Comparing meson-meson and diquark-antidiquark creation operators for a $\bar{b}b\bar{u}u$ tetraquark

“38th International Symposium on Lattice Field Theory”

Marc Wagner
Goethe-Universität Frankfurt, Institut für Theoretische Physik
mwagner@itp.uni-frankfurt.de
https://itp.uni-frankfurt.de/~mwagner/

in collaboration with Pedro Bicudo, Antje Peters, Sebastian Velten

July 28, 2021

Basic idea: lattice QCD + BO

- Study heavy-heavy-light-light tetraquarks $\bar{b}bqq$ in two steps.

  1. Compute potentials of two static quarks $\bar{b}b$ in the presence of two lighter quarks $qq$ ($q \in \{u, d, s, c\}$) using lattice QCD.

  2. Check, whether these potentials are sufficiently attractive to host bound states or resonances (→ tetraquarks) by using techniques from quantum mechanics and scattering theory.

((1) + (2) → Born-Oppenheimer approximation).

---

Marc Wagner, “Comparing meson-meson and diquark-antidiquark creation operators for a $\bar{b}bud$ tetraquark”, July 28, 2021
Previous work on $\bar{b}bqq$ tetraquarks

- Lattice QCD static potentials and Born-Oppenheimer approximation.

- Full lattice QCD ($b$ quarks with Non Relativistic QCD) [list not complete]:
   [arXiv:1607.05214 [hep-lat]]]

- Other approaches: quark models, effective field theories, QCD sum rules ... [list not complete]:
   [arXiv:1902.03044]]
Outline

• $\bar{b}bqq / BB$ potentials.

• Stable $\bar{b}bqq$ tetraquarks.

• Structure of a $\bar{b}bqq$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ (meson-meson versus diquark-antidiquark structure).

Marc Wagner, “Comparing meson-meson and diquark-antidiquark creation operators for a $\bar{b}b ud$ tetraquark”, July 28, 2021
At large $\bar{b}\bar{b}$ separation $r$, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.

Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).

Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of $qq$ on

- the “light” quark flavors $q \in \{u,d,s,c\}$ (isospin, flavor),
- the “light” quark spin (the static quark spin is irrelevant),
- the type of the meson $B, B^*$ and/or $B_0^*, B_1^*$ (parity).

→ Many different channels: attractive as well as repulsive, different asymptotic values ...

Marc Wagner, “Comparing meson-meson and diquark-antidiquark creation operators for a $\bar{b}b\bar{u}d$ tetraquark”, July 28, 2021
**$\bar{b}\bar{b}qq / BB$ potentials (2)**

- To determine potentials, compute temporal correlation functions of operators

$$O_{BB,\Gamma} = 2N_{BB}(C\Gamma)_{AB}(C\tilde{\Gamma})_{CD}(\bar{Q}_C^a(-r/2)\psi_A^{(f)a}(-r/2))(\bar{Q}_D^b(+r/2)\psi_B^{(f')b}(+r/2)).$$

- The most attractive potential of a $B(\ast)B^\ast$ meson pair has $(I, |j_z|, P, P_x) = (0, 0, +, -)$:
  - $C = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $\psi(f)\psi(f') = ud - du$, $\Gamma \in \{(1 + \gamma_0)\gamma_5, (1 - \gamma_0)\gamma_5\}.$
  - $\bar{Q}\bar{Q} = \bar{b}\bar{b}$, $\tilde{\Gamma} \in \{(1 + \gamma_0)\gamma_5, (1 + \gamma_0)\gamma_j\}$ (irrelevant).

- Parameterize lattice results by

$$V_{qq,j_z,P,P_x}(r) = -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0$$

(1-gluon exchange at small $r$; color screening at large $r$).

Stable $\bar{b}bqq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}b$ using the previously computed $\bar{b}bqq / BB$ potentials,

$$\left( \frac{1}{m_b} \left( - \frac{d^2}{dr^2} + \frac{L(L+1)}{r^2} \right) + V_{qq, j_z, P_x}(r) - 2m_{sl} \right) R(r) = ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}bqq$ tetraquarks.

- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}b$:

  - Binding energy $-E = 38(18)$ MeV with respect to the $BB^*$ threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.  

- No further bound states.

Structure of the $\bar{b}bqq$ tetraquark (1)

- Two types of operators, which probe the same sector:

  - **Meson-meson operator** ($BB$):
    \[
    O_{BB,\Gamma} = 2N_{BB}(C\Gamma)_{AB}(C\tilde{\Gamma})_{CD}\left(\bar{Q}_C^a(-r/2)\psi_A^{(f)^a}(-r/2)\right)\left(\bar{Q}_D^b(+r/2)\psi_B^{(f')^b}(+r/2)\right)
    \]
    with $\Gamma \in \{(1 + \gamma_0)\gamma_5, \gamma_5\} \rightarrow (j_z, \mathcal{P}, \mathcal{P}_x) = (0, -, +))$.

  - **Diquark-antidiquark operator** ($Dd$):
    \[
    O_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\left(\psi_A^{(f)^b}(z)(C\Gamma)_{AB}\psi_B^{(f')^c}(z)\right)
    \]
    \[
    \epsilon^{ade}\left(Q_C^{f}(-r/2)U^{f'd}(-r/2; z)(C\tilde{\Gamma})_{CD}\bar{Q}_D^{g}(+r/2)U^{g'e}(+r/2; z)\right)
    \]
    with $\Gamma \in \{(1 + \gamma_0)\gamma_5, \gamma_5\} \rightarrow (j_z, \mathcal{P}, \mathcal{P}_x) = (0, -, +))$.

- $\psi(f)\psi(f') = ud - du$ ($\rightarrow I = 0$).
- $\tilde{\Gamma} = (1 + \gamma_0)\gamma_3$ (essentially irrelevant).
- Compute the $4 \times 4$ correlation matrix
  \[
  C_{jk}(t) = \langle \Omega | O_j^\dagger(t)O_k(0) | \Omega \rangle.
  \]
Structure of the $\bar{b}bqq$ tetraquark (2)

- Effective energies corresponding to diagonal elements of the correlation matrix,

$$V_j^{\text{eff}}(r,t) = -\frac{1}{a} \log \left( \frac{C_{jj}(t)}{C_{jj}(t-a)} \right) \quad \text{(no sum over } j\text{)}.$$

- For large $\bar{b}b$ separations (right plot $r \approx 0.79$ fm), $BB$ effective energies reach plateaus at smaller $t$ separations than $Dd$ effective energies.

$\to BB$ dominates at large $r$, $Dd$ not important (energetically disfavored due to flux tube).

- For small $\bar{b}b$ separations (left plot $r \approx 0.16$ fm), $BB$ and $Dd$ effective energies similar.

$\to$ More detailed investigation at small $r$ necessary.
• Differences of effective energies corresponding to diagonal elements of the correlation matrix at small temporal separation $t = 2a$ as functions of the $\bar{b}b$ separation $r$,

$$V_j^{\text{eff}}(r, t = 2a) - V_k^{\text{eff}}(r, t = 2a).$$

• $BB$ versus $Dd$ (left): $Dd$ dominates for $r \lesssim 3.15a \approx 0.25$ fm, while $BB$ dominates for $r \gtrsim 3.15a \approx 0.25$ fm.

• $BB$ operators (center): $\Gamma = (1 + \gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Expected. Via a Fierz transformation one can show that $\Gamma = (1 + \gamma_0)\gamma_5$ generates exclusively ground state mesons, while $\gamma_5$ also generates parity excitations.)

• $Dd$ operators (right): $\Gamma = (1 + \gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Interesting. In the literature mostly $\gamma_5$ is discussed.)
Structure of the $\bar{b}bqq$ tetraquark (4)

- Optimize trial states

$$|\Phi_{b,d}\rangle = b|\Phi_{BB,(1+\gamma_0)\gamma_5}\rangle + d|\Phi_{Dd,(1+\gamma_0)\gamma_5}\rangle$$

((|\Phi_j\rangle = \mathcal{O}_j|\Omega\rangle)) by minimizing effective energies

$$V_{b,d}^{\text{eff}}(r,t) = -\frac{1}{a} \log \left( \frac{C_{[b,d][b,d]}(t)}{C_{[b,d][b,d]}(t-a)} \right), \quad C_{[b,d][b,d]}(t) = \left( \begin{array}{c} b \\ d \end{array} \right) \dagger C_{jk}(t) \left( \begin{array}{c} b \\ d \end{array} \right)_{k}.$$ with respect to $b, d \in \mathbb{C}$.

- Since norm and phase of $b$ and $d$ are irrelevant, consider relative weights of $BB$ and $Dd$,

$$w_{BB} = \frac{|b|^2}{|b|^2 + |d|^2}, \quad w_{Dd} = \frac{|d|^2}{|b|^2 + |d|^2} = 1 - w_{BB}.$$

- For fixed $\bar{b}b$ separation $r$, $w_{BB}$ and $w_{Dd}$ depend only weakly on $t$.
  \(\rightarrow w_{BB}$ and $w_{Dd}$ estimate the percentage of $BB$ and of $Dd$.  

Marc Wagner, “Comparing meson-meson and diquark creation operators for a $\bar{b}b$ tetraquark”, July 28, 2021
Structure of the $\bar{b}bqq$ tetraquark (5)

- $w_{BB}$ and $w_{Dd}$ as functions of the $\bar{b}b$ separation $r$ (for two ensembles, $a \approx 0.079$ fm and $a \approx 0.063$ fm).

- $r \lesssim 0.2$ fm: Clear diquark-antidiquark dominance.

- $0.2$ fm $\lesssim r \lesssim 0.3$ fm: Diquark-antidiquark dominance turns into meson-meson dominance.

- $0.5$ fm $\lesssim r$: Essentially a meson-meson system.
Structure of the $\bar{b}bqq$ tetraquark (6)

- Generalized eigenvalue problem (GEVP)

$$C_{jk}(t)v^{(n)}_k(t) = \lambda^{(n)}(t)C_{jk}(t_0)v^{(n)}_k(t) \quad , \quad n = 0, \ldots, N - 1$$

for $t_0/a \geq 1$ and $t/a > t_0/a$ with corresponding effective energies

$$V^{\text{eff},(n)}(r, t) = -\frac{1}{a} \log \left( \frac{\lambda^{(n)}(t)}{\lambda^{(n)}(t - a)} \right).$$

- Eigenvector components $v^{(n)}_j(t)$ (which we always normalize according to $\sum_j |v^{(n)}_j(t)|^2 = 1$) contain information about the relative importance of the operators. For large $t$ and $t_0$,

$$|n\rangle \approx \sum_j v^{(n)}_j(t)|\Phi_j\rangle,$$

where $\approx$ denotes an approximate expansion of the energy eigenstate $|n\rangle$ in terms of the trial states $|\Phi_j\rangle$. 

Marc Wagner, “Comparing meson-meson and diquark-antidiquark creation operators for a $\bar{b}b\bar{d}d$ tetraquark”, July 28, 2021
One can show: For $t_0 = t - a$, optimizing trial states by minimizing effective energies (as on previous slides) is equivalent to solving a GEVP, i.e.

$$(w_{BB}, w_{Dd}) = (|v_{BB}^{(0)}(1+\gamma_0)\gamma_5|^2, |v_{Dd}^{(0)}(1+\gamma_0)\gamma_5|^2)$$

(might offer another perspective on GEVP eigenvector components).

→ Results for $w_{BB}$ and $w_{Dd}$ can also be interpreted as GEVP results.


• In the literature typically small values for $t_0$ are used, e.g. $t_0/a = 1$ (instead of $t_0 = t - a$ as used to obtain $w_{BB}$ and $w_{Dd}$ on previous slides).

• Similar results also for $t_0/a = 1$, when using a $2 \times 2$ correlation matrix (left plot).

• Consistent results, when using a $4 \times 4$ correlation matrix (right plot).
Structure of the $\bar{b}bqq$ tetraquark (8)

- Define the $r$ dependent $BB$ and $Dd$ percentages,

$$p_{BB}(r) = w_{BB}, \quad p_{Dd}(r) = w_{Dd}$$

and use the probability density of the $\bar{b}b$ separation

$$p_r(r) = 4\pi |R(r)|^2$$

obtained from the BO wave function $R(r)/r$, to estimate the total $BB$ and $Dd$ percentages of the $\bar{b}bud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$:

$$\%BB = \int dr \, p_r(r) p_{BB}(r), \quad \%Dd = \int dr \, p_r(r) p_{Dd}(r) = 1 - \%BB.$$ 

- We find $\%BB = 0.58$, $\%Dd = 0.42$.

- Using $|v_{BB,(1+\gamma_0)\gamma_5}^{(0)}|^2, |v_{Dd,(1+\gamma_0)\gamma_5}^{(0)}|^2$ instead of $w_{BB}, w_{Dd}$ we find $\%BB = 0.60$, $\%Dd = 0.40$.

- Results are in qualitative agreement with a GEVP result we obtained in a full lattice QCD computation, where the $\bar{b}$ quarks are treated within NRQCD.


[Talk by M. Pflaumer, today in this session at 6:45 a.m.]
Summary

- The hadronically stable $\bar{b}b\bar{d}d$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ is neither exclusively a meson-meson system nor a diquark-antidiquark pair.

- $r \lesssim 0.2$ fm: Clear diquark-antidiquark dominance.

- $r \gtrsim 0.3$ fm: Clear meson-meson dominance.

- Total $BB$ and $Dd$ percentages: $\%BB \approx 0.60$, $\%Dd \approx 0.40$. 