad = \quad \ldots \quad = \quad \#\Big\langle W_{(R,T)}\Big\rangle$$

$$V(R) \quad = \quad -\frac{1}{a}\lim_{T\rightarrow\infty}\bigg(\ln\Big\langle W_{(R,T)}\Big\rangle - \ln\Big\langle W_{(R,T-1)}\Big\rangle\bigg).$$

- In practice we have "moderate T separations" instead of $T \rightarrow \infty$:
  - Wilson loops work well for $R < R_{\mathrm{sb}}$.
  - Wilson loops fail for $R > R_{\mathrm{sb}}$, i.e. the potential is still linearly rising and there seems to be no string breaking.

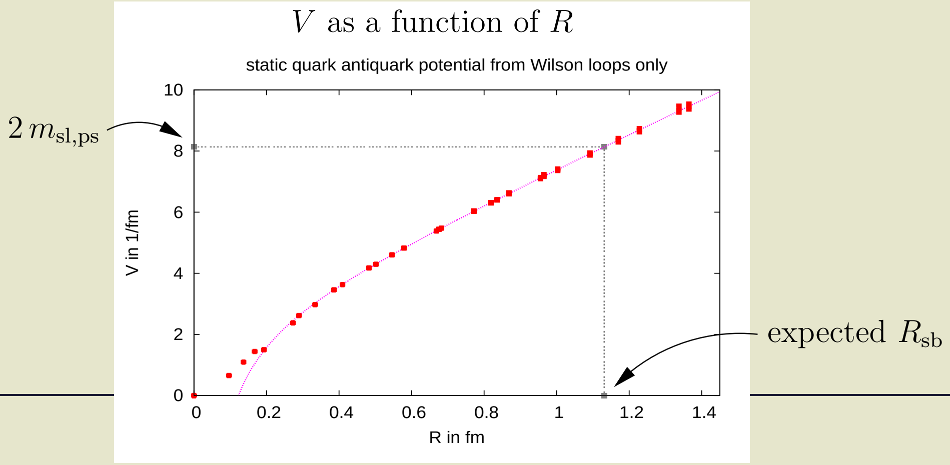


The static quark antiquark potential (3)

- **Why do Wilson loops fail?**
  - $\bar{Q}(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)Q(\mathbf{y},0)|\Omega\rangle$ has poor overlap to the ground state state for $R > R_{\text{sb}}$ (a "two meson state").

- However, the pure Wilson loop potential can be used to cross check the mass of the lightest static-light meson (pseudoscalar): $2\,m_{\text{sl,ps}} \approx V(R_{\text{sb}})$.

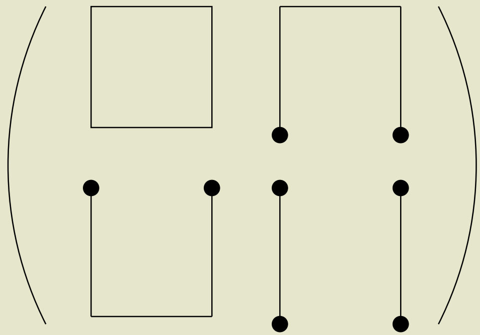
- Expectation for the string breaking distance: $R_{\text{sb}} \approx 1.13\,\text{fm}$ (in agreement with results from lattice computations and from phenomenological models).



The static quark antiquark potential (4)

- **Computation of the "full static quark antiquark potential"**:
  - Instead of a single state use a whole set of states containing not only "string states" but also two meson states, i.e. $\bar{Q}(\mathbf{x},0)\gamma_5 q(\mathbf{x},0)\bar{q}(\mathbf{y},0)\gamma_5 Q(\mathbf{y},0)|\Omega\rangle$.
  - Extract the potential from the corresponding correlation matrix

$\mathcal{C}(T) \quad =$



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Summary

- Static-light meson masses have been computed on the ETMC twisted mass gauge field configurations at a small value of the lattice spacing ($a \approx 0.097\,\mathrm{fm}$) and a small value of the pion mass ($m_\mathrm{ps} \approx 278\,\mathrm{MeV}$):
  - Total angular momentum $J = 1/2\,,\, 3/2\,,\, 5/2$.
  - Parity $P = +\,,\, -$.
  - Ground states and first excited states.
- The plateaux quality of effective meson masses seems pretty good although, at the moment, computations have only been performed on $\approx 20\%$ of the available gauge field configurations.
- The resulting masses are consistent with the pure Wilson loop static quark antiquark potential and with results from literature.

Outlook

- Reduce statistical errors by a factor of ≈ 2 by considering all available gauge configurations.
- Perform computations at different lattice spacings and different lattice volumes.
- Perform computations at different light quark masses.
- **String breaking**: compute correlation matrices both from string states and from two meson states to extract the full static quark antiquark potential.