Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks based on correlation functions with scattering interpolating operators both at the source and at the sink

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Introduction

- LQCD study of a stable $\bar{b}\bar{b}ud$ tetraquark with $I(J^P) = 0(1^+)$.
- Theoretically simple (compared to other tetraquark candidates).
- $\bar{c}\bar{c}ud$ counterpart T_{cc} recently discovered by LHCb.
- Existing LQCD studies of $\bar{b}\bar{b}qq$ systems use only local four-quark operators ...

  [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214]]
  [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D **99**, 054505 (2019) [arXiv:1810.10550]]
  [P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285]]
  [R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, Phys. Rev. D **102**, 114506 (2020) [arXiv:2006.14294]]
  [P. Mohanta and S. Basak, Phys. Rev. D **102**, 094516 (2020) [arXiv:2008.11146]]
- ... or local and scattering four quark operators, but the latter only at the sink.

  [L. Leskovec, S. Meinel, M. Pflaumer and M. Wagner, Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197]]
  [S. Meinel, M. Pflaumer and M. Wagner, [arXiv:2205.13982]]
- This work: scattering operators both at the source and at the sink.
  - → Precise determination of finite-volume energy levels not only for bound states, but also for scattering states.
  - → Particularly important for related $\bar{Q}\bar{Q}qq$ systems, where bound states and scattering states are very close, or where bound states do not exist (i.e. for tetraquark resonances).
  - → Example: $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$.

    [Poster by Jakob Hoffmann, "Inclusion of heavy quark spin effects in the $I(J^P) = 0(1^-)$ four-quark channel in ..."]

Interpolating operators

- Local interpolating operators:

$$\mathcal{O}_1 = \mathcal{O}_{[BB^*](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_5 d(\mathbf{x})\, \bar{b}\gamma_j u(\mathbf{x}) - (d \leftrightarrow u)$$

$$\mathcal{O}_2 = \mathcal{O}_{[B^*B^*](0)} = \epsilon_{jkl} \sum_{\mathbf{x}} \bar{b}\gamma_k d(\mathbf{x})\, \bar{b}\gamma_l u(\mathbf{x}) - (d \leftrightarrow u)$$

$$\mathcal{O}_3 = \mathcal{O}_{[Dd](0)} = \sum_{\mathbf{x}} \bar{b}^a \gamma_j \mathcal{C} \bar{b}^{b,T}(\mathbf{x})\, d^{a,T} \mathcal{C}\gamma_5 u^b(\mathbf{x}) - (d \leftrightarrow u).$$

- Scattering interpolating operators:

$$\mathcal{O}_4 = \mathcal{O}_{B(0)B^*(0)} = \bigg(\sum_{\mathbf{x}} \bar{b}\gamma_5 d(\mathbf{x})\bigg) \bigg(\sum_{\mathbf{y}} \bar{b}\gamma_j u(\mathbf{y})\bigg) - (d \leftrightarrow u)$$

$$\mathcal{O}_5 = \mathcal{O}_{B^*(0)B^*(0)} = \epsilon_{jkl} \bigg(\sum_{\mathbf{x}} \bar{b}\gamma_k d(\mathbf{x})\bigg) \bigg(\sum_{\mathbf{y}} \bar{b}\gamma_l u(\mathbf{y})\bigg) - (d \leftrightarrow u).$$

Lattice setup

- $2+1+1$-flavor HISQ ensembles generated by the MILC collaboration.

  [A. Bazavov *et al.* [MILC], Phys. Rev. D **87**, 054505 (2013) [arXiv:1212.4768]]

ensemble	a [fm]	$L^3 \times T$	$m_\pi^{(\mathrm{sea})}$ [MeV]	$m_\pi^{(\mathrm{val})}$ [MeV]	N_{conf}
a12m310	0.1207(11)	$24^3 \times 64$	305.3(4)	310.2(2.8)	1053
a12m220S	0.1202(12)	$24^3 \times 64$	218.1(4)	225.0(2.3)	1020
a12m220	0.1184(10)	$32^3 \times 64$	216.9(2)	227.9(1.9)	1000
a12m220L	0.1189(09)	$40^3 \times 64$	217.0(2)	227.6(1.7)	1030
a09m310	0.0888(08)	$32^3 \times 96$	312.7(6)	313.0(2.8)	1166
a09m220	0.0872(07)	$48^3 \times 96$	220.3(2)	225.9(1.8)	657

- Wilson clover u, d and s valence quarks (i.e. a mixed action setup).

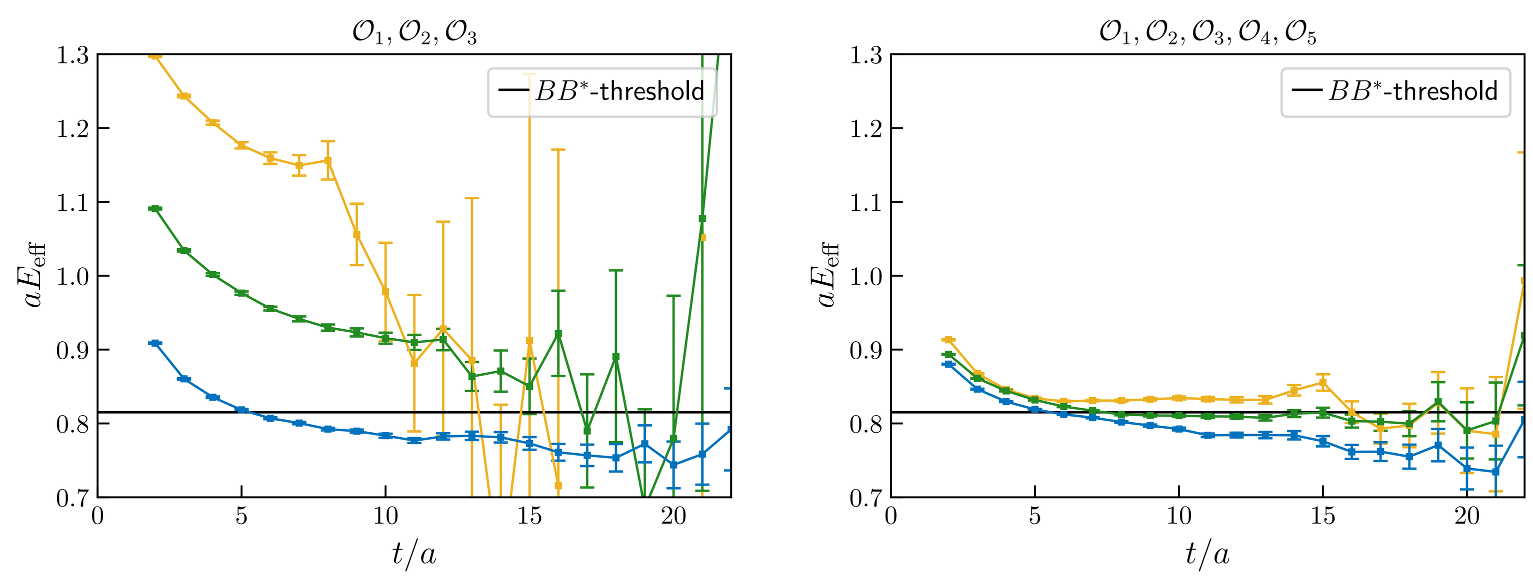
  [T. Bhattacharya *et al.* [PNDME], Phys. Rev. D **92**, 094511 (2015) [arXiv:1506.06411 [hep-lat]]
  [R. Gupta, Y. C. Jang, B. Yoon, H. W. Lin, V. Cirigliano and T. Bhattacharya, Phys. Rev. D **98**, 034503 (2018) [arXiv:1806.09006]]
- b valence quarks via lattice NRQCD.
- Quark propagators used for the computation of correlation functions:
  - Point-to-all, if local operator at the source.
  - Stochastic timeslice-to-all, if scattering operator at the source.
- APE gauge link smearing, Gaussian quark field smearing.
- Analysis of correlation matrices by two independent methods:
  - Generalized eigenvalue problem (GEVP).
  - Athens Model Independent Analysis Scheme (AMIAS).

    [C. Alexandrou, T. Leontiou, C. N. Papanicolas and E. Stiliaris, Phys. Rev. D **91**, 014506 (2015) [arXiv:1411.6765]]

"Local" versus "local and scattering" operators

- Expected low-lying states:
  2nd excitation: B^*B^* scattering state, $\approx 50\,\mathrm{MeV}$ above BB^* threshold.
  1st excitation: BB^* scattering state, close to BB^* threshold.
  Ground state: stable tetraquark, $\approx 100\,\mathrm{MeV}$ below BB^* threshold.
- Plots show effective masses from a GEVP:
  - Left: 3×3 matrix,
    only local operators ($\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$).
    → Strong discrepancy to expectation.
  - Right: 5×5 matrix,
    local operators ($\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$) and scattering operators ($\mathcal{O}_4$, $\mathcal{O}_5$).
    → Consistent with expectation.

→ Scattering operators essential for a precise determination of scattering states.



Scattering analysis

- Determine the mass of the $\bar{b}\bar{b}ud$ tetraquark at infinite volume (at fixed a and m_π):
  (1) Compute the two lowest energy levels in a finite spatial volume (see discussion of operators and GEVP plots above).
  (2) Compute the corresponding phase shifts $\delta_0(k_0)$, $\delta_0(k_1)$ using Lüscher's finite volume method.
  (3) Parameterize $\delta_0(k_0)$, $\delta_0(k_1)$ using the "effective range expansion",

$$k\cot(\delta_0(k)) = \frac{1}{a_0} + \frac{r_0}{2}k^2 \quad \text{(2 parameters, } a_0 \text{ and } r_0\text{).} \qquad (1)$$

  (4) The mass of the $\bar{b}\bar{b}ud$ tetraquark (and the energy of the first excitation) at infinite spatial volume corresponds to a pole in the scattering amplitude

$$T_0(k) = \Big(\cot(\delta_0(k)) - i\Big)^{-1}.$$

- Left plot:
  - Gray data points: LQCD finite volume energy levels (ground state and 1st excitation), three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but same $a \approx 0.12\,\mathrm{fm}$ and $m_\pi \approx 220\,\mathrm{MeV}$ (a12m220S, a12m220, a12m220L).
  - Orange curve: two lowest finite volume energy levels as functions of the spatial extent L computed with Lüscher's finite volume method from the effective range expansion (1).
  - Rather small differences of finite-volume and infinite-volume energy levels.
    - ⋆ A consequence of the large binding energy, $\Delta E_0 = \mathcal{O}(100\,\mathrm{MeV})$.
    - ⋆ Scattering analyses are expected to be more important for smaller binding energies (e.g. $\bar{b}\bar{b}su$) and essential for tetraquark resonances (e.g. $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$).
- Right plot: u/d quark mass extrapolation based on 6 ensembles results in

$$\Delta E_0(m_{\pi,\mathrm{phys}}) \approx (-103 \pm 8)\mathrm{MeV}.$$

(slightly smaller, but consistent with previous lattice studies [see introduction]).